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The importance of spectral magnitude in signal through a noisy channels in data transmission

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In this paper we present a novel technique that can be applied in data transmission noisy channels. The method is based on the reconstruction of one or two dimensional signal with finite support from only its Fourier transform magnitude computed from the converted original signal. This method is the Saxton and Gershberg iterative algorithm for reconstructing a non minimum phase sequence from magnitude, using this method we have proved that spectral magnitude is less sensitive to additive white Gaussian noise than spectral phase and signal. The algorithm is operated on converted non minimum to minimum phase signal by concentrating the higher energy on the left hand end sequence than the right hand end sequence, thus dealing with magnitude we have achieved a good performance at all SNR levels with limited extra computational complexity.

1. INTRODUCTION

When a digital signal is transmitted over a typical band limited, noise and interference often arise and are the main impairments to reliable communication. Under this circumstances the signal is seriously degraded, a novel technique is presented to prevent the degradation of the signal in the presence of highly additive Gaussian noise. When the channel introduces only minor variations in the signal, a widely used method is to use an adaptive linear filter to equalize the channel distortion [1]. In the case of channels introducing a large amount of noise this method is very long, resulting in slow convergence of adaptive algorithm. An improved solution is to use the spectral magnitude instead of signal.

Signal reconstruction from the Fourier transform magnitude (FTM) has been the topics of much interest in nature. Such as reconstruction procedure would have application in wide range of fields, including optical astronomy and microscopy. This attention has been mentioned by the discovery that most two dimensional signals of finite extent can be reconstructed from its partial information [2, 3]. Several algorithm have been proposed for reconstruction from FTM, pioneered by the work of Fienup on the iterative algorithms [4, 5]. After, efforts have been made to provide estimates for these iterative procedures often becomes converging to a solution [3, 5]. Iterative algorithms similar to those we discuss have

been useful in a number of areas where partial information in the two domains is available. In particular, the algorithm presented in this paper is similar in style to the Monson H. Hayes [3].

By long-term exposure to atmospheric turbulence, the distorting signal is known to have a phase function which is approximately zero and consequently the phase of the noisy image is very similar to that of the original image. In such cases, the blind convolution problem may be viewed as problem in which image reconstruction is desired from its phase function and thus the algorithm proposed here may be applicable [11]. Even though the results performed by this algorithm have important theoretical significance, they are limited practice since they are based on the assumption that the exact phase is available. In many potential application problems, the available phase may have been degraded by measurement noise, quantization noise. To understand the effects of phase degradation on the reconstructed sequence, a series of experiments has been performed.

In this paper, we present an experimental results and propose a technique that reduces the signal degradation effects. The main of this work, is how to reduce the signal distortion in the presence of noise in data transmission channel and show the utility of the spectral magnitude function as less sensitive parameter in the signal. Due to its interests, the magnitude only signal reconstruction still the subject of several research [7, 8, 14], from these interest, we will show that the magnitude is less sensitive to additive Gaussian noise than phase or signal under certain conditions. Some results has been performed to reconstruct a signal from a noisy phase [9], but there is not enough results in the case of noisy magnitude. When the blurred function is not zero phase, the reconstructed image will be degraded. In the present work, the problem is viewed as the image reconstruction from its noisy magnitude. We show when the noisy function is superposed to spectral magnitude, the image reconstructed from noisy magnitude will have a good quality instead of the noisy phase reconstruction. The reconstruction is not possible from a degraded phase because the major information containing in signal (image, speech) [6] are losses. The results proposed show under certain conditions the importance of magnitude in data transmission. Another area in which the importance of magnitude in image coding appear, both phase and magnitude are typically coded and transmitted. In developing coding schemes for the phase and magnitude, it has been found that assigning considerably more bits to coding the phase and magnitude is important in the success of Fourier image coding. In this case the magnitude require less bit rate than phase, its preferably to reconstruct the phase and image from the coded magnitude.

In general, a signal cannot be uniquely specified by only the phase or magnitude of its Fourier transform. However, one condition under which the magnitude and phase are related is the minimum phase condition under this condition a signal can be uniquely recovered from the magnitude of its Fourier transform. Since the algorithm of Hayes [3] is not applicable in the case of magnitude only reconstruction and when the signal is not a minimum phase. In the section 2, we discuss a number of conditions for a signal to be minimum phase [10]. In section 3, we present the iterative algorithm which reconstruct a non minimum phase signal from its spectral magnitude after applying modifications on signal without information losses, and the method to change the signal from a non minimum phase to minimum phase. The zeros

of signals lying outside the unit circle will interring inside the unit circle. In section 4, we show the effect of additive Gaussian noise on the image and separately on the magnitude and phase, after the reconstructed image from the degraded phase is obtained by applying the iterative algorithm [3]. The signal reconstruction from noisy magnitude is achieved when the signal is modified and thus the algorithm must be ameliorated to deal with a non minimum phase signal. In this section, it is clearly shown the importance of magnitude. In section 5 we concludes with a summary.

2. THE MINIMUM PHASE CONDITION

In the following condition, we restrict the z transform of the sequence $h(n)$ to be rational function which express in the form

$$H(z) = Az^{n_o} \frac{\prod_{k=1}^{M_i} (1 - a_k z^{-1}) \prod_{k=1}^{M_o} (1 - b_k z)}{\prod_{k=1}^{P_i} (1 - c_k z^{-1}) \prod_{k=1}^{P_o} (1 - d_k z)}, \quad (1)$$

where $|a_k|$, $|b_k|$, $|c_k|$ and $|d_k|$ are less than or equal to unity, z^{n_o} is a linear phase factor, and A is a scale factor.

M_i and P_i are both the number of zeros and poles inside the unit circle respectively.

M_o and P_o are both the number of zeros and poles outside the unit circle respectively.

When, in addition, $h(n)$ is stable, i. e., $\sum_n |h(n)| < \infty$, $|c_k|$ and $|d_k|$ are strictly less than one.

A complex function $H(z)$ of a complex variable z_i defined to be minimum phase if its reciprocal $H^{-1}(z)$ are both analytic for b , $|z| \geq 1$.

A minimum phase sequence is then defined as a sequence whose z transform is minimum phase. For $H(z)$ rational, as in (2), the minimum phase condition exclude poles or zeros on or outside the unit circle in the z -plane or at infinity. As a consequence, the factors of the form $(1 - b_k z)$ corresponding to zeros on or outside the unit circle and the factors of the form $(1 - d_k z)$ corresponding to poles on or outside the unit circle will not be present. Furthermore, in (1), $n_o = 0$ to exclude poles or zeros at infinity. Thus, for $H(z)$ minimum phase, (1) reduces to

$$H(z) = A \frac{\prod_{k=1}^{M_i} (1 - a_k z^{-1})}{\prod_{k=1}^{P_i} (1 - c_k z^{-1})}, \quad (2)$$

where $|a_k|$ and $|c_k|$ are both strictly less than unity.

From (2) other conditions can be formulated for a signal to be minimum phase. Two conditions in particular which we discuss below are particularly useful in the context of the iterative algorithm to be discussed in sections 3 and 4.

Minimum Phase Condition A

Consider $h(n)$ and $H(z)$ rational in the form of (1) with no zeros on the unit circle. A necessary and sufficient condition for $h(n)$ to be minimum phase is that $h(n)$ be causal, i. e., $h(n) = 0$, $n < 0$, and n_0 in (1) be zero.

From (2), it follows that these conditions are necessary. To show that they are sufficient, we want to show that they force (1) to reduce to (2). Clearly, factors of the form $(1 - d_k z)$, $|d_k| < 1$ in the denominator introduce poles outside the unit circle which would violate the causality condition since $h(n)$ is restricted to be stable with $n_0 = 0$ in (1), factors of the form $(1 - d_k z)$ would introduce positive powers of z in the Laurent expansion of $H(z)$, requiring $h(n)$ to have some non zero values for negative values of n , thereby again violating the causality condition. Therefore, the factors cannot be present and with $n_0 = 0$, (1) reduces to (2). Finally, because our condition assumes $h(n)$ is stable and that $H(z)$ has no zeros on the unit circle, $h(n)$ is minimum phase.

The above minimum phase conditions require that $h(n)$ be causal and that the unwrapped phase function have no linear phase component. Another slightly different set of necessary and sufficient conditions for a signal to be minimum phase can be stated as follows.

Minimum Phase Condition B

Consider $h(n)$ stable and $H(z)$ rational in the form of (1) with no zeros on the unit circle. A necessary and sufficient condition for $h(n)$ to be minimum phase is to be causal i. e., $h(n) = 0$, $n < 0$ and $h(0) = A$ is the scale factor of the (1). Again, from (2) it follows that these conditions are necessary since (2) has no poles or zeros outside the unit circle or at infinity, guaranteeing causality, and from the initial value theorem

$$h(0) = \lim_{z \rightarrow \infty} H(z) = \lim_{z \rightarrow \infty} A z^{n_0} \prod_{k=1}^{M_0} (1 - b_k). \quad (3)$$

Since $h(0) = A$,

$$\lim_{z \rightarrow \infty} z^{n_0} \prod_{k=1}^{M_0} (1 - b_k z) = 1. \quad (4)$$

3. NUMERICAL ALGORITHM

Consider $h(n)$ and $H(z)$ rational in the form of (1) with no zeros on the unit circle. A necessary and sufficient condition for $h(n)$ to be minimum phase is that $h(n)$ be causal, i. e. $h(n)=0$, $n<0$, and n_0 in (1) be zero. According to the iterative [11], or direct [13] algorithm, the phase only reconstruction signal $h(n)$ is within a scale factor β , but this algorithm does not converge in the magnitude only reconstruction case, the solution is as follows.

Empirically, we show if we increase the factor $h(0)$ to the largest sample of the sequence $h(n)$ all the zeros and poles of $H(z)$ lie inside the unit circle and $h(n)$ becomes minimum phase sequence [1] as illustrated in fig. 1. When we increase $h(N-1)$ (N is the length of $h(n)$) to the largest sample of the sequence, all the zeros and poles of $H(z)$ lie outside the unit circle and $h(n)$ becomes maximum phase as illustrated in fig. 2. We conclude in the first case that the major information of the signal are concentrated in the first sample $h(0)$, in contrast with the second case. Then the magnitude only signal reconstruction algorithm converge. As an example, the magnitude only signal reconstruction is established.

First, let the sequence, $h(n) = \{2, 6, 8, 1, 4, 2, 9\}$ and calculating its zeros, the first sample takes the following values $h(0)$, $10 \times h(0)$, $100 \times h(0)$

$$h(n) = \{h(0), h(1), h(2), \dots, h(N-1)\}. \quad (5)$$

In the second case, the final sample takes the following values $h(N-1)$, $10 \times h(N-1)$, $100 \times h(N-1)$.

The fig. 1 (a, b, c) shows the minimum phase condition on the signal, and the fig. 2 (a, b) shows the maximum phase condition on the signal through the dispersion of the zeros of $H(z)$ inside and outside the unit circle.

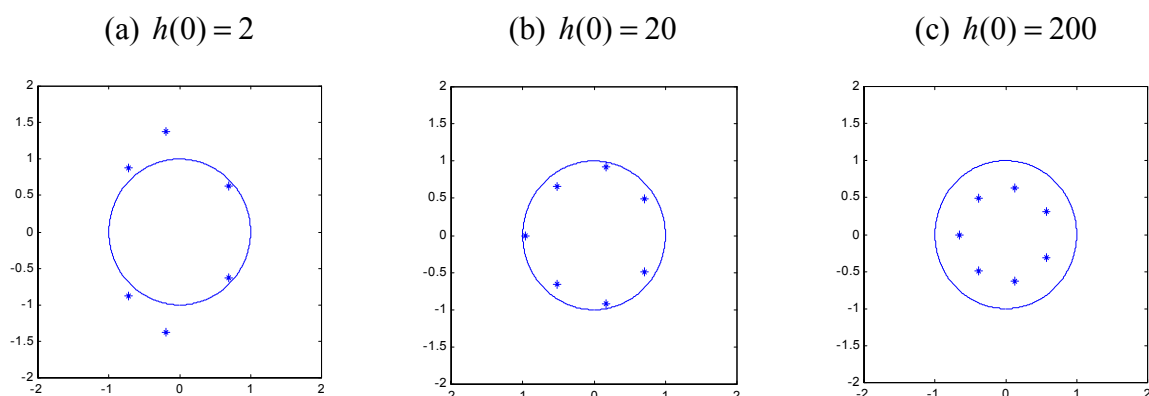


Fig. 1. Zeros location of $h(n)$ (minimum phase condition)

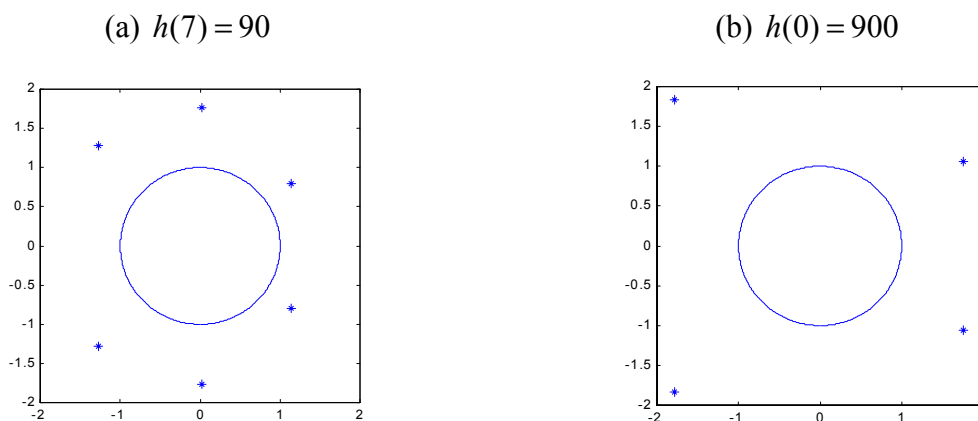


Fig. 2 Zeros location of $h(n)$ (maximum phase condition)

As an example, we apply the iterative algorithm for the magnitude only reconstruction for a non minimum phase image (276×260 pixels [12]) and replace the first sample pixel $h(0,0)=15$ by the following values $h(0,0)$, $100 \times h(0,0)$ and $1000 \times h(0,0)$.

As illustrated in fig. 3, the algorithm gives a good results when we used one (1) iteration.

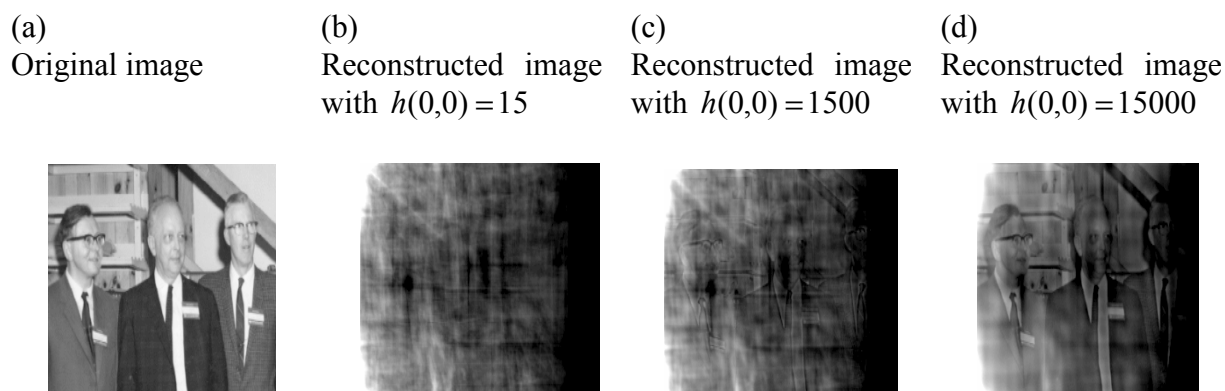


Fig. 3. Image reconstruction from spectral magnitude (for 1 iteration)

The following fig. 4 shows the efficiency of the algorithm when we increase the iteration number, 4 iterations are sufficient to achieve a perfect reconstruction instead of increasing the first sample.

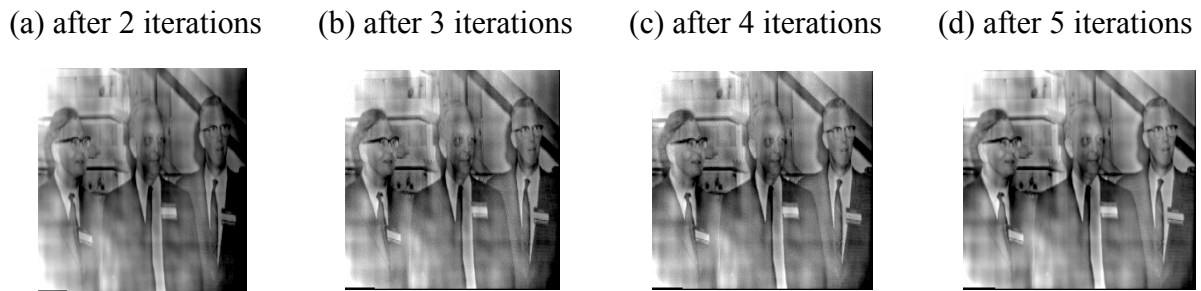


Fig. 4. Image reconstruction from spectral magnitude (for $h(0,0)=15000$)

The fig. 5 shows the case of perfect reconstruction $h(0,0)=1500000$, the number of iteration is 2.



Fig. 5 The perfect reconstruction of image $h(0,0)=1500000$, (for 2 iterations)

4. THE EFFECT OF ADDITIVE NOISE ON SPECTRAL PHASE AND MAGNITUDE

To understand the effect of magnitude degradation on the reconstructed sequence, a series of experiments has been performed. In this paper, we present the experimental results and propose a technique that reduces the signal degradation.

When the Fourier transform phase is degraded by additive noise the equation (6) can be written as (7) and when the Fourier transform magnitude is degraded by additive noise the equation (6) can be written as (8):

$$X(\omega) = |X(\omega)| \cdot e^{j\theta_x(\omega)} \quad (6)$$

$$X_1(\omega) = |X(\omega)| \cdot e^{j[\theta_x(\omega) + w(\omega)]} \quad (7)$$

$$X_2(\omega) = |X(\omega) + w(\omega)| \cdot e^{j\theta_x(\omega)} \quad (8)$$

where $w(\omega)$ represents the additive noise in the phase or magnitude and $x_1(n)$, $x_2(n)$ are the reconstructed sequences from the degraded phase and the degraded magnitude respectively.

For a non zero additive noise $w(\omega)$, $x_1(n)$ in (7) is different from $x(n)$, and the objective of this paper is to replace the degraded phase by a degraded magnitude.

In the case of bidimensional signal, the noisy phase image and the noisy magnitude image $x_1(n,m)$, and $x_2(n,m)$ are compared to the original image $x(n,m)$ and the noisy image in terms of intelligibility, the noise is generated by a random process [12] (zero-mean Gaussian density).

The fig. 7 presents the degraded image by a noisy function $w(n,m)$, fig. 9 presents the reconstructed image from the degraded phase, it is clearly shown that there is no information in this figure, thus a completely loss of information. When the same noisy function is added to the magnitude, the reconstructed image from the degraded magnitude is achieved after 1 iterations of the algorithm (see fig. 8).

Now, if we increase the amplitude of the blurring function, the same work is done and the results is illustrated in figures 10, 11, and 12. Furthermore, additional experiments showed that the reconstructed image from a degraded magnitude is more efficient than the reconstructed image from the degraded phase. If we increase the iteration number the results still the same as it is shown in figures 13, 14, 15, 16, 17, and 18.

1 iteration

Noise intensity : $W_1(n,m) = 50 \times \text{rand}(n,m)$



Fig. 6. Original image



Fig. 7. Noisy image



Fig. 8. Reconstructed image from noisy magnitude

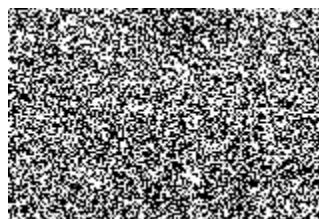


Fig. 9. Reconstructed image from noisy phase

1 iteration

Noise intensity: $W_2(n,m) = 2000 \times \text{rand}(n,m)$



Fig. 10.
Noisy image

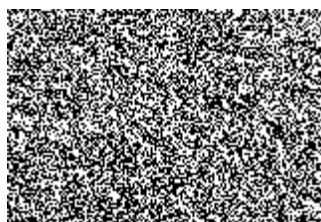


Fig. 11.
Reconstructed image from
noisy phase



Fig. 12.
Reconstructed image from
noisy magnitude

2 iterations

Noise intensity : $W_1(n, m) = 50 \times \text{rand}(n, m)$



Fig. 13.
Noisy image

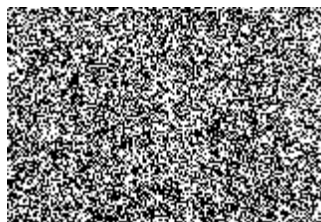


Fig. 14.
Reconstructed image from
noisy phase



Fig. 15.
Reconstructed image from
noisy magnitude

2 iterations

Noise intensity : $W_2(n, m) = 2000 \times \text{rand}(n, m)$



Fig. 16.
Noisy image



Fig. 17.
Reconstructed image from
noisy phase



Fig. 18.
Reconstructed image from
noisy magnitude

5. CONCLUSION

In this paper, we have studied the effects of phase degradation on the image reconstruction using the iterative signal reconstruction algorithm. The usefulness of magnitude only reconstruction in Fourier transform image coding was, then, considered as an application example that illustrates how the results of this paper may be used in practice. Our analysis suggests that reconstructing an image from the coded magnitude using the magnitude only reconstruction algorithm is considerably more efficient in the bit rate than reconstructing the image from the coded phase and magnitude.

Finally, to reduce the noise effect on image, a technique is developed to reconstruct the image from the degraded magnitude. This technique can significantly reduces signal degradation and may be used in several applications.

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