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A comparative survey for an adaptive FIR filter design in image compression by wavelets decomposition

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In this paper, adaptive filtering, wavelets, and lossy image compression are considered. Performances of the subband adaptive digital filter are discussed. We propose a transform coding method in which the low pass filters in the wavelets decomposition tree are time-varying. The idea is to decompose a digital image using an adaptive FIR[1] filter in each low frequency subband and compare it with an invariant filter of Daubechies. We employ a framework that includes the main comparison parameters. It is clearly shown here that it provides better performances than invariant filters in most applications. This work consists of applying an adaptive FIR[1] filter by the ADFFLS[2] algorithm to digital images. The filter performances on the different stages of the data image compression chain are then valued and compared with an invariant and biorthogonal filter of Daubechies. This is done studying of the main parameters, namely the covariance matrix, the subband coding gain, the concentration of energy in the low frequency subband, the signal to noise ratio, the correlation, the bit allocation, and the compression ratio. This study resides in the meticulous choice of comparison parameters, and the different stages of comparison. Results show that the PSNR[3], the correlation between original and reconstructed images, and the compression ratio are better with the adaptive filter for different lengths, quantifiers, and quantification levels.

INTRODUCTION

The study of image compression method has been an active area of research. For transformation coding, the KLT[4] is the optimal linear transformation in the sense that it minimizes the mean squared reconstruction error. Many adaptive methods have been used in lossy composed of a wavelets decomposition tree, different quantifiers, and the Huffman encoding. These methods go from conventional adaptive filtering up to complex combinations, where some of or all the elements of the chain are time varying [1, 2, 3, 4]. This paper proposes an approach to adaptation transform coding in which the low pass filters in the wavelets decomposition tree are time varying. This paper is organised as follows: Section 1 reviews the techniques of adaptive digital filtering: The ADFLMS[5] and the ADFFLS[2] algorithms are considered. Section 2 presents the application of the conceived adaptive filter in the lossy source coding chain, and the mean comparison parameters. Performances of this method for compressing images, results and discussion are investigated in Section 3. Finally, the last section concludes the paper.

1. ADAPTIVE DIGITAL FILTERING

Features in adaptive filtering are time varying according to certain predefined criteria. The principle of an adaptive digital filter is shown in Figure 1. The error signal $e(n)$ is used to obtain the best coefficients of the programmable filter, according to some predefined criteria. Two categories appear according to the choice of reference $y(n)$: If the input signal is taken as reference then, the filter is called of prediction, otherwise, if $y(n)$ is different of the input signal then the adaptive filter realizes a modelling of the system that has produced this reference. The main parameters [2, 5] common to the different algorithms for the coefficients update are:

- The number of coefficients;
- The number of iterations;
- The choice between prediction and modelling;
- The addition of a noise signal to the input signal or to the reference signal;
- The value of the adaptation step or that of the length of the observation temporal window.

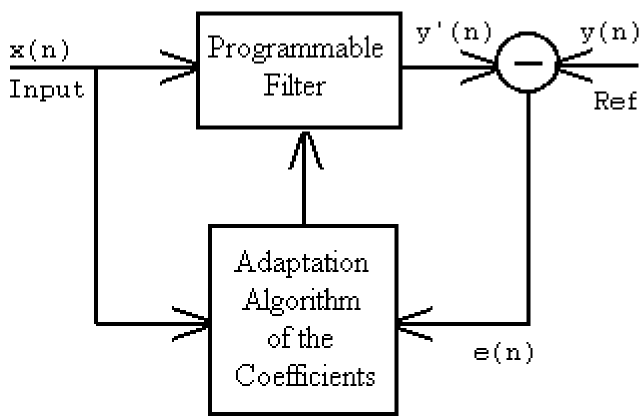


Fig. 1. Structure of adaptive filter

The most important option in adaptive filtering is the algorithm for updating the coefficients. The gradient algorithm is the most used in 1-D signals. The fast least square algorithms continue to develop and may present some considerable advantages in processing speed and in adaptation precision [6, 5].

1.1. The ADFLMS[5] algorithm

In this conception, there is an appearance of two filter structures: The transverse and the lattice structures. Many versions of the gradient algorithm exist in the literature [2, 5]. In the basis version of the transverse structure, the system equations [2, 5] are the following:

$$e(n+1)=y(n+1)H^t(n)x(n+1); \tag{1}$$

$$H(n+1)=H(n)+\delta\, x(n+1)e(n+1). \tag{2}$$

An example of an adaptive FIR[1] prediction filter of the second order (see Fig. 2), [5, 7], is the following:

$$e(n+1)=x(n+1)-a_1(n)x(n)-a_2(n)x(n-1); \tag{3}$$

$$\begin{bmatrix} a_1(n+1) \\ a_2(n+1) \end{bmatrix} = \begin{bmatrix} a_1(n) \\ a_2(n) \end{bmatrix} + \delta \begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} e(n+1); \tag{4}$$

$$x(n)=\sin\left(\frac{n\pi}{4}\right)+b(n); \tag{5}$$

$\delta_b^2 = 5\cdot 10^{-5}$; $\delta_x^2 = 0.5$; $\delta = 0.05$.

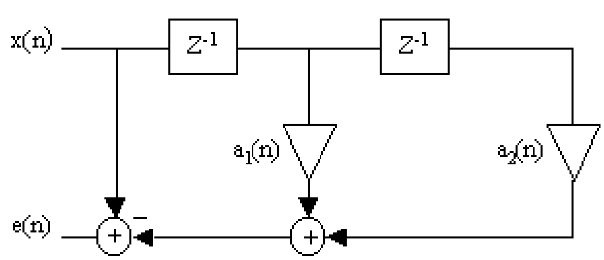


Fig. 2. Predictive filter of second order

The obtained results are shown in Fig. 3. The upper boundary of the adaptation step δ has been taken at $2/P_x$, applying relation (1) for $\ell=1$ and considering that the prediction errors in the subsets are lower in power than that of the input signal.

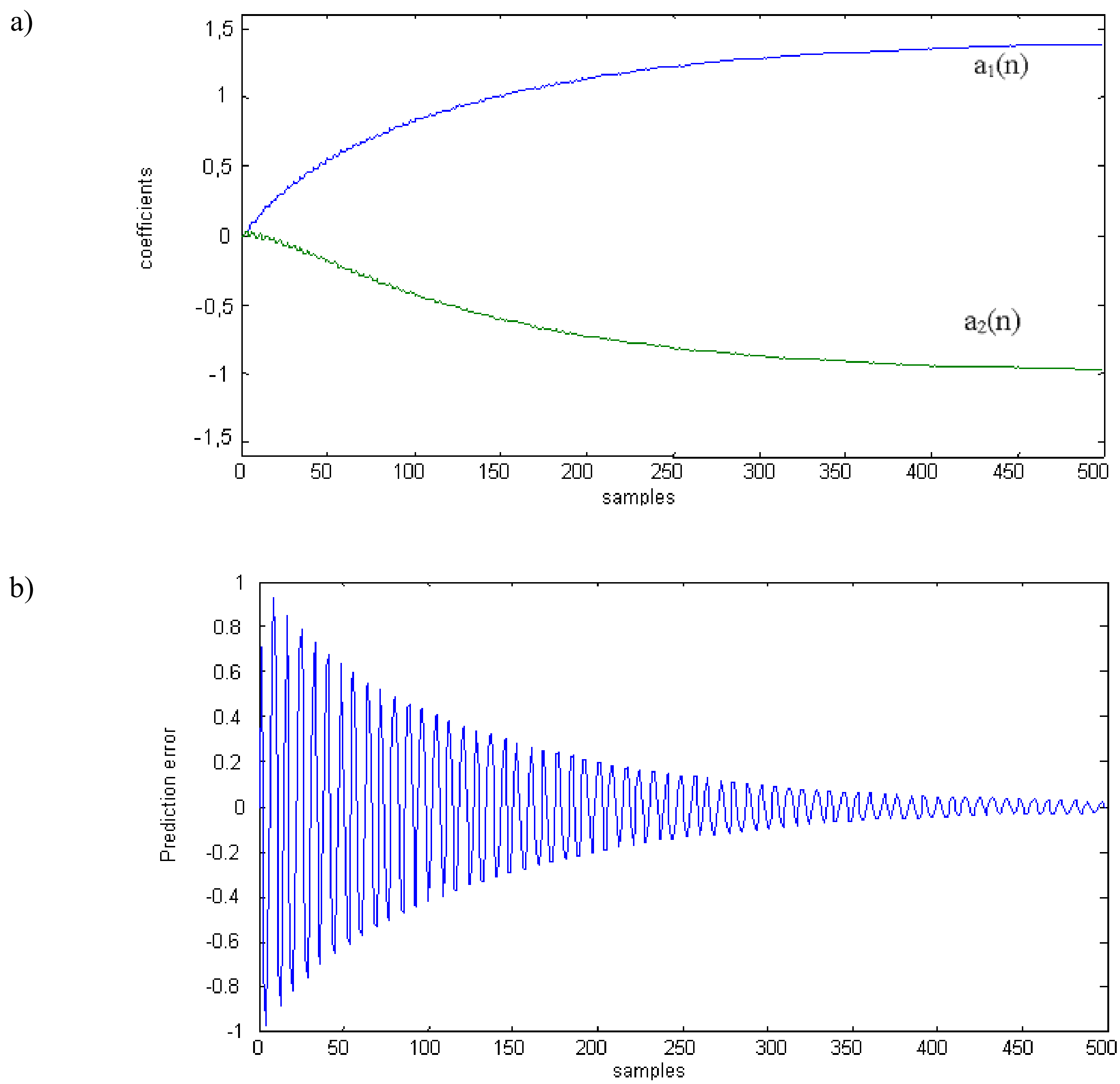


Fig. 3. Prediction filter obtained by the gradient method (ADFLMS[5]):
a) Filter coefficients, b) Prediction error

The interpretation of these results is that, starting with coefficients of zero values, the prediction error stabilises around zero. The advantages of the gradient algorithms are their simplicity and hardiness; the inconvenience is their limited performances. In some cases, the least square algorithms can bring an important improvement in time of initial convergence and in vestigial error after convergence [5].

1.2. The ADFFLS[2] algorithm

In this conception, as in ADFLMS[2] (see section 1.1), appear the transverse and the lattice structures. In addition, two more parameters intervene: the weighting factor W and the initial energy of the prediction error E_0. In transverse structure, the coefficients update is obtained using the following equation [5, 6]:

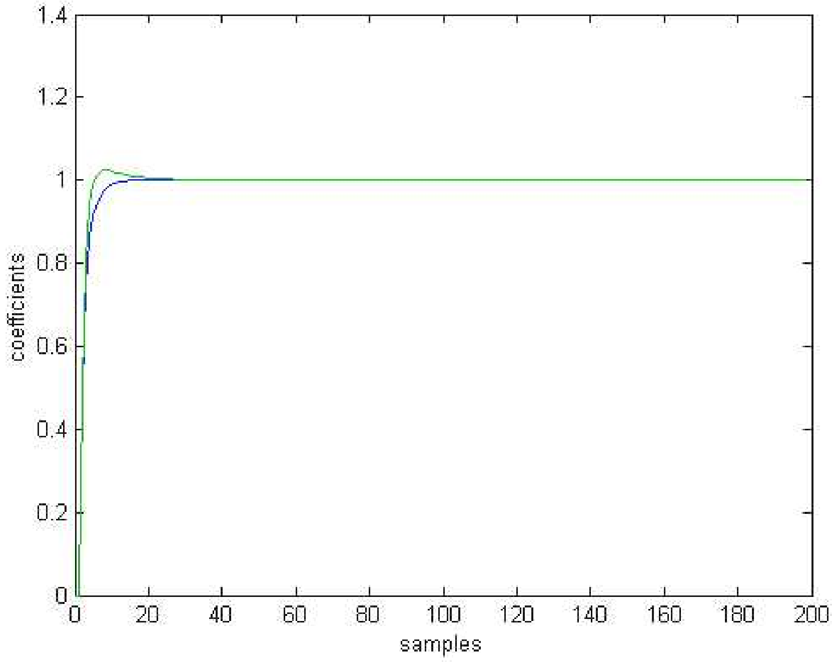
$$H(n+1)=H(n)+R_L^{-1}(n+1)x(n+1)e(n+1). \tag{6}$$

Some simulation examples in the modelling of the lattice structure show clearly the performances improvement obtained with the least square algorithm with regard to the gradient algorithm. Reference [6] presents in details, the principles, structures and algorithms of the adaptive filtering, as well as some related applications. Fig. 4 shows that the convergence of the filter is faster than that of the gradient.

2. APPLICATIONS

In this paper, the work has been applied to a lossy source coding chain (see Fig. 5), consisting of a two levels structure wavelets decomposition (see Fig. 6). Three types of scalar quantifiers and a Huffman encoding have been used [1]. The ADFFLS[2] algorithm (see section 1.2) has been chosen for the design of the adaptive FIR[1] filter. This latter has been applied in every low frequency subband of the decomposition tree (see Fig. 6). Performances of the adaptive filter (A_d) are compared with those of the biorthogonal and invariant filter of Daubechies (bior) [8, 9]. Simulations are made using the popular (512×512) “Lena” image (see Fig. 7).

a)



b)

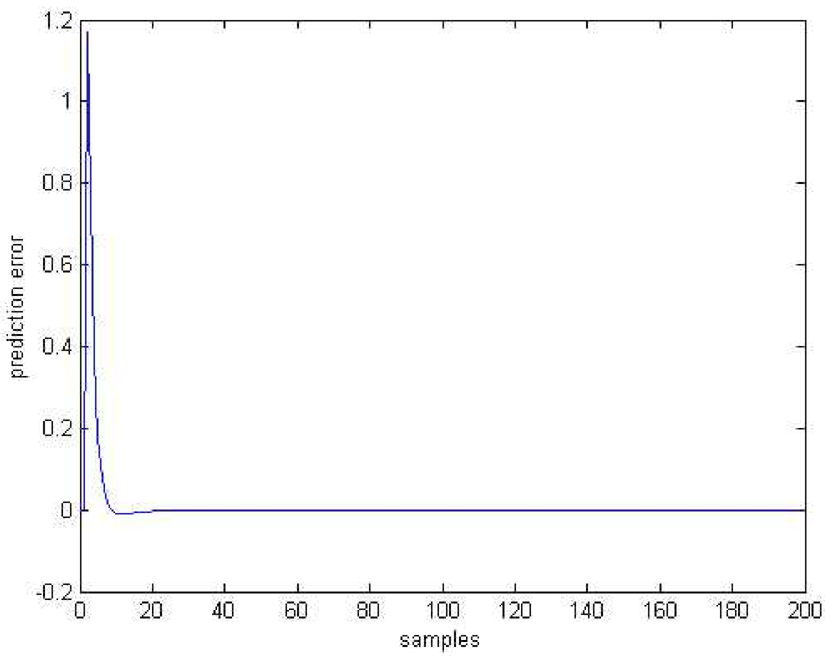
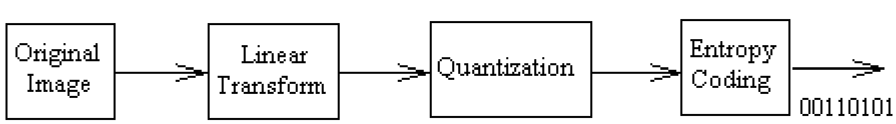


Fig. 4. Prediction filter by the least square method (ADFELS)
a) Filter coefficients, b) Prediction errors

a)



b)

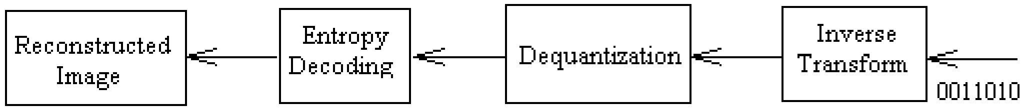


Fig. 5. Block diagram of a typical transform coding system:
a) Encoder block diagram, b) Decoder block diagram

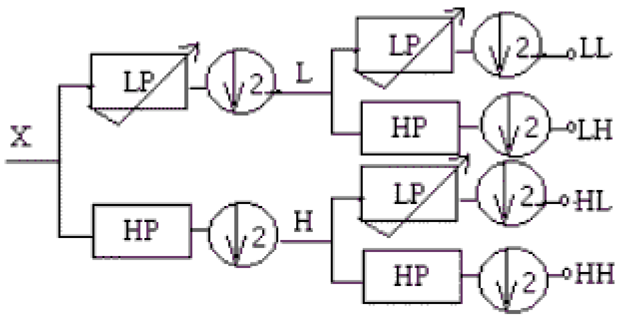


Fig. 6. Tree structure for 4-band separable subband filter

X – Input signal; LP – Low pass filter; HP – High pass filter; L – Low; H – High LL – Low-low subband; LH – Low-high subband; HL – High-low subband; HH – High-high subband

Fig. 7. Original (256×256) "Lena" image at 8b/pixel

In order to evaluate the performances of the designed adaptive filter, this latter has been compared to a wavelet filter of Daubechies using the following parameters [8, 9]:

2.1. Covariance

The covariance between two images X and Y is given by the following equation:

$$\operatorname{cov}(x,y)=\frac{\sum_{i=0}^{N-1}\sum_{j=0}^{M-1} X_c(i,j)Y_c(i,j)}{NM}. \tag{7}$$

2.2. Covariance matrix

The covariance matrix M_C is calculated for the four sub-images after decomposition (see Fig. 6). Two interesting parameters are derived from this matrix: the subband coding gain G_S and the concentration of energy C_E in the low frequency subband *LL*. The covariance matrices of "Lena" obtained after two decomposition levels using adaptive filtering (A_d) and invariant filtering (Bior) of length $\ell = 16$, are given in expressions (10) and (11) respectively.

$$G_S = \frac{1}{4}\frac{\sum_{i=1}^{4} M_C(i,i)}{\sqrt[4]{\prod_{i=1}^{4} M_C(i,i)}}; \tag{8}$$

$$C_E = \frac{M_C(i,i)}{\sum_{i=1}^{4} M_C(i,i)} \times 100. \tag{9}$$

$$M_{C(A_d)} = \begin{array}{c} \\ LL \\ LH \\ HL \\ HH \end{array} \begin{array}{c} \begin{array}{cccc} LL & LH & HL & HH \end{array} \\ \begin{bmatrix} 12196 & 86 & 20 & 58 \\ -86 & 244 & 1 & 12 \\ 20 & 1 & 161 & 9 \\ 58 & 12 & 9 & 68 \end{bmatrix} \end{array}, \tag{10}$$

$$M_{C(bior)} = \begin{array}{c} \\ LL \\ LH \\ HL \\ HH \end{array} \begin{array}{c} \begin{array}{cccc} LL & LH & HL & HH \end{array} \\ \begin{bmatrix} 15889 & 2 & 113 & -9 \\ 2 & 84 & 3 & 2 \\ 113 & 3 & 467 & 3 \\ -9 & 2 & 3 & 3 \end{bmatrix} \end{array}. \tag{11}$$

2.3. Correlation

The normalised and centred correlation is defined as

$$C(x,y) = \frac{E(xy) - E(x)E(y)}{E(x^2)E(y^2)}. \tag{12}$$

In practice, the correlation is estimated using relation (13):

$$C(x,y) = \frac{1}{N}\frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-\bar{y})^2}}. \tag{13}$$

2.4. PSNR[3]

The maximum signal to noise ratio is given by the following equation:

$$\mathrm{PSNR} = 10\log\frac{255^2}{\mathrm{QME}}. \tag{14}$$

2.5. Quantization and encoding

For the entire obtained subband, scalar quantifiers have been chosen for their easiness. They are either uniform or adapted to the coefficients distribution (using Lloyd-Max for example) [10], for these:

- The three scalar quantifiers have been studied separately: Uniform, Lloyd-Max and DPCM[6];
- The PSNR[3] of the low frequency subband *LL* for different quantification levels has been studied;
- The compression rate *R* and a Huffman encoding have been studied [11].

3. RESULTS AND DISCUSSION

The obtained results are summarised in Fig. 8, 9, 10, 11 and Tables 1, 2. The covariance matrices of “Lena” image obtained after two decomposition levels using adaptive filtering (A_d) and invariant filtering (Bior) of length $\ell = 16$ are given in expressions (13) and (14) respectively.

The correlation between the original image and the subband *LL*, as well as the PSNR[3] are better with an adaptive filter (A_d), and this whatever its length is (see Fig. 8 and Tab. 1). The reconstructed image is obtained after two wavelets decomposition levels, a scalar quantification, and a Huffman coding (see Fig. 11). The subband *LL* has been quantified differently for every reconstructed image (uniform, Lloyd-max and DPCM[6]). Each of the subbands *LH*, *HL*, and *HH* has been through a uniform scalar quantification (see Fig. 6). Quantification levels of 22, 10, 19 and 8 are affected respectively to subbands *LL*, *LH*, *HL* and *HH*. The same work has been realised for quantification levels of 22, 9, 7 and 3 (see Fig. 11). These results show the importance of the choice of the quantifier, mainly for the low frequency subband and the quantification levels.

Table 2 shows the bit allocation performances of the adaptive filter (A_d), comparing it with a Daubechies biorthogonal filter ($\mathrm{bior}_{2\text{-}2}$) of length $\ell = 6$. In this comparison, the Lloyd-Max and the DPCM[6] algorithms work the same way and offer the same performances in the two cases.

CONCLUSIONS

A comparative survey to adaptive compression is proposed in this paper, based on an adaptive digital FIR[1] filter. The results presented herein have shown that the adaptive filtering can outperform the globally optimal linear transform. The same image was coded using both the invariant filter of Daubechies and the adaptive filter. For our approach, the correlation, the PSNR[1], the compression rate are better, and the image quality was improved. A better compromise for the main comparison parameters is obtained for a Lloyd-Max quantifiers in the low frequency subband *LL*. Finally, the obtained results show indeed the interest of the use of the adaptive filter. In fact, the correlation and the PSNR[3] are better whatever its length is. The different quantifiers offer a better compression rate, and a better compromise for the correlation, and the PSNR[3], in addition the compression ratio is obtained for a Lloyd-max quantifier in the low frequency subband *LL*.

Table 1. Coding results at various lengths ℓ, obtained on the standard (256×256) "Lena" image with adaptive (A_d) and Daubechies (bior) filters

ℓ		2	6	10	12	16
Bior	C	0.9757	0.9952	0.9926	0.9898	0.9648
	PSNR	15.0928	15.2383	15.4254	15.5495	14.4132
	G_S (dB)	5.4552	9.8707	8.0266	6.1462	11.8782
	C_E (%)	93.7468	96.9758	96.3398	95.1909	97.5067
A_d	C	0.9973	0.9995	0.9991	0.9994	0.9998
	PSNR	34.4336	23.0104	22.4231	17.1168	11.8373
	G_S (dB)	2.6875	6.3252	5.4458	5.8983	20.0584
	C_E (%)	83.0891	94.3893	93.8984	95.0819	98.7191

Table 2. "Lena" image bitrate "bpp" comparison between the adaptive (A_d) and the Daubechies (bior) filters, with length $\ell = 6$, different quantizations and quantification levels

Quantification Levels	22-9-7-3		22-10-19-8	
Filter	A_d	$Bior_{2.2}$	A_d	$Bior_{2.2}$
Uniform and Scalar Quantization	0.3173	2.0749	0.3757	1.8258
DPCM	0.3060	2.0281	0.3644	1.7790
Lloyd-Max	0.3197	2.1230	0.3782	1.8740

a)

b)

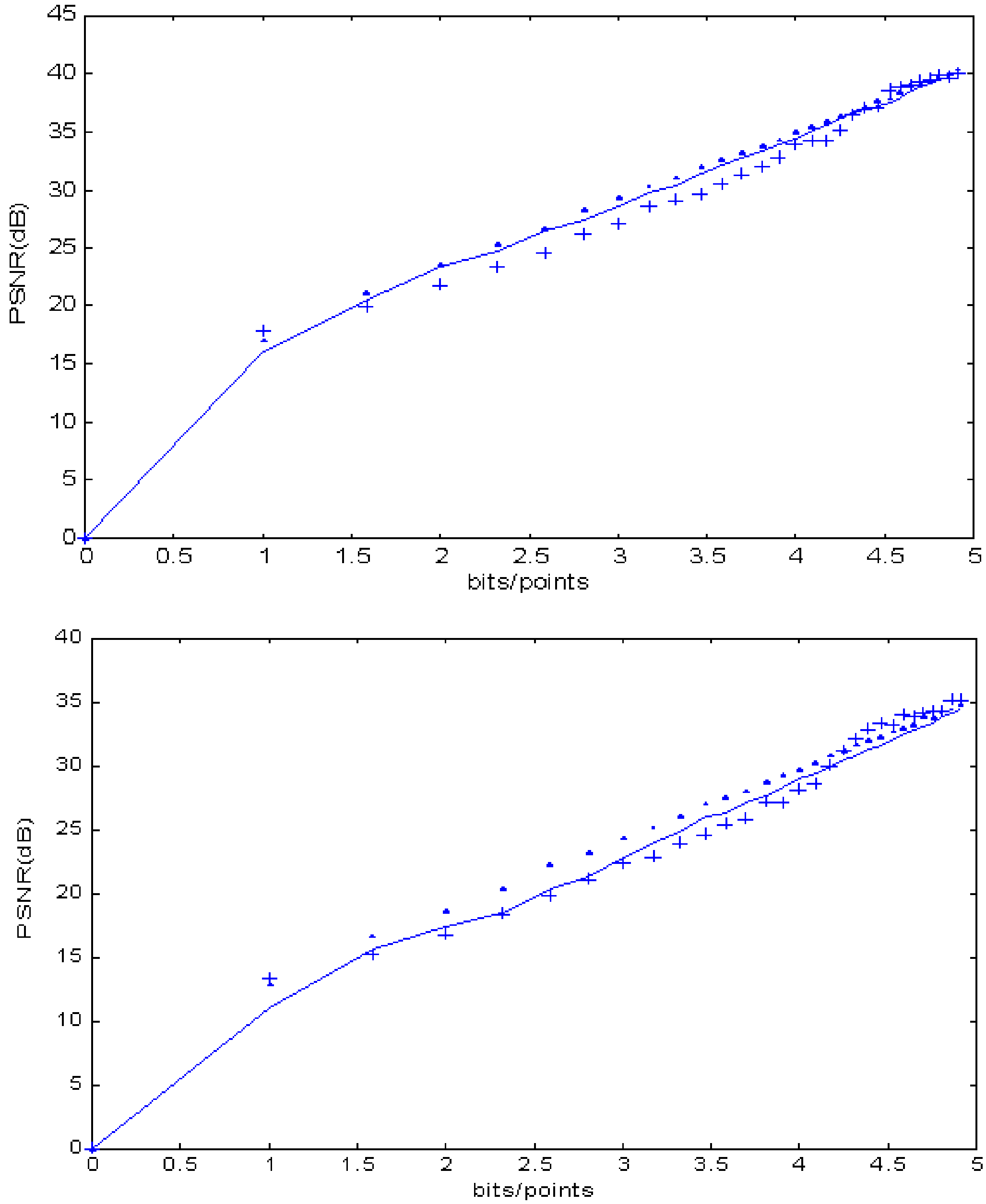


Fig. 8. Rate-Distortion performance of the “*LL*” subband “Lena” for different quantizations with: a) Adaptive filter, b) Daubechies filter

——*Uniform*, ⋯⋯ *Lloyd-Max*, + + *DPCM*

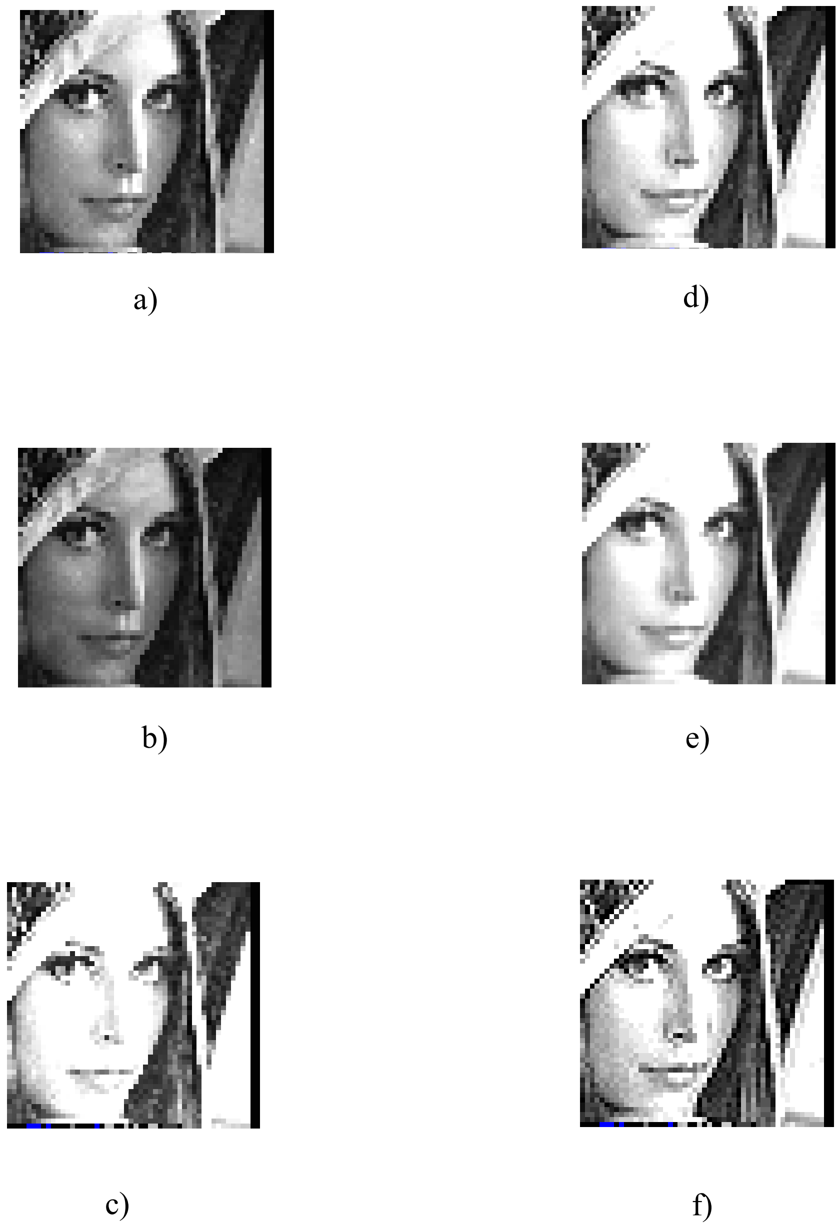


Fig. 9. The *LL* subband of "Lena" image region (100×100) obtained with adaptive (A_d) and Daubechies (Bior) filters, for different lengths ℓ:
a) A_d, $\ell = 6$; b) A_d, $\ell = 2$; c) A_d, $\ell = 16$;
d) $Bior_{1-1}$, $\ell = 6$; e) $Bior_{2-2}$, $\ell = 2$; f) $Bior_{3-7}$, $\ell = 16$

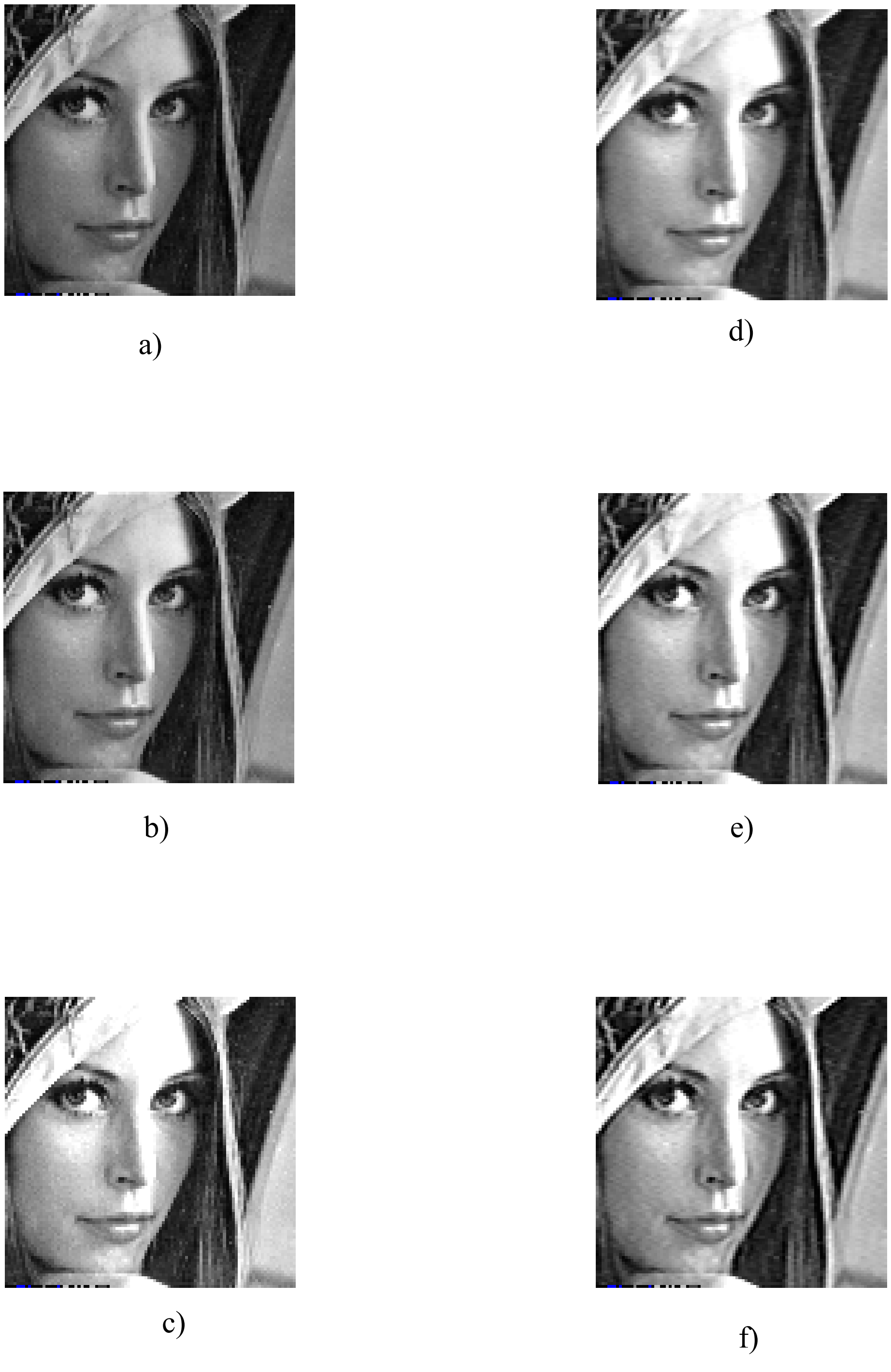


Fig. 10. “Lena” image region (100×100), obtained with adaptive (A_d) and Daubechies (Bior) filters, for different lengths ℓ:
a) A_d, $\ell = 6$; b) A_d, $\ell = 2$; c) A_d, $\ell = 16$;
d) $Bior_{1\text{-}1}$, $\ell = 6$; e) $Bior_{2\text{-}2}$, $\ell = 2$; f) $Bior_{3\text{-}7}$, $\ell = 16$

a) b) c) d) e) f)

Fig. 11. Reconstructed “Lena” image obtained with the following parameters:
a) C =0.9953, PSNR=33.8820, bpp=2.0749; b) C =0.9948, PSNR=33.4857, bpp=2.0281;
c) C =0.9955, PSNR=34.1115, bpp=2.1230; d) C =0.9864, PSNR=29.2545, bpp=1.8258;
e) C =0.9859, PSNR=29.1151, bpp=1.7790; f) C =0.9866, PSNR=29.3322, bpp=1.8740

APPENDIX A. SYMBOL AND NOTATIONS

δ	Adaptation step
δ_b^2	Power of the noise signal b(n)
δ_x^2	Power of the useful input signal
$a_i(n)$	Filter coefficients
A_d	Adaptive
$b(n)$	Noised signal
bpp	Bits per pixel
Bior	Biorthogonal
C_E	Concentration of energy
$C(x,y)$	Correlation
$e(n)$	Error signal
E_0	Prediction error
E	Expectation
G_S	Subband coding gain
HL	High-Low subband
HH	High-High subband
$H(n)$	Vector of the l filter coefficients
$H^t(n)$	Transposed vector of $H(n)$
LH	Low-high subband
LL	Low-low (low frequency subband)
ℓ	Filter length
M	Number of columns
M_C	Covariance Matrix
P_x	Power of the input signal x(n)
N	Number of lines
QME	Quadratic mean error
$R_L(n)$	Estimated auto-correlation matrix of the input signal
W	Weighting factor
$x(n)$	Vector of the ℓ^{th} most recent input data
$y(n)$	Reference signal
x	Original image
y	Reconstructed image
X_C	Image x centred with regard to the mean
Y_C	Image y centred with regard to the mean
$\bar{x}$	Mean value of the original image
$\bar{y}$	Mean value of the reconstructed image

[1]Finite Impulse Response
[2]Adaptive Digital Filter with Fast Least Square
[3]Pic Signal Noise Ratio
[4]Karhunen-Loève Transformation
[5]Adaptive Digital Filter with the Least Mean Square
[6]Differential Pulse Coded Modulation

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