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A note on calibrational technique for reusable piezoelectric film strain gauges using stepped beam with variable cross section[2]

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The analytical solution, which gives the dependence between strain and displacement of beam free end at first eigen mode in cantilever, stepped beam was yielded. The dependence can be used as a basis for calibration of PVDF reusable strain gauges in practice. A value of calibrational accuracy for reusable PVDF strain gauges using conventional vibrational equipment was estimated and proofed. The characteristics of rig were obtained, reported and certified.

Получено аналитическое решение, определяющее зависимость смещения свободного конца консольной составной балки переменного сечения и деформации ее основания на первой частоте собственных колебаний. Полученная зависимость может быть использована для калибровки пьезотензодатчиков многократного применения на основе пьезополимерной пленки. Получено и подтверждено значение погрешности калибровки пьезотензодатчиков с применением стандартной аппаратуры для вибрационных измерений. Приведенные характеристики установки подтверждены метрологически.

INTRODUCTION

The strain of vibrating structure becomes an important parameter for estimation of energy flows propagated via beams such as vehicle frames, shafts, water-filled pipeline and engineering junctions. Data about dynamic strain can be also implemented when strength problem, machinery diagnostics and NDE (non-destructive evaluation) have to be resolved. Measurement of vibrational forces and moments in structures also requires data about dynamic strain. The advantage of reusable strain gauges in contrast to conventional single-used wire ones is obvious. Firstly the gauges compatible to conventional equipment for vibrational measurement, secondly they can be implemented many times and thirdly the gauges are used with known sensitivity in measurement procedure. Theirs application is resemble to conventional rule when measurement of acceleration are performed.

A calibration with given accuracy becomes essential when the transducers must be delivered to user. The customer needs the reliable data about transducer sensitivity to dynamic strain, which induced in the structure due to its oscillation. However there is no standard value for dynamic strain similar to standard of mass, length, time etc. Therefore the purpose of this work to develop the reliable method and set-up with proofed accuracy using conventional equipment available to user.

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[1]This work has been done at Krylov Shipbuilding Research Institute
[2]The article was not subjected to circumstantial lingual examination by English speaking expert. Editorial staff is not responsible for lingual mistakes in the article

1. THEORY OF STEPPED BEAM

A theory of beam with variable cross-section (stepped beam) was well developed and can be found in [1]. The shape of beam cross-section is shown in Fig 1.

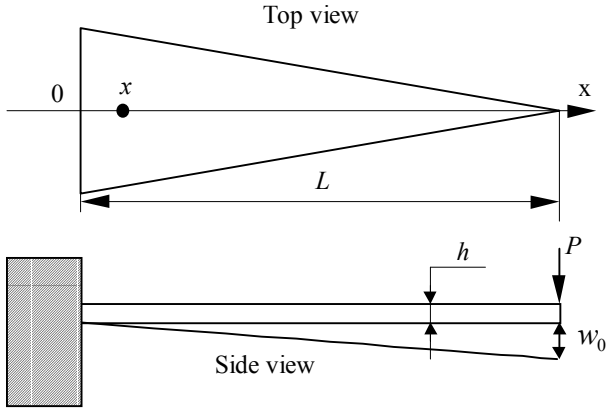

Fig.1. Ordinary beam with variable cross-section

Dynamic strain can be reproduced in the cantilever beam undergone the bending load operative at free end. There is a simple functional relationship between free end displacement, beam geometry and strain at the observation point x as [1]

$$\varepsilon(x) = const \quad \varepsilon = \frac{h}{L^2} w_0 = \frac{h}{\omega^2 L^2} \ddot{w}_0 . \tag{1}$$

It can be seen that variable cross-section provides the strain which is independent on observation point, namely, co-ordinate x. Equation (1) is valid for static loading of the beam at free end as well as for dynamic due to the same shape of static and vibrational mode. Unfortunately the beam shape shown in Fig. 1 is impractical because a measuring the displacement exactly at free end using accelerometer or other contact means is impossible. The non-contact optical technique is quite expensive. The more practical solution to beam shape that normally used in calibration technique of measurement channels for static strain by means of wire gauges is depicted in Fig. 2. The beam geometry allows placing the accelerometer at free end and making the measurements by means of conventional electronic equipment such as the combination of charge preamplifier and FFT dual channel analyzer. However, the equation for displacement, strain at point of interest, beam thickness and length is different from equation (1). Here we have to derive a corrected formula for stepped beam. Note that symbols and notation are given in Appendix A.

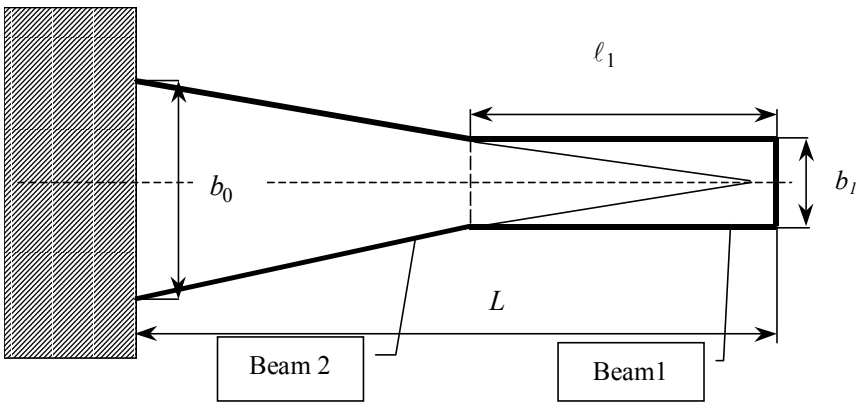

Fig. 2. The sketch of stepped beam with variable cross section

It is clear that the displacement of free end in the stepped beam will be different than the value of ordinary beam. The problem is to get functional relationship between displacement of free end w_0, strain at beam part of variable width ε_0 and geometry parameter l_1, b_1, b_0, L.

The equation of static bend for neutral axis of ordinary beam with variable cross section can be written in the form:

$$E\,I(x)w'' = P(L-x)\,. \tag{3}$$

Here the moment of inertia of beam cross section is:

$$I(x) = I_0\left(1-\frac{x}{L}\right) = \frac{b_0 h^3}{12}\left(1-\frac{x}{L}\right). \tag{4}$$

Substituting (4) to (3) and integrate the equation using appropriate boundary conditions for a clamped end

$$w'\big|_{x=0} = w\big|_{x=0} = 0$$

we will obtain the equation of beam displacement w as a function of current co-ordinate x

$$w = \frac{P\,L^3}{2E\,I_0}\left(\frac{x}{L}\right)^2, \tag{5}$$

and rotational angle will be

$$\varphi = \frac{\partial w}{\partial x} = \frac{P\,L^2}{E\,I_0}\left(\frac{x}{L}\right). \tag{6}$$

Let determine the rotational angle when $x = L - l_1$:

$$w\big|_{x=L-l_1} = \frac{P\,L^3}{2E\,I_0}\left(\frac{x}{L}\right)^2 = \frac{P\,L^3}{2E\,I_0}\left(1-\frac{l_1}{L}\right)^2, \tag{7}$$

$$w'\big|_{x=L-l_1} = \frac{P\,L^2}{E\,I_0}\left(\frac{x}{L}\right) = \frac{2P\,L^3}{2E\,I_0}\left(1-\frac{l_1}{L}\right)^2. \tag{8}$$

The displacement of the free end for ordinary beam at the point $x = L$ will be expressed as:

$$w_0 = \frac{P\,L^3}{2E\,I_0}; \quad P = \frac{2E\,I_0}{L^3}w_0\,. \tag{9}$$

The displacement of the free end for stepped beam will comprises free values.

- Displacement of beam 1 with constant cross section:

$$w_1 = \frac{P\,l_1^3}{3E\,I_1}\,. \tag{10}$$

- Displacement of the beam 2 with variable cross section:

$$w_2 = w_0\left(1-\frac{l_1}{L}\right)^2. \tag{11}$$

- Displacement due to rotational angle of the beam 2:

$$w_3 = \varphi\, l_1 = l_1 \frac{2w_0}{L}\left(1 - \frac{l_1}{L}\right). \tag{12}$$

Finally we shall have for free end:

$$w\big|_{x=L} = w_1 + w_2 + w_3 = \frac{2I_0 l_1^3}{3I_1 L^3} w_0 + w_0\left(1 - \frac{l_1}{L}\right)^2 + 2\frac{l_1}{L} w_0\left(1 - \frac{l_1}{L}\right). \tag{13}$$

After simplifying and using denotation $l_1/L = a$ we shall have:

$$w\big|_{x=L} = \left(\frac{2}{3}(a)^3 \frac{b_0}{b_1} + 1 - a^2\right) w_0 . \tag{14}$$

Substituting the expression (1) to formula for the strain we obtain

$$\varepsilon = \frac{h}{L^2} w_0 \left(\frac{2}{3} a^3 \frac{b_0}{b_1} + 1 - a^2\right). \tag{15}$$

Equation (15) can be served as a start point for calibration of reusable PVDF strain gauges.

2. EXPERIMENTAL RESULTS AND ACCURACY ESTIMATION

The point where calibrated strain gauge places should be positioned close to the beam clamp in order to avoid the effect of vertical acceleration which could impact the strain gauge calibration accuracy due to non-zero gauge sensitivity to vertical acceleration. This arrangement is shown in Fig. 3.

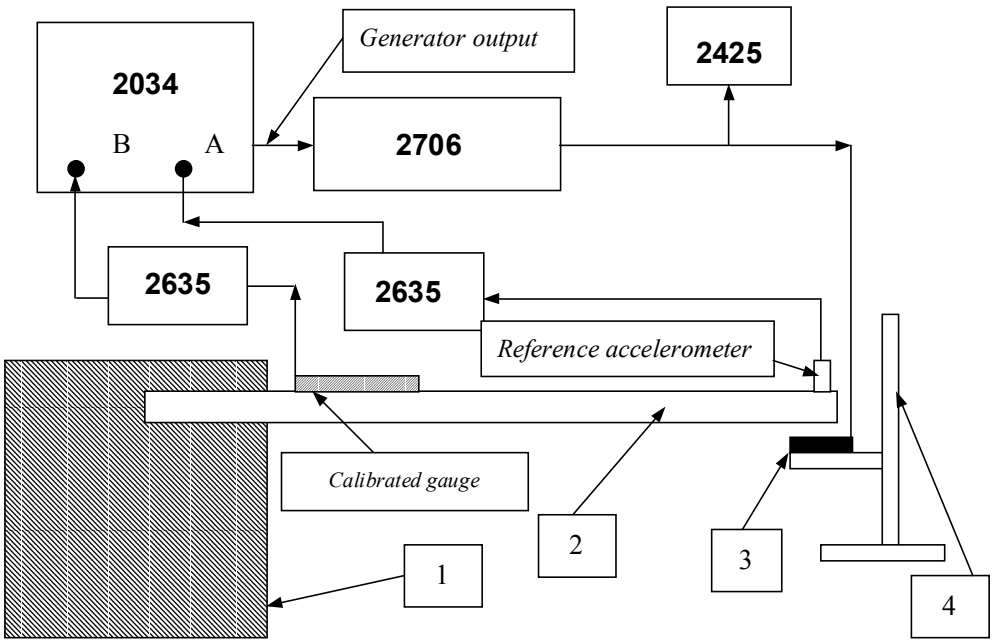

Fig. 3. Experimental set-up with stepped cantilever beam
1 – clamp affixed to heavy foundation, 2 – stepped beam,
3 – non-contact electromagnetic exciter, 4 – rack

2.1. DETERMINATION OF THE BEAM PARAMETERS

The practical realization of mechanical part of the calibrational rig with stepped beam is depicted in Fig. 4.

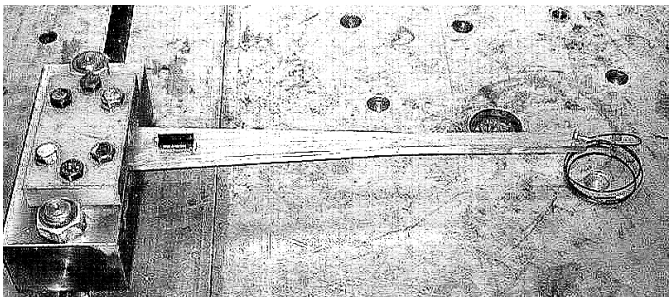

Fig. 4. Mechanical part of calibrational rig with stepped beam

There are two main reasons, which can impact the calibrational accuracy:

- Mode shape at first eigen frequency could be distorted due to weak clamp;
- The beam loading by excitational fixture and relatively heavy accelerometer would change the behavior of beam which considered in theoretical model.

Two measures were undertaken to alleviate above problems.

The acceleration at beam clamp should be about of −80 dB than at free end one. The 80 dB is the dynamic range of FFT analyzer. Therefore the beam clamp was bolted to the rig with high mechanical impedance.

In order to reduce the mass loading effect by excitational system the magnet part of headphones was implemented as non-contact electromagnetic shaker for such purpose. A miniature "B&K" accelerometer model 8307 (0,4 grams) was utilized for measuring the acceleration (displacement) at beam free end.

The first eigen mode was measured using transfer mechanical mobility of the beam involving impact excitation technique. The plot of the mechanical mobility and mode shape is presented in Fig. 5 and 6, respectively.

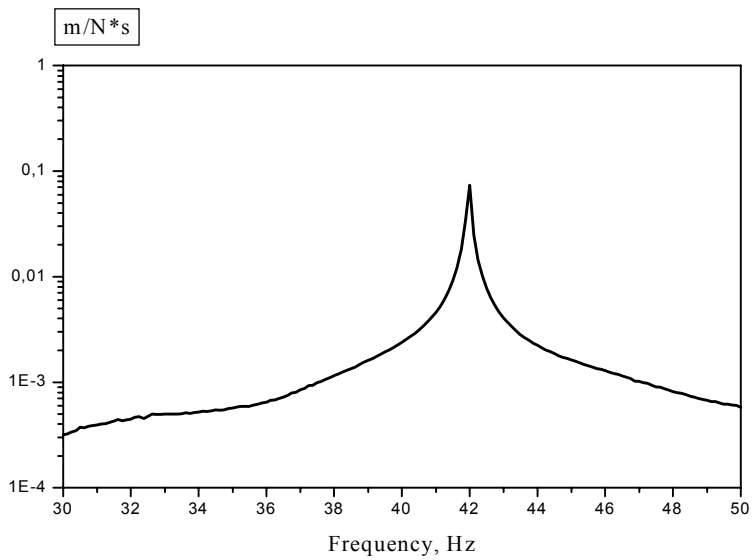

Fig. 5. Transfer mechanical mobility of stepped beam (magnitude)

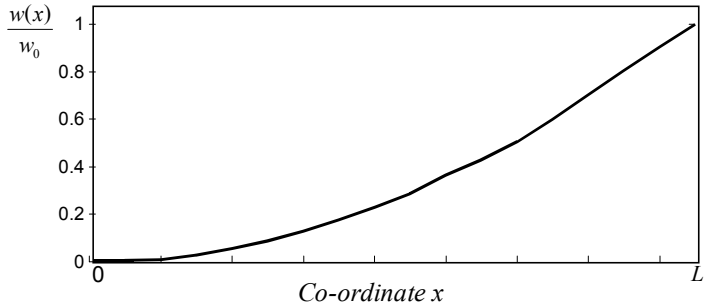

Fig. 6. The mode shape of the beam

It has been found that acceleration at the clamped end is −80 dB less than at free one.

2.2. ACCURACY ESTIMATION AND RIG PARAMETERS

Real sizes of the beam are shown in Fig. 7.

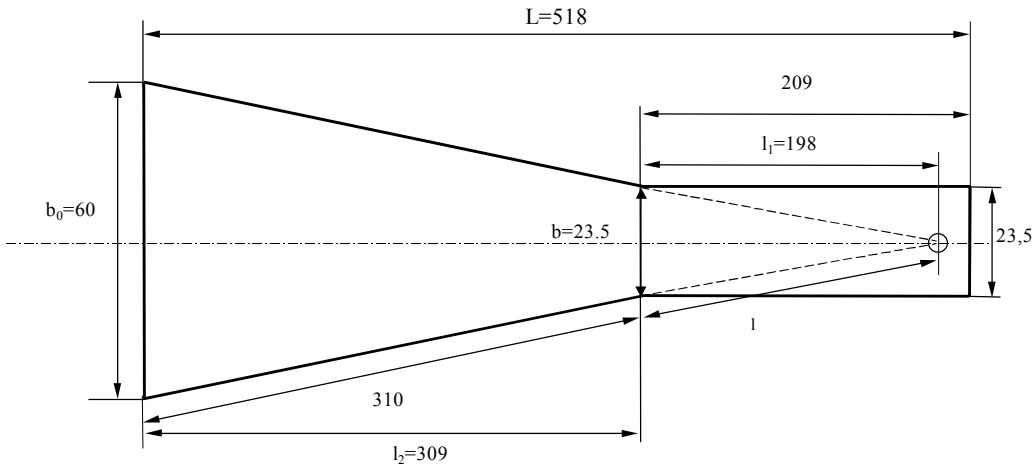

Fig. 7. Sizes of the stepped beam in mm

Substituting the sizes to the beam equation we have obtained

$$\varepsilon = \frac{h}{L^2} w_0 \left(\frac{2}{3} \left(\frac{l_1}{L_1} \right)^3 \frac{b_0}{b_1} + 1 - \left(\frac{l_1}{L_1} \right)^2 \right) = \frac{h}{L^2} w_0 \left(\frac{2}{3} \left(\frac{198}{507} \right)^3 \frac{60}{23{,}5} + 1 - \left(\frac{198}{507} \right)^2 \right) = 0{,}945 \frac{h}{L^2} w_0 \ . \tag{16}$$

The total error of strain reproduction can be calculated as

$$\delta_\Sigma = \pm K \cdot \sqrt{\delta_h^2 + \delta_{2034}^2 + \delta_{ny}^2 + \delta_a^2 + \delta_o^2 + 4 \cdot (\delta_L^2 + \delta_{f_0}^2)} = 1{,}1\sqrt{0{,}05^2 + 0{,}1^2 + 0{,}1^2 + 3^2 + 0{,}5^2 + 4 \cdot (0{,}1^2 + 0{,}15^2)} = 3{,}6\,\%, \tag{17}$$

where δ_h — measurement error of the beam thickness (0,05%), δ_{2034} — measurement error of dual channel analyzer (0,1%), δ_{2035} — measurement error of charge preamplifier (0,1%), δ_a — error due to analyzer frequency response ripple (3% or 0,3 dB), δ_{accel} — accuracy of accelerometer

calibration (0,5%), δ_L – measurement error of the beam length (0,1%), δ_f – measurement error of the first eigen frequency (0,15%) and K assumed to be equal 1,1. The K – here is the quotient of Gauss probability distribution for confidence level 95%. Data about accuracy of "B&K" electronic equipment can be found in [2].

The following parameters have been obtain during certification:

- Range of dynamic strain at beam first eigen mode: from 8,9 to 0,089 μm/m (10^{-6} m/m).
- Limits of relative error for strain reproduction in specified strain range ± 3,6% at confidence level 0,95.
- Beam first eigen frequency: 41,875 Hz.

CONCLUSIONS

The basic equation for calibrational technique was derived using rigorous solution. The calibrational rig was manufactured and metrologically certified according to requirements of Russian Standard Committee (Gosstandart). The rig can be used for calibration of the standard PVDF strain gauges, which serve to transferring a measurement unit from one calibrational set-up to another. The technical and accuracy parameters are proofed.

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APPENDIX A. SYMBOL AND NOTATIONS

E – Young modulus of beam material (Pa)

$I(x)$ – moment of inertia of beam cross section as a function of coordinate (m^4)

L – beam length (m)

b_0 – beam width at the clamped end (m)

h – beam thickness (m);

$I_0 = \frac{b_0 h^3}{12}$ – moment of inertia of beam variable cross section at $x = b_0$ (m^4)

$I_1 = \frac{b_1 h^3}{12}$ – moment of inertia of beam constant cross section (m^4)

w – beam displacement (m)

$\dot{w}$ – time derivative (m/s)

w' – co-ordinate derivative (dimensionless)

ε – strain of beam at the part with variable cross section (μm/m)

P – force operative at the beam free end (N)

ω – angular frequency (s^{-1})

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