Electronic Journal «Technical acoustics»
http://webcenter.ru/~eeaa/ejta

1 (2001) 3.1–3.16

A. G. TROSHIN

"Izmeron Ltd" Tool Plant, Laboratory of linear measurements and electronic devices
Novogorodskaya street 13, Saint-Petersburg, 193144, Russia, e-mail: eeaa@online.ru

Uncertainty in structural intensity measurement (review)*

received 10.06.2001, published 30.06.2001

The purpose of the paper is to point out the main sources of errors for widespread experimental methods used in estimation of vibrational energy flows. This paper comprises the list of the main errors and ways of errors estimation using real data obtained during measurements. The errors analysis was carried out to show a methods sensitivity to various sources of errors. The main sources of uncertainty for the most popular and state-of-art techniques namely Direct Method, Strain Gauge Method, Complex Transmissibility Technique, Mobility Energy Flow and Finite Difference Technique are emphasized and reviewed. The practical examples of errors estimation with knowledge about structure materials, real experimental data and channels features are shown in the paper. The analysis can help researcher in determining the confidence level of measured data. The practical formulae and measurement set-ups are presented.

Целью данной статьи является анализ основных источников погрешностей широко распространенных экспериментальных методов, используемых для оценки потоков колебательной энергии. В статье дан перечень погрешностей и способы их оценки с использованием данных, полученных в процессе эксперимента. Проведен анализ погрешностей, чтобы показать чувствительность методов к различным источникам ошибок. Сделан обзор основных источников неопределенности для наиболее распространенных методов, которые внедрены в практику измерений, а именно, непосредственного метода, метода с применением датчиков деформации, метода комплексного перепада, метода податливости и метода конечных разностей. Приведены практические примеры оценки погрешностей с использованием данных о свойствах материалов, конструкций, характеристиках каналов и экспериментальных результатов. Анализ погрешностей может помочь исследователю при оценке доверительных интервалов данных измерений. Приведены формулы и описаны измерительные тракты для практического применения.

*This paper had no linguistic support from English speaking expert. Editorial staff is not responsible for lingual mistakes in the article

PREFACE

The experimental structural intensity which used last 40 years is quite modern method that deals with measurement of vibrational energy flows. The advantage of experimental structural intensity is ability to reveal the noise sources and quantification of energy transfer paths. However the actual question of all experimental research works concerns the measurement accuracy. This topic causes sometimes a negative reaction of experts who deal with pure empirical approaches and they are really believing that measurement is "absolutely accurate". Most of them are saying that accuracy prediction can not be carried out due to lack of the standard value for energy flow.

Another reaction comes from pure theoretical experts who can not imagine that measurement error in structural intensity or acoustic measurements could be more than 100%. Sometimes the discrepancies between calculated and measured data are considered as a "fault of measurer" or disagreement caused by «lousy experiment» but mathematical model is perfect. We will not open something new if reminds that acoustics, structural acoustics and experimental dynamics have the objectives of an extremely complicated nature. The final result depends on statistic properties of random fields and signal time history, multidirectional oscillations in wide frequency and dynamic ranges including interaction between media and structures. For example, interaction between moving liquid in pipes and pipe walls, sound fields and barriers, partitions, flanking transmission etc. Some authors call the vehicle and ship acoustics as a "black art". All aspects of vibration fields are "terra incognita" for each individual case. The errors are functionally dependent on measured parameters such as coherence function, phase angles, number of samples (observation time) as well as real multidirectional vibration field. Therefore in most cases the total error of measurements can not be predicted "a-priori" despite the high quality measurement equipment and transducers are available to researcher. The purpose of the paper to make survey of probable source of errors and uncertainties in order to help researcher in answering the questions "why there is no and will not be" a good agreement between measured and calculated results. Measurements without any information about error value or at least its limits make user blind. Therefore another purposes of the paper to make help user how to perform the error estimation for individual cases by use of different technique.

UNCERTAINTY DEFINITION AND PRESENTATION OF RESULTS

The measurement result can be considered as a «very good» if total error does not exceed 1–3 dB over all frequency and dynamic ranges. The error within 3–6 dB can be regarded as a «good». We could say that error (or uncertainty) about of 10 dB or more makes situation undetermined. These values are relative and subjective of course. All of them referred somehow to human hearing but unfortunately an uncertainty relation for experimental structural acoustics was not formulated yet. It is matter of the future and I hope that smart minds will be found soon.

It would be expedient to represent the experimental data showing the confidence level. Consider two basic measurement channels 1 for force and 2 for acceleration. The confidence level 95% is useful for most cases unless it is associated to human health and security. Latter referred to 99% of confidence. The level of structural energy with confidence 2σ (95% for Gaussian probability distribution) can be indicated according to equation

$$L_{2\varepsilon_{tot}} = 10 \cdot \log_{10}\left(\frac{\overline{W} \pm 2 \cdot \Delta_{tot}}{W_0}\right). \qquad (1)$$

Here $\overline{W}$ is a value of averaged energy. There are five potential source of errors. The total measurement error of energy flow can be written as

$$\Delta_{tot} = \sqrt{\Theta_{rand}^2 + \Theta_{bias}^2} \ . \tag{2}$$

Here Θ_{rand} – is random error of measurement and Θ_{bias} is the total bias error that can be written as

$$\Theta_{bias} = \pm 1{,}1 \cdot \sqrt{\Theta_{cal1}^2 + \Theta_{cal2}^2 + \Theta_{FRF1}^2 + \Theta_{FRF2}^2 + \Theta_{trans1}^2 + \Theta_{trans2}^2 + \Theta_{\varphi}^2} \ , \tag{3}$$

where

Θ_{cal} – calibrational error of transducers;

Θ_{FRF} – error due to frequency response ripple of measurement chain;

Θ_{trans} – error due to transverse sensitivity of transducers;

Θ_{φ} – error due to channels phase mismatch.

All the formulae given above are referred to Russian standards. The random and phase mismatch errors depend on measured phase, coherence function and phase mismatch between channels. It means that those values must be acquired during measurement session and are represented as a function of frequency. Data about them will be used for error estimation further on. The most difficult task is acquiring data about phase mismatch between channels including preamplifiers and transducers. The components of errors listed above have to be considered as main ones and should be taken into account during implementation of all methods.

DIRECT METHODS USING FORCE GAUGE AND ACCELEROMETER

This technique can be used in specific cases when structure excited by attached shaker and oscillation is more or less unidirectional. It should be noted that force gauge has sensitivity to moment and base strain. The latest phenomena do not allow to perform a correct measurement using measuring flanges and platforms where force transducers incorporated into metal structure. However, the correct energy input can be measured not only at single point but for the case of structure excitation by shakers attached to number of points. It was named multiple point excitation or excitation with the system of forces. The equation for the input energy flow is quite simple:

$$W_i(f) = \mathrm{Re}\left[\frac{G(F_i(f) \cdot a_i(f))}{j\omega}\right] = -\frac{\mathrm{Im}\, G(F_i(f) \cdot a_i(f))}{\omega} \ , \tag{4}$$

if structure undergone the rectilinear force,

$$W_i(f) = \mathrm{Re}\left[\frac{G(M_i(f) \cdot \ddot{\theta}_i(f))}{j\omega}\right] = -\frac{\mathrm{Im}\, G(M_i(f) \cdot \ddot{\theta}_i(f))}{\omega} \ , \tag{5}$$

if structure undergone the moment. The list of symbols and notations is given at the end of the paper.

The possible measurement set-up is shown in Fig. 1. The details how to avoid stinger resonance and how to affix shaker can be found in [1].

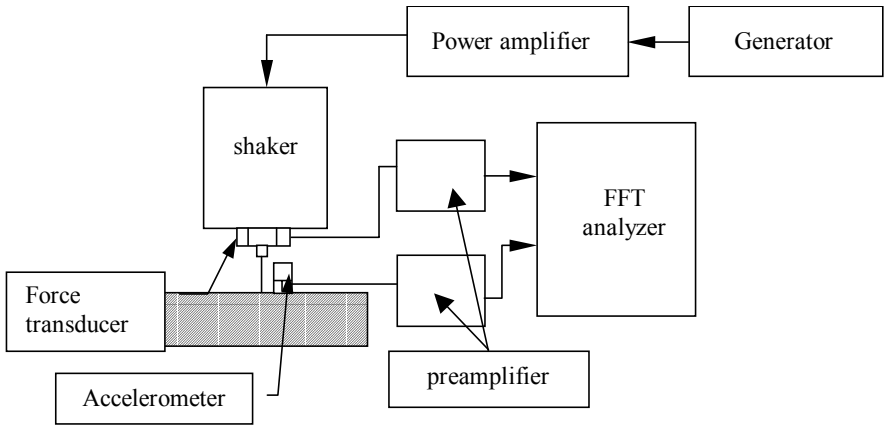

Fig. 1. Set-up for direct method using force transducer and accelerometer

Note here that an impedance head can not be recommended for slender beams, plates and lightly damped structure due to mass loading effect. This circumstance lead to a negative sign of energy injected into the structure or the same one in real part of mobility. The following errors inherent the method:

Error due to phase and magnitude mismatch between channels is equal to

$$\Theta_{bias}(\varphi, f) = \frac{1}{(1+\Theta_F(f))(1+\Theta_a(f))}\left(\varphi^*(f)\cot(\varphi_{Fa}(f))+1\right). \qquad (6)$$

Here Θ_F and Θ_a are the total bias error of force channel and the total bias error of acceleration channel, respectively.

This error dominates when phase angle between force (moment) and linear (angular) acceleration become of the same order as channel phase mismatch. It means that the structure is lightly damped. The plot for this error is shown in Fig. 2.

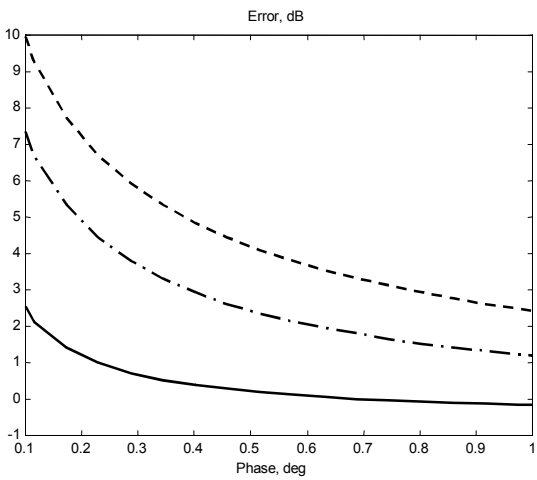

Fig. 2. Bias error due to phase and magnitude mismatch vs. wave phase, Θ_1=20%, Θ_2=5%
(—) – φ^* = 0,1 *degree*, (-·-·-) – φ^* = 0,5 *degree*, *(- - -)* – φ^* = *1 degree*

The error can not be compensated because the phase φ^* depends on frequency. Any mechanical set-up can not provide getting pure phase in wide frequency range due to transverse motion of calibration table and transducer transverse sensitivities as well as base strain. The phase mismatch comprises pure electronic phase mismatch and mechanical one. Hence calibration should be performed for total chain — transducer plus charge preamplifier and channel of FFT analyzer. For example, the phase mismatch between two delta-shear accelerometers 4370 "B&K" including charge preamplifiers 2635 and FFT analyzer model 2032 is shown in Fig. 3. Accelerometers were placed and removed ten times from calibrational table model 4815 equipped with reference one (model 8305). Phase characteristics were measured with respect to reference accelerometer. Experiment explains that error caused by phase (or magnitude) mismatch can not be compensated because of the random nature of accelerometers affixing to measurement object. It means that phase mismatch error of measured structural intensity has to be classified as random one.

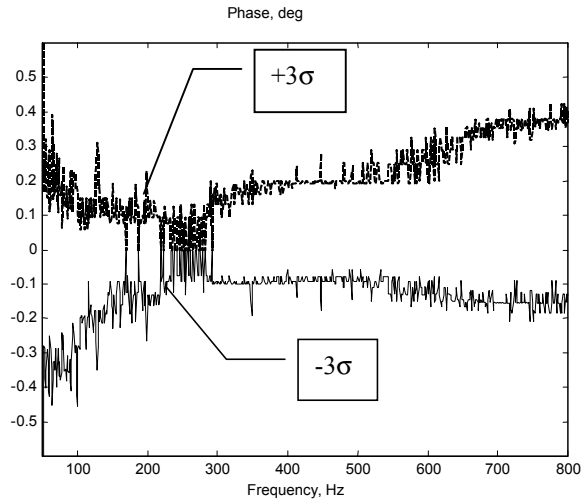

Fig 3. Typical phase mismatch between accelerometers
(—) – *phase expectation N = 10,* (- - - - -) – *upper and lower limits with confidence 95%* (3σ)

<u>Random errors due to low coherence function and small measured phase angle</u> is equal to

$$\Theta_{rand}(\gamma^2,\varphi,f)=\sqrt{\frac{1}{n_d}\left(\frac{1}{\gamma_{Fa}{}^2(f)}+\frac{1-\gamma_{Fa}^2(f)}{2\cdot\gamma_{Fa}^2(f)}\cot^2(\varphi_{Fa}(f))\right)}. \tag{7}$$

The error is common one and occurs in all measurement methods based on imaginary part of cross spectra between force and acceleration (moment and angular acceleration) or acceleration pairs. An equation (7) was derived by [2] for two microphone measurement techniques. A three dimensional (3D) plot of random error is shown in Fig. 4. It is clear that error of measured imaginary part for cross spectrum more sensitive to small phase angle rather than a coherence when both parameters come close to zero.

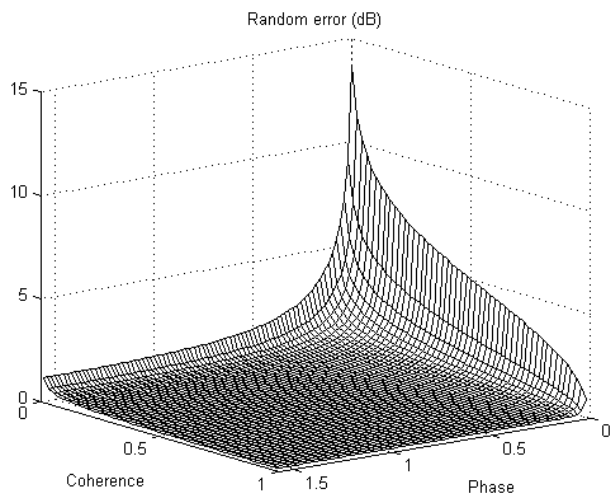

Fig. 4. Random error of direct method (number of samples 1000)

Error due to calibrational accuracy of accelerometer and force transducer

Typically the error of accelerometer calibration and force transducer is no more than 5%. The frequency response ripple of FFT analyzer Θ_{FRF} is usually about of 0,3 dB (3%) or less. This value is given for FFT analyzer 2034/32. The errors due to transverse sensitivity are quite different for accelerometer and force transducers. It is possible to find high quality accelerometers with transverse sensitivity 2% and even 0,5% but force transducers still have this value about of 20 % or higher. We would say that transverse sensitivity error is most questionable one. It is very difficult to predict the error without any data about real vibrational field "in advance". The feature of transverse sensitivity vs. frequency is also difficult to acquire. The total error can be estimated using equations from (1) to (3).

COMPLEX TRANSMISSIBILITY TECHNIQUE (CTT)

Technique for estimation of energy flows propagated via machinery supports and pipe structure and hangers was well developed by J. Verheij [3], R. Pinnington, R. White [4], A. Moorhouse and B. Gibbs [12].

The method used the data about blocked impedance matrix of isolator and complex valued transmissibility of acceleration measured by accelerometers placed upstream and downstream of isolator. The forces and moments acting via isolator can be estimated using Hook's law. The force (moment) can be multiplied with appropriate linear (angular) velocity. The relationships between isolators inputs and outputs can be written in the conventional form for harmonic oscillation as:

$$\begin{cases} F_1 = Z_{11}^F \cdot v_1 - Z_{12}^F \cdot v_2 \\ F_2 = -Z_{21}^F \cdot v_1 + Z_{22}^F v_2 \\ M_1 = Z_{11}^M \cdot \dot{\theta}_1 - Z_{12}^M \cdot \dot{\theta}_2 \\ M_2 = -Z_{21}^M \cdot \dot{\theta}_1 + Z_{22}^M \cdot \dot{\theta}_2 \end{cases} \qquad (8)-(12)$$

Those equations in energy terms can be represented for vertical direction Z in the form as

$$W_{input}(f) = G^z_{a1\,a1}(f)\cdot \mathrm{Re}\,Z^z_{11}(f) - \mathrm{Re}\,G^z_{a1\,a2}(f)\cdot \mathrm{Re}\,Z^z_{a1\,a2}(f) - \mathrm{Im}\,G^z_{a1\,a2}(f)\cdot \mathrm{Im}\,Z^z_{a1\,a2}(f)),$$
$$W_{transf}(f) = G^z_{a2\,a2}(f)\cdot \mathrm{Re}\,Z^z_{22}(f) - \mathrm{Re}\,G^z_{a2\,a1}(f)\cdot \mathrm{Re}\,Z^z_{a2\,a1}(f) - \mathrm{Im}\,G^z_{a2\,a1}(f)\cdot \mathrm{Im}\,Z^z_{a2\,a1}(f)). \quad (13)-(14)$$

More detail derivation of eq. (13) and (14) were reported in [12]. Transducers arrangement is shown in Fig. 5.

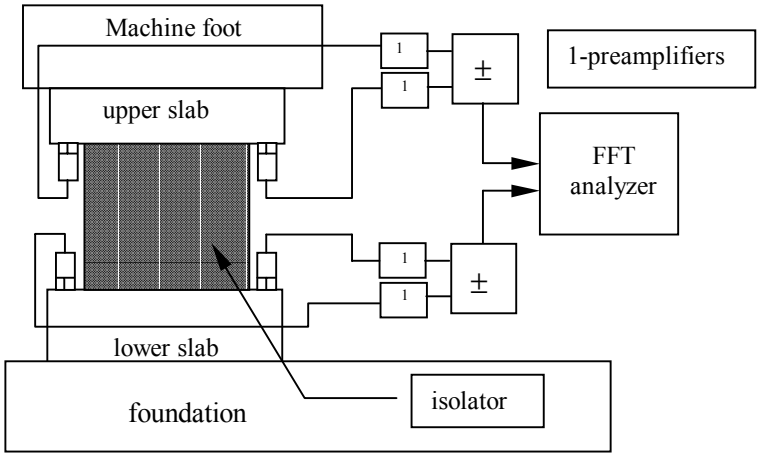

Fig. 5. Accelerometers arrangement upstream and downstream of isolator

The equations similar to (13) and (14) can be rearranged for all degrees-of-freedom linear and rotational taking into account data about appropriate isolator blocked impedances. The uncertainty of technique depends on measurement accuracy of isolator blocked impedance values. Inaccuracy in blocked impedance value will lead to proportional error in power estimation so that the error of blocked impedance determination about of 25% will cause the equal error for power. The usage of accelerometers pairs to select translational and rotational component simultaneously is preferable for elimination the extra phase mismatches error caused by accelerometers offset from mount centerline at high frequencies. The random error of autospectra, real and imaginary parts of cross spectra between accelerations can be expressed by formulae:

$$\Theta_{rand1} = \Theta_{rand}(G_{aa}) = \frac{1}{\sqrt{n_d}}, \quad (15)$$

$$\Theta_{rand2} = \Theta_{rand}(\mathrm{Im}\,G_{a1\,a2}) = \sqrt{\frac{1}{n_d}\left(\frac{1}{\gamma_{a1\,a2}{}^2(f)} + \frac{1-\gamma^2_{a1\,a2v}(f)}{2\gamma^2_{a1\,a2v}(f)}\cot^2(\varphi_{a1\,a2}(f))\right)}, \quad (16)$$

$$\Theta_{rand3} = \Theta_{rand}(\mathrm{Re}\,G_{a1\,a2}) = \sqrt{\frac{1}{n_d}\left(\frac{1}{\gamma_{a1\,a2v}{}^2(f)} + \frac{1-\gamma^2_{a1\,a2v}(f)}{2\gamma^2_{a1\,a2v}(f)}\tan^2(\varphi_{a1\,a2v}(f))\right)} \quad (17)$$

and total random error is

$$\Theta^{total}_{rand} = \sqrt{\Theta^2_{rand1} + \Theta^2_{rand2} + \Theta^2_{rand3}}. \quad (18)$$

The error due to phase mismatch between transducers can be found in [12] and equation has been derived by A. Moorhouse and B. Gibbs:

$$\Theta_{bias}(\varphi,\varphi^*,\eta)=1-\frac{\sin(\varphi+\varphi^*)-\eta\cos(\varphi+\varphi^*)}{\sin\varphi-\eta\cos\varphi}\approx 1-\frac{\varphi^*}{R-\eta-\varphi^*}. \quad (19)$$

If reactive field is strong R=0,01 the bias error could be about of 10 dB. When reactivity index more than 0,1 error become smaller than 1 dB.

The error seems to be a function of isolator loss factor, phase mismatch and reactivity index. The ratio R introduced by authors of [12] and resembles to reactivity index for sound intensity implies the quotient $R=\mathrm{Im}\,G_{12}/\mathrm{Re}\,G_{12}=\sin(\varphi+\varphi^*)/\cos(\varphi+\varphi^*)$. Here the imaginary part is proportional to potential energy (elastic forces) and real part to active energy (dissipative forces).

Another error cuased by seating response and can be expressed as [12]

$$\Theta_{bias}^{resp}(\sigma,\eta,\varphi)=\frac{\eta\cdot\sigma}{\sin(\varphi-\eta)+\eta\cdot\sigma}. \quad (20)$$

Here $\sigma=G_{22}(f)/|G_{12}(f)|$ implies the coherent transmissibility. It can be seen that if the σ at least 5 dB the error will be less than 1 dB except of cases when reactivity is high (R < 0.01).

MOBILITY ENERGY FLOW (MEF)

The method needs a knowledge about velocities at machine's feet measured when machine is freely suspended, calculated or measured data about mobility matrix of the foundation, and mobility matrix of freely suspended machine. The method comprises numerous limitations.

A heavy machine can not be freely suspended. Although automated system used a measurement of entire mobility matrix takes a lot of time. The errors of mobility measurement lead to ill-conditioned mobility matrix and therefore the final result will be uncertain due to inversion of experimentally determined mobility matrix. Expected error can not be predicted in analytical form. Such method can be recommended for limited number of degree-of freedom and connection points. This method was verified by its comparison with complex transmissibility technique using small-sized fan. Details can be found in [10]. A good agreement for tonal component where discrepancies between two methods are within 3–6 dB in energy terms was shown [10]. The equations for MEF method are following:

$$W_{input}=|v_0|^2\,\mathrm{Re}\left([A]-[A]^{-1}\cdot\left[[A]+j\omega[K]^{-1}+[B]\right]^{-1}\cdot[A]\right),$$
$$W_{trans}=|v_0|^2\left|\left[[A]+j\omega[K]^{-1}+[B]\right]^{-1}\right|^2\mathrm{Re}([B]). \quad (21)-(22)$$

If machine is rigidly connected the term $j\omega[K]$ will be equal zero. The MEF is considered to be impractical in the case when machine connected to numerous contact points and where diversity of degree-of-freedom has to be taken into account. However MEF is useful in computer modeling of structure-borne sound propagating if database about machinery and isolator mobility matrices, isolator stiffness values are at hand.

FINITE DIFFERENCE TECHNIQUE (FD)

Methods for measurement of energy flow in structure of pipe wall were proposed by J. Verheij [3] and G. Pavic [7] and they are based on conventional transducers array (accelerometers) and finite difference technique. This is most erroneous technique especially for longitudinal and bending component. Some authors try to simplify problem and use two or three point technique for measurement of bending component. They also ignore other components

(longitudinal and torsional) making excuses that at low frequencies the bending waves are dominant. In real practice (e.g. for ship pipeline) above statement is groundless because ship shafts and pipelines are long enough and total oscillatory behavior will be determined by longitudinal and torsional eigen frequencies as well. In addition the real structures are excited by machine in all directions that is why regarding the bending waves only makes situation uncertain. The example for water filled pipeline is shown in Fig. 6.

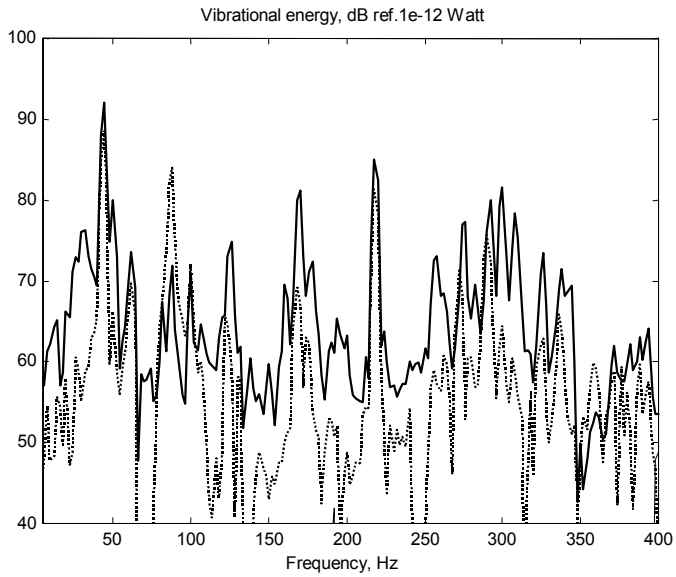

Fig. 6. Relationship between component of structural energy (measured by reusable strain gauges)
(—) - *longitudinal,* (·····) – *bending*

Another simplification for two or three point technique uses the far-field approximation [9]. Corresponding error can be estimated by equation (23) and dependence vs. dimensionless wave distance D and reflection quotient R is depicted in Fig. 7.

$$\left|\Theta_{near\ field}\right| = \frac{e^{-k_B x}\cdot\left[\sin k_B x - \cos k_B x\right]}{1-R} = \frac{e^{-2\pi D}\cdot\left[\sin 2\pi D - \cos 2\pi D\right]}{1-R}, \tag{23}$$

where $k_B = \dfrac{2\pi}{\lambda_B}$ and $D = \dfrac{x}{\lambda_B}$.

In fact this bias error is not compensated because correction factor depends on frequency, distance and reflection coefficient in a complicated manner. The lack of data about reflection quotient makes also additional uncertainty. The transducer array can not be placed more than 0,1 of bending wavelength far from pipe hangers, valves and shaft journal bearings in most cases because there is no appropriate distance on the curved pipelines or if shaft bearing are closely spaced.

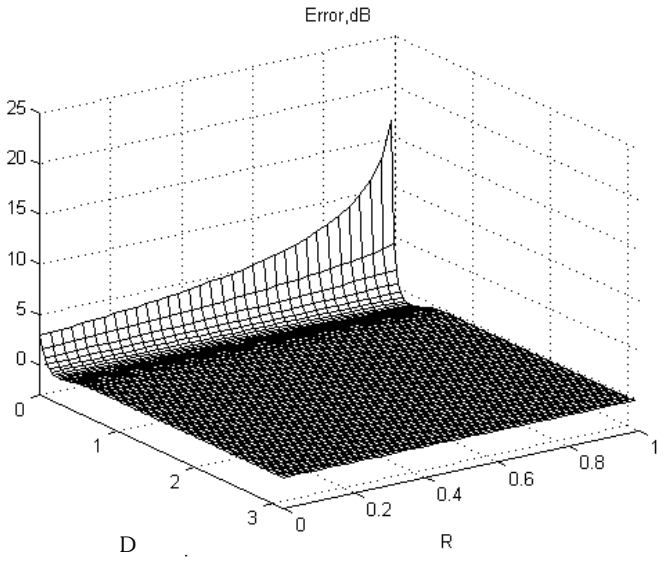

Fig. 7. Bias error due to far-field approximation vs. reflection coefficient *R* and dimensionless wave distance *D*

The longitudinal component can be measured in narrow frequency band only due to high speed of the longitudinal waves. The measurement in low frequency range requires a long measurement distance which should provide a sufficient phase angle between transducers and is not feasible. This requires the long straight parts of pipelines and shaft that sometimes can not be found in reality. The FD technique using conventional accelerometer array can be successfully implemented for measurement of torsional component of energy flow propagated along shaft and pipelines. Its frequency limitations substantially are not so strict in comparison with technique for longitudinal component. The speed of torsional waves is lower than longitudinal one. Low and upper frequency limits can be calculated according to formulae:

$$f_{long}^{low} = \frac{3\varphi^*}{2\pi\,\Delta_l}\sqrt{\frac{E}{\rho}}, \quad f_{long}^{high} = \frac{0{,}25}{\Delta_l}\sqrt{\frac{E}{\rho}}, \qquad (24)-(25)$$

$$f_{tors}^{low} = \frac{3\varphi^*}{2\pi\,\Delta_t}\sqrt{\frac{E}{2(1+\nu)\rho}}, \quad f_{tors}^{high} = \frac{0{,}25}{\Delta_t}\sqrt{\frac{E}{2(1+\nu)\rho}}. \qquad (26)-(27)$$

The calculation results for various measurement distances and steel structure according to (24)–(27) are shown in Fig. 8 and Fig. 9.

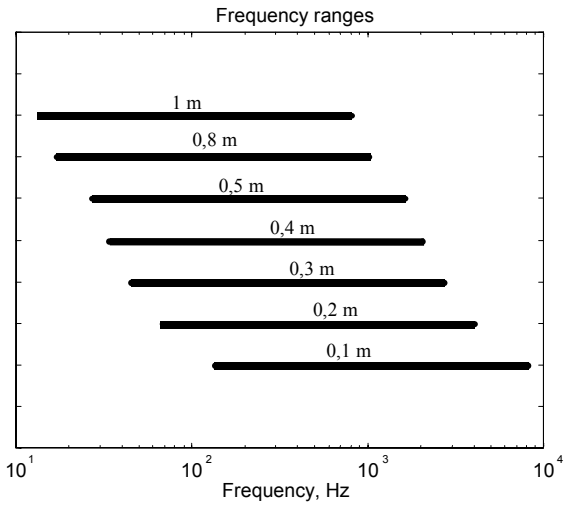

Fig. 8. Frequency ranges with tolerance 1 dB for torsional component by FD technique
Phase mismatch is 1 degree, measurement distance is varying

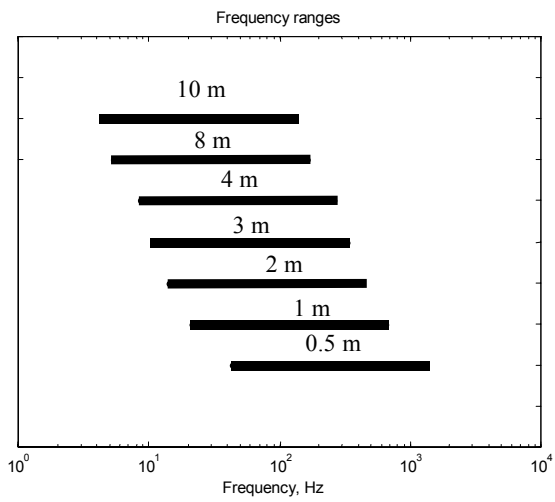

Fig. 9. Frequency ranges with tolerance 1 dB for longitudinal component by FD technique
Phase mismatch is 1 degree, measurement distance is varying

Figure 9 demonstrates the impossibility of measurement of the longitudinal component by conventional FD technique using accelerometer arrays with short measurement distance. This fact limits the implementation of FD technique for longitudinal components.

TECHNIQUE BASED ON REUSABLE PIEZOELECTRIC FILM STRAIN GAUGES

Developing a new generation of electromechanical transducer – reusable PVDF (polyvinyl idenfluoride) strain gauges reveals a number of advantages for measuring vibrational energy flows, vibration induced strains, forces and moments in pipe and shaft structures by measuring surface strain. PVDF strain gages produce electrical charge proportional to dynamic strain and therefore it is possible to use them jointly with traditional measurement equipment namely charge preamplifiers and FFT analyzer which channels equipped according to ICP standard. The PVDF strain gauges make possible measurement of extremely small magnitudes of strain about 10^{-5} με (μm/m according to denotation accepted by Russian standard). In fact it can be explained that PVDF film is more elastic than wire and therefore can produce more powerful signal. Despite the lower values of piezoelectric modulii comparing to ceramic and quartz the cross coupling mechanic to electric

efficiency of PVDF film is higher than traditional peizoelectrically active material. This phenomenon makes film very useful in design of transducers, actuators and other practical applications.

The reusable PVDF strain gauges allow to measure strain and force directly which is make possible to reduce the order of derivative approximated by FD and give an opportunity to alleviate drawbacks of parametric methods with conventional wire strain gauges which become poor for measurement of small dynamic (vibrational) strain. Advantage of reusable PVDF strain gauges is their high sensitivity to strain and possibility to calibrate and use them having at hand sensitivity vs. frequency as for usual vibrational transducer. The formulae for measurement of individual components in total energy flow become simpler in comparison with traditional FD approach. This circumstance gives an additional advantage for overcoming the specific error inherent the FD for example narrow frequency range and long measurement distance. All technique discussed earlier (see for example [8] and [9]) were based on single used PVDF gauges and they had implemented in so-called "academic structures" such as free-free beams and beams with anechoic ends.

Equations for determination of energy flows in terms of cross spectra are given below:

For longitudinal force component

$$W_{F_x}(f) = \frac{E\,S}{4\,\omega}\left\{\operatorname{Im} G_{\Delta\varepsilon_x a_x}(f)\right\}, \tag{28}$$

where $\Delta\varepsilon_x = \varepsilon_{1x} + \varepsilon_{2x}$ and $a_x = a_{1x} + a_{2x}$.

For bending moment component

$$W_{M_\theta}(f) = \frac{E\,I_{bend}}{D_{outer}^2\,\omega}\left\{\operatorname{Im} G_{\Delta\varepsilon_\theta\,\ddot{\theta}}(f)\right\}, \tag{29}$$

where $\Delta\varepsilon_\theta = \varepsilon_{1x} - \varepsilon_{2x}$ and $\ddot{\theta} = a_{1x} - a_{2x}$.

There are two advantages of technique employing equation (28) and (29). One of them is an opportunity to make measurement both longitudinal and bending moment components simultaneously. Another one lack of lower and upper frequency limitation due to finite difference method because of direct measurement of longitudinal force and bending moment. Nevertheless bias errors due to channel phase mismatch and random error inherent this technique. The example of total error estimation in form of frequency dependent plots is show in Fig. 10 and 11. The formulae for error estimation in this technique are exactly the same as for Direct Method.

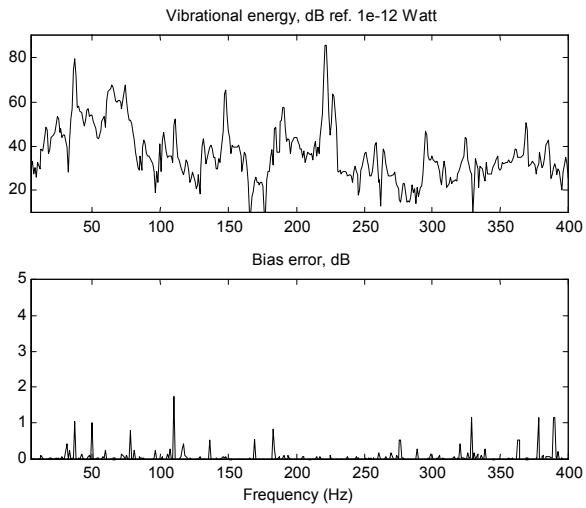

Fig 10. Bias error due to phase mismatch of 1 degree vs. frequency when pump is running
Longitudinal component measured with PVDF technique

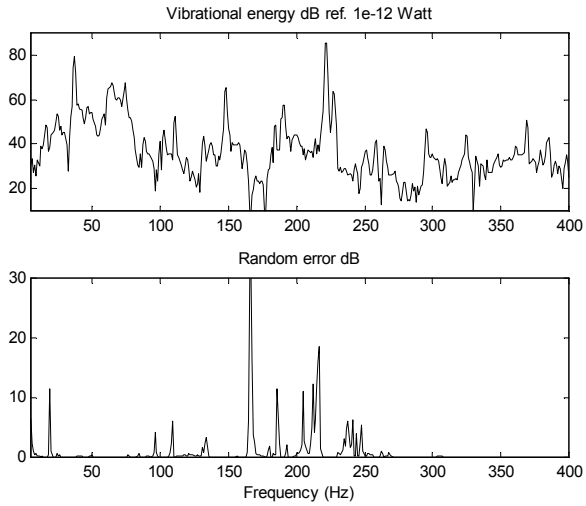

Fig 11. Random error (number of samples 500) vs. frequency when pump is running.
Longitudinal component measured with PVDF technique

For shear force component

$$W_{Q_z}(f) = \frac{E\,I_{bend}}{D_{outer}\;\Delta_{bend}\;\omega}\left\{\operatorname{Im} G_{\Delta\varepsilon_z\,a_z}(f)\right\}, \tag{30}$$

where $\Delta\varepsilon_z = \varepsilon_{1x} - \varepsilon_{2x} - \varepsilon_{3x} + \varepsilon_{4x}$.

Transducer arrangements on structure are depicted in Fig. 12 and 13.

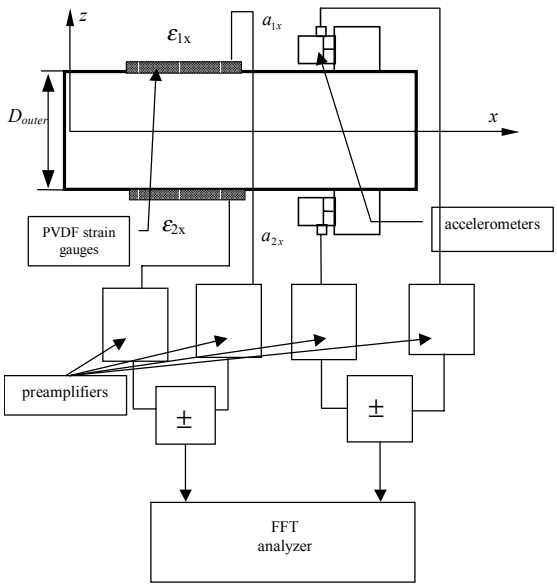

Fig. 12. Accelerometers and strain gauges arrangement on structure for measurement of bending moment and longitudinal force components

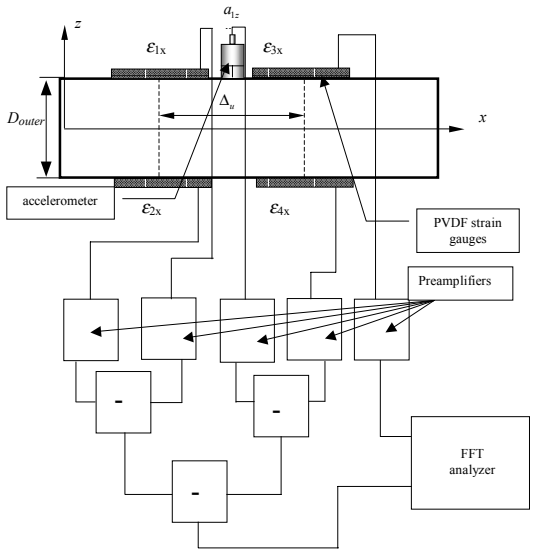

Fig. 13. Accelerometers and strain gauges arrangement on structure for measurement of shear force components

DISCUSSION

The errors in measurement of structural intensity by state-of-art techniques are discussed. It is pointed out that MEF (mobility energy flow) method and FD (finite difference) technique can be implemented in limited number of cases due to theirs sensitivity to various errors. Making the estimation of confidence level is impossible due to lack of information about degree of matrices ill-conditioning for MEF method. FD technique can not provide a tolerant results in a wide frequency range. In structural acoustics (or vibroacoustics) there is no absolute accurate experimental technique which gives precise results over all dynamic and frequency ranges. It would be noted that calculational models are just a models and nothing else. The aim of correctly performed experiment is elimination of raw errors and estimating the measurement error in order to show results with upper and lower confidence level rather than showing only one plot vs. frequency. Another aim is explanation to model designer why the absolute accuracy can not be achieved at specific frequencies or in the frequency bands. One reason is that observation time (or number of independent samples) can not be infinite. Another one is the extremely small magnitudes of wave motion that diminishes the coherence function between force and velocity at those frequencies. The small phase between them due to small structural dampening also impacts the measurement accuracy. It is recommended to implement CTT (complex transmissibility technique), Strain Gauge method and Direct Method in practical applications. MEF and FD technique have to be used accurately for limited number of applications in order to avoid raw errors. The FD technique for measuring the torsional component of energy flow can be used with reasonable accuracy in practical frequency range on real structures.

ACKNOWLEDGEMENT

Special thanks go to Andrej V. Popov and Jurij I. Kuznetzov from Vibroacoustic laboratory at Krylov Institute, Russia, for their friend attitude and useful discussions. Author gratefully appreciates to Dr. Claes Fredo from Ingemansson AB, Sweden for his criticism, Dr. Michael Sanderson from Lear Corporation, USA for his optimism and hard working. Thanks go to Dr. Leif Kari and Prof. Anders Nilsson from KTH MWL, Sweden, for theirs interest and support. All discussions, contacts and research projects made this paper possible.

LIST OF SYMBOLS AND NOTATIONS

F_i — force operative at the point i

M_i — moment operative at the point i

a_i — vibration acceleration at the point i

ε_i — vibrational strain at the point i

$[A]$ — mobility matrix of machine

$[K]$ — blocked stiffness matrix of isolator

$[B]$ — mobility matrix of receiving structure (foundation)

D_{outer} — outer diameter of pipe or shaft

S — cross section area

I_{bend} — bending moment of inertia of cross section

Δ_{bend} — measurement distance between strain gauges

Δ_l — measurement distance between accelerometer pairs (longitudinal component);

λ_B — bending wavelength

$k_B = \dfrac{2\pi}{\lambda_B}$ — bending wave number

$G_{xy}(f)$ — cross spectrum between processes $x(t)$ and $y(t)$

$G_{xx}(f)$ — autospectrum of process $x(t)$

$\gamma_{xy}^2(f)$ — coherence function;

R — reactivity index or reflection quotient

φ_{xy} — phase angle

φ^* — channel phase mismatch

f — frequency

$\omega = 2\pi f$ — angular frequency

$\ddot{\theta}_i$ — angular acceleration at the point i

$\dot{\theta}_i$ — angular velocity at the point i

v_0 — vibrational velocity at machine feet (machine is freely suspended)

Z_{12}^F — point-to-point force blocked impedance

Z_{12}^M — point-to-point moment blocked impedance

Z_{11}^F, Z_{22}^F — driving point force blocked impedance;

Z_{11}^M, Z_{22}^M — driving point moment blocked impedance

W_i — vibrational power at the point i

W_{transf} — vibrational power emitted to foundation

W_{input} — vibrational power emitted to isolator

$W_0 = 1 \cdot 10^{-12}\ Watt$ — reference value for vibrational energy

η — isolator loss factor

ρ — material density

E — Young modulus

ν — Poisson's ratio

Θ_{bias} — bias error

Θ_{rand} — random error

Δ_t — measurement distance between accelerometer pairs (torsional component)

n_d — number of samples

REFERENCES

1. D. Ewins, Modal testing: theory and practice, Academic Press, Wiley & Son Publishing 1979.

2. A. F. Seybert, Statistic errors in acoustic intensity measurement. J. of Sound and Vibration, 75, 1981, 585–595.

3. J. W. Verheij, Multi path sound transfer from resiliently mounted shipboard machinery. Teschnisch Physische Dienst. TNO-TH Delft, 1982.

4 R. J. Pinnington and R. G. White, Power flow through machine isolators to resonant and non resonant beams. J of Sound and Vibration, 75, 1981, 179–197.

5. F. J. Fahy, Measurement of acoustic intensity using the cross-spectral density of two microphone signals. JASA, 62(4), 1977, 1057–1059.

6. J. V. Chung, Cross-spectral method of measurement acoustic intensity without error caused by instrument phase mismatch. JASA 64(6), 1978, 1613–1616.

7. G. Pavic, Technique for determination of vibration transmission mechanism in structures. Dissertation. Institute of sound and Vibration Research, Faculty of Engineering and Applied Science, University of Southampton, 1976.

8. A. G. Troshin and V. I. Popkov, Measurement of Vibration Power flow in rod structures by using piezo-electric film sensors. 4-th International Congress on Structural Intensity Technique, Senlis, France 1993, 169-174.

9. C. R. Helkyard and B. R. Mace, A wave approach to structural intensity in beams. 4-th International congress on intensity technique. Senlis, France, August 31–September 2, 1993, 183–191.

10. A. G. Troshin, M. A. Sanderson and L. Ivarsson, Vibration Isolation of Structure borne power transmission by fans in building: a theoretical and experimental examination using Mobility and Complex Transmissibility methods. Report S 97-06, Department of Applied Acoustics, Chalmers University of Technology, Gothenburg, Sweden, 1997.

11. A. G. Troshin, Summation-subtraction device for a six degrees of freedom of motion transducer comprised of six linear accelerometers. Report S 97-05, Department of Applied Acoustics, Chalmers University of Technology, Gothenburg, Sweden, September 1997.

12. A. T. Moorhouse and B. M. Gibbs, Measurement of structure borne sound emission from resiliently mounted machine *in situ*. J of Sound Vibration, 180(1), 1995, 143–161.

13. A. G. Troshin and M. A. Sanderson, Structural energy flow in a resiliently coupled T-shaped beam by wave intensity and Mobility Approaches. Acustica United with Acta Acoustica, Vol. 84, 5, 860–869.