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Flow-induced vibration of a Y-shaped tube conveying fluid

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This work investigates out of plane vibration of a Y-shaped tube conveying fluid with clamed ends conditions. The mathematical model is based on the equation of motion of each tube coupled with matched boundary conditions at the junction of the three segments. The resulting equations are then resolved using Galerkin approach. The resulting eigen-values, eigen-function and shape modes are found numerically. A stability analysis of the solution is then performed. The effect of geometrical and flow parameters on the vibration of the Y-shaped tube conveying fluid is investigated. Results show that for small length of branching side compared to the supplying tube and for zero branching angle then the first three non-dimensional frequency is close to those of straight single tube with clamped-clamped conditions. Moreover, neutral stability regions were observed in first, second, and third modes for large range of dimensionless flow velocity. Results further demonstrate that an increase in dimensionless flow velocity results in decreasing of the non-dimensional frequency for the first three modes. Effect of branching angle and geometrical configuration on the mode shape and frequency is also investigated.

1. INTRODUCTION

The most common usage of pipes is to convey fluids. As such the study of the dynamics of tubes conveying fluid has many practical Engineering applications that include heat exchangers, automatic control, hydraulic lines, power lines, and aircraft lines. Moreover, the respiratory system in human bodies is regarded as an important application of such systems. On the other hand, the analysis of pipes conveying fluid has important theoretical application since they represent good models for gyroscopic systems weather linear of non-linear. In general, the existence of Coriolis forces in such systems categorize them as non-conservative, which results in the distortion of the mode, shapes. Furthermore, pipes conveying fluids are generally categorized as self-excited system and suffer from instability problems at certain conditions.

Consequently, such interesting problem was studied since the middle of the previous century. In fact, Ashly and Haviland [1] were the first to attempt to describe the vibration in the Trans Arabia pipelines. However, their formulation of the problem was erroneous as demonstrated by Housner [2] who derived the correct equation of motion for a tube conveying fluid and analyzed the case of simply supported tube at both ends. The cantilever case was first studied by Benjamin [3] as a limiting case of articulated pipes conveying fluids. Theoretically, Gregory & Paidoussis [4], investigated the same problem of Benjamin [3] while assuming that the gravity forces are inoperative. After that many outstanding researches on the subject was conducted some of them are listed below.

Paidoussis and Sundarajan [5], used two methods to analyze the parametric and combination resonance of pipes conveying fluids. Lundgren et al. [6] made theoretical and experimental study of self-induced non-parallel vibration of a flexible tube conveying fluids. They considered a tube fixed from one end while the flow is ejected at an angle from the free end. Paidoussis [7] reviewed the state of art of two classes of vibrating systems namely the vibration of cylindrical tubes induced by cross-flow an by axial flow while earlier Hunnoyer & Paidoussis [8] put forward the general theory for dynamics of slender non-uniform axi-symmetric beams subjected to internal of external flow or both simultaneously.

An important work was presented by Silva [9] when a variational formulation followed by Galerkin approximation is applied to a simply supported cantilever pipes. In this work lumped ad rotational inertia were introduced resulting in a substantial modification the magnitude of the frequency. Moreover, the influence of eccentricity is found to be of greater importance then that of mass for constant eccentricity factor. Also the work emphasize the need of ensuring research on coupled out-of-plane bending and twisting of the beam. Gorman [10] developed an analytical solution for the in-plane and out-of-plane vibration of the U-tube. He developed the required interface boundary conditions in details.

Nonlinear effects are quite important, Namchivaga [11] examined the nonlinear behavior of supported pipes conveying fluids in the vicinity of sub-harmonic resonance. The method of averaging is used to yield a set of autonomous equations when the parametric excitation frequency is twice the natural frequency of the system. Numerical procedure for the dynamical stability of pipes conveying fluid is introduced by Dang [12]. Furthermore, Aithai and Steven [13] investigated internal damping effect on the vibration of curved tubes. They proposed the model and investigated the stability of the system.

Going through all the previous researches there was no work on the vibration of Y-shaped tubes conveying fluid. This problem is of great practical importance to Engineering applications where the flow is branched into tow directions. Clearly this system is also quite important to medical field since it resembles the flow direction in the human respiratory system. The present work is based on Euler beam theory for out-of-plane vibration of such pipes. The global effect of structure-structure and structure-fluid interaction on vibration and instability is investigated.

2. PROBLEM FORMULATION

In this section the problem of out-of-plane vibration of a Y-shaped tube conveying fluid is formulated. The main goal is to study the effect of the branching angle, length ratio, fluid velocity, and mass ratio on the vibration characteristics of such tubes.

2.1. EQUATIONS OF MOTION

Figure 1 demonstrates the general system of a Y-shaped tube under consideration. The three ends are clamed to the corresponding walls as shown in the figure. For simplicity three coordinate systems are chosen coinciding with each of the three segments of the tube. These coordinates are illustrated in Figure 2 where $Y_1(X_1,t)$ describes the out of plane deflection of the first segment while $Y_{2,3}(X_{2,3},t)$ represent the out of plane deflection of the other two branched segments. Here X_1, X_2, and X_3 are along the three span segments in the direction of the flow; X_1 begins from the wall upstream of the junction and ends at the junction, while X_2 and X_3 begin from the junction and end at the clamped ends of each segment. In the present work the equation of motion of each segment is formulated separately and then the segment are coupled through the boundary conditions at the junction by insuring the continuity of displacement, slopes, bending moments, and shear forces.

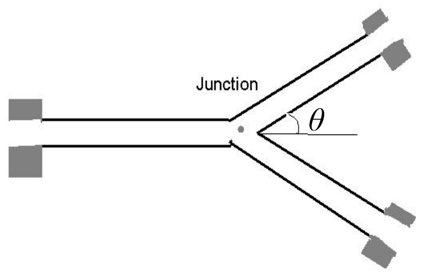

Figure 1. Y-shaped tube conveying fluid

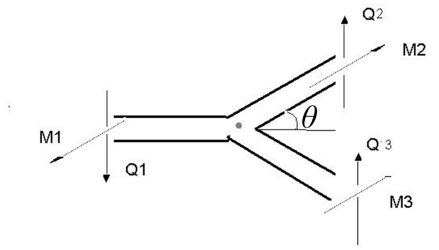

Figure 2. Bending Moments and Shear Forces near junction

Furthermore, the present analysis is based on the following assumptions: steady flow in the tube, small deflection, zero fluid acceleration, negligible damping, negligible effect of rotary inertia and transverse shearing, negligible pressure gradient, equal tubes cross sectional area, negligible fluid losses at junction, isotropic tube material, and finally inoperative gravity forces. Based on these assumptions the equation of motion of each straight segment of the tube can be written as

$$EI\frac{\partial^4 Y}{\partial X^4}+(m_f V^2+PA-\widehat{T})+2m_f V\frac{\partial^2 Y}{\partial X\partial t}+(m_f+m_p)\frac{\partial^2 Y}{\partial t^2}=0\,. \tag{1}$$

This equation represents the balance between elastic forces, centrifugal forces, Coriolis forces, and the inertia forces of the fluid and the tube.

By introducing the following non-dimensional parameters:

$$\eta_i=\frac{Y_i}{L}, \quad (2) \qquad \xi_i=\frac{X_i}{L}, \quad (3) \qquad U_i=\sqrt{\frac{m_f}{EI}}V_i L\,, \quad (4)$$

$$\gamma = \frac{PA}{EI} L^2 , \quad (5) \qquad \beta = \frac{m_f}{m_p + m_f} , \quad (6) \qquad \tau = \sqrt{\frac{EI}{m_p + m_f}} \frac{t}{L^2} , \quad (7)$$

$$\omega_i = \sqrt{\frac{EI}{m_p + m_f}} \Omega L^2 , \quad (8) \qquad \overline{T} = \frac{l_i \hat{T}}{EI} , \quad (9) \qquad RL_1 = \frac{l_1}{L} , \quad (10)$$

$$RL_2 = \frac{l_2}{L} , \quad (11)$$

Where l_1 is the length of the first signet and l_2 is the length of the second segment.

Noting that

$$l_1 + l_2 = L \quad (12)$$

and

$$RL_1 + RL_2 = 1.0 . \quad (13)$$

For similarly branched tubes

$$l_2 = l_3 .$$

The equation of motion in non-dimensional form for each segment then be comes:

$$\frac{\partial^4 \eta_i}{\partial \xi_i^4} + (U_i^2 + \gamma - \overline{T}) \frac{\partial^2 \eta_i}{\partial \xi_i^2} + 2\sqrt{\beta} U_i \frac{\partial^2 \eta_i}{\partial \xi_i \partial \tau} + \frac{\partial^2 \eta_i}{\partial \tau^2} = 0 . \quad (14)$$

Seeking the Galerkin solution for the PDE in the form

$$\eta_i(\xi_i, \tau) = \varphi_i(\xi_i) \exp(j\omega\tau) , \quad (15)$$

where the index i = (1, 2, 3) corresponds to the three segments, respectively, and $j = \sqrt{-1}$.

Substituting the Galerkin solution into equation (14), which represents the governing equation of each segment the following ordinary differential equation, is obtained

$$\frac{d^4 \varphi_i}{d\xi_i^4} + (U_i^2 + \gamma - \overline{T}) \frac{d^2 \varphi_i}{d\xi_i^2} + 2j\sqrt{\beta}\omega U_i \frac{d\varphi_i}{d\xi_i} - \omega^2 \varphi_i = 0 , \quad (16)$$

where ω is the dimensionless frequency of the tube related to the circular frequency of motion as mentioned in equation (8).

Equation (16) represent three fourth-order ordinary differential equation with constant coefficient governing the motion of each segment of the pipe. These equations are coupled at the boundary conditions as it is explained later. However, the form of the solution can be discussed at an early stage even before discussing the boundaries. Actually the general solution to equations (16) is in the form

$$\varphi_1 = \sum_{k=1}^{4} c_k \exp(j\alpha_{1,k}\xi_1) , \quad (17)$$

$$\varphi_2 = \sum_{k=5}^{8} c_k \exp(j\alpha_{2,k}\xi_1) , \quad (18)$$

$$\varphi_3 = \sum_{k=9}^{12} c_k \exp(j\alpha_{3,k}\xi_1)\,, \tag{19}$$

where the k index corresponds to the number of roots of each equation (noting that each of equations (16) has four roots).

Substituting equations (17)–(19) in equation (16) results in the following characteristics equation(s):

$$\alpha_{i,k}^4 - (U_i^2 + \gamma - \bar{T})\alpha_{i,k}^2 - 2\sqrt{\beta}U_i\omega\alpha_{i,k} - \omega^2 = 0\,. \tag{20}$$

In fact equation (20) represent the root of three equations; for $i = 1$, $k = 1,\ldots,4$, for $i = 2$ $k = 5,\ldots,8$, and for $i = 3$, $k = 9,\ldots,12$. To simplify the problem further one can consider the case where the pressure and the tension are negligible since they are not the focus of the present discussion. Furthermore, noting that for our case $U_1 = 2$, $U_2 = U$, $U_2 = U_3 = U/2$ then equation(s) (20) can be written for the main branch as

$$\alpha_{1,k}^4 - (U^2)\alpha_{1,k}^2 - 2\sqrt{\beta}U\ \omega\alpha_{1,k} - \omega^2 = 0\,. \tag{21}$$

While for the two other branches

$$\alpha_{2,k}^4 - (U^2)\alpha_{2,k}^2/4 - 2\sqrt{\beta}U\ \omega\alpha_{2,k} - \omega^2 = 0\,, \tag{22}$$

$$\alpha_{3,k}^4 - (U^2)\alpha_{3,k}^2/4 - 2\sqrt{\beta}U\ \omega\alpha_{3,k} - \omega^2 = 0\,. \tag{23}$$

Since equation (23) is the same as that of (22) there are actually eight distinct values of α. In general the frequency is a function of U and β. Through the coupling at the junction boundary conditions and by specifying the problem's configuration the characteristics of the Y-shaped tube vibration can then be determined as it is demonstrated below.

2.2. BOUNDARY CONDITIONS

The complete solution of the system requires 12 boundary conditions to be satisfied. For clapped ends case the following boundary conditions are to be imposed at the three walls to fulfill the requirements of the continuity of displacements slopes, bending moments and shear forces at the rigid junctions and their clamped ends. For zero slopes and deflection at the ends the following boundary conditions are used:

$$Y_1(0) = 0\,, \tag{24}$$

$$Y_1'(0) = 0\,, \tag{25}$$

$$Y_2(l_2) = 0\,, \tag{26}$$

$$Y_2'(l_2) = 0\,, \tag{27}$$

$$Y_3(l_2) = 0\,, \tag{28}$$

$$Y_3'(l_2) = 0\,. \tag{29}$$

Nonetheless, at the junction the following boundary conditions can be utilized to as shown in figure 2.

The continuity of lateral deflection

$$Y_1(l_1) = Y_2(0), \tag{30}$$

$$Y_1(l_1) = Y_3(0). \tag{31}$$

To insure continuity of slopes projection between the three segments

$$Y_1'(l_1) - Y_2'(0)\cos(\theta) = 0 \tag{32}$$

and

$$Y_1'(l_1) - Y_3'(0)\cos(\theta) = 0. \tag{33}$$

To balance the bending moment

$$M_1(l_1) = \left(M_2(0) + M_3(0)\right)\cos(\theta) \text{ or } Y_1''(l_1) = \left(Y_2''(0) + Y_3''(0)\right)\cos(\theta). \tag{34}$$

To balance the shear forces

$$Q_1(l_1) = \left(Q_2(0) + Q_3(0)\right) \text{ or } Y_1'''(l_1) = \left(Y_2'''(0) + Y_3'''(0)\right)\cos(\theta). \tag{35}$$

Substituting the dimensionless for of Y_i and taking into consideration equations (15) and (17)–(19) the boundary conditions (24)–(35) can be put in the following form:

Equations (36)–(47) represent a system of 12 linear equations that can be put in the form:

$$\sum_{i=1}^{4} c_i = 0, \tag{36}$$

$$\sum_{i=1}^{4} \alpha_{1,i} c_i = 0, \tag{37}$$

$$\sum_{i=5}^{8} c_i \exp(jRL_2\alpha_{2i}) = 0, \tag{38}$$

$$\sum_{i=5}^{8} c_i\alpha_{2,i} \exp(jRL_2\alpha_{2i}) = 0, \tag{39}$$

$$\sum_{i=9}^{12} c_i \exp(jRL_2\alpha_{2i}) = 0, \tag{40}$$

$$\sum_{i=9}^{12} c_i\alpha_{2,i} \exp(jRL_2\alpha_{2i}) = 0, \tag{41}$$

$$\sum_{i=1}^{4} c_i \exp(jRL_1\alpha_{1i}) = \sum_{i=5}^{8} c_i, \tag{42}$$

$$\sum_{i=1}^{4} c_i \exp(jRL_1\alpha_{1i}) = \sum_{i=9}^{12} c_i, \tag{43}$$

$$\sum_{i=1}^{4} c_i\alpha_{1,i} \exp(jRL_1\alpha_{1i}) = \sum_{i=5}^{8} \alpha_{1,i} c_i \cos(\theta), \tag{44}$$

$$\sum_{i=1}^{4} c_i\alpha_{1,i} \exp(jRL_1\alpha_{1i}) = \sum_{i=9}^{12} \alpha_{1,i} c_i \cos(\theta), \tag{45}$$

$$\sum_{i=1}^{4} c_i \alpha_{1,i}^2 \exp(jRL_1\alpha_{1i}) = \left(\sum_{i=5}^{8} \alpha_{2,i}^2 c_i + \sum_{i=9}^{12} \alpha_{2,i}^2 c_i \right) \cos(\theta), \tag{46}$$

$$\sum_{i=1}^{4} c_i \alpha_{1,i}^3 \exp(jRL_1\alpha_{1i}) = \sum_{i=5}^{8} \alpha_{2,i}^3 c_i + \sum_{i=9}^{12} \alpha_{2,i}^3 c_i , \tag{47}$$

$$[D] \cdot [c_i] = 0 , \tag{48}$$

where the matrix $[D]$ is 12×12 complex matrix.

For equation (48) to have a non-trivial solution then

$$Det[D] = f_1(\alpha_{1,i}, \alpha_{2,i}, RL_2, \theta) = 0 . \tag{49}$$

Equation (49) is coupled with equations (21) and (22) to from the dispersion relation for the system. The system is then resolved numerically as explained below.

3. NUMERICAL PROCEDURE

Equations (21), (22), and (49) form a set of three nonlinear complex algebraic equations which are resolved numerically to find the values of ω, α_{1i}, and α_{2i} for certain values of U, β, θ, and RL_2. A numerical iteration scheme is set to conduct such calculation by first assuming a certain value of ω to find the four corresponding roots of α_{1i}, and α_{2i} from equations (21) and (22) using Bairstow's method [14]. These roots are two real pair and tow complex conjugate pairs. The error in the polynomials is found to be in the order of 10^{-15}. The resulting values of α_{1i}, and α_{2i} are then substituted in equation (49) where an LU factorization scheme is utilized to fined the determinants of $[D]$ via the IMSL subroutine DLFDCG [15]. A sweeping search is then conducting bay repeating the process until the sign of the determinant of $[D]$ changes after-which a bisection method is used to converge for the desired accuracy of ω. Once ω is found then the Eigen Functions can be calculated from equations (48) utilizing the IMSL subroutine DLSACG [15]. The entire procedure is the repeated for any values of U, β, θ, and RL_2.

4. RESULTS AND DISCUSSION

Since there are no previous results to compare with the present calculations are checked by comparing it to the results of a clamped-clamped straight tube case. For the Y-shaped tube to be similar to the straight tube case θ should be zero and RL_2 should be as small as possible. Nonetheless, RL_2 cannot be too small for the Euler beam theory to be valid. As such, RL_2 is chosen to be 0.1, β to be 0.2 and the resulting calculation of the non-dimensional frequency is demonstrated in figures 3–4. The effect of the dimensionless flow velocity on the first-mode frequency is shown in figure 3 for zero θ and is compared to the straight tube case. The comparison is quite acceptable taking into consideration that the slight increase in ω is due to the slight increase in the stiffness due to the branching at $RL_1 = 0.9$. Similar argument can be said about the second and the third modes of vibration as shown in figures 4 and 5, respectively. For large enough values of U neutral stability points are reached as shown in figures 3–5.

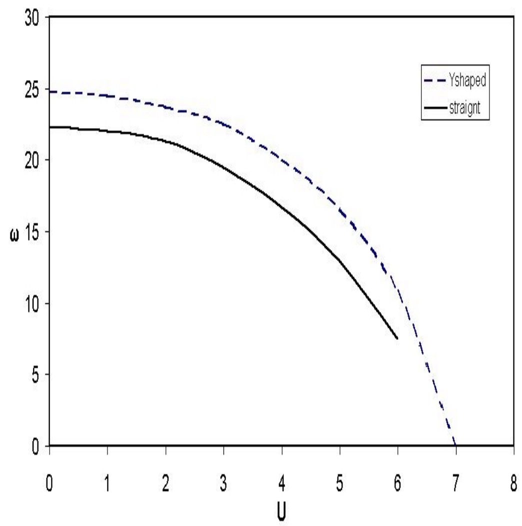

Figure 3. Comparison to straight pipe.
First mode, $RL_2 = 0.2$, $\theta = 0$

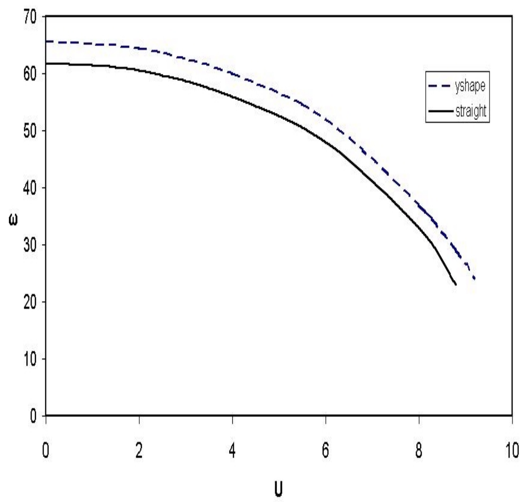

Figure 4. Comparison to straight pipe.
Second mode, $RL_2 = 0.2$, $\theta = 0$

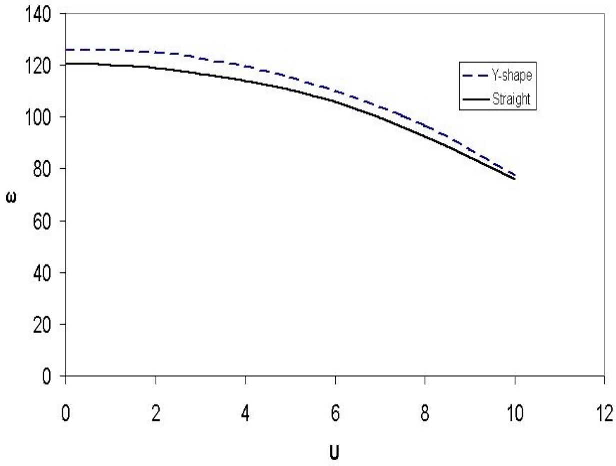

Figure 5. Comparison to straight pipe.
Third mode, $RL_2 = 0.2$, $\theta = 0$

For no-flow conditions figure 6 shows the influence of RL_2 on first mode frequency for three values of θ and no flow conditions. It is clear that ω has a maximal around $RL_2 = 0.2$ and a minimal when $RL_2 = 0.8$. Furthermore, the effect of θ is coupled with the effect of RL_2 as shown in the figure. On the other hand. Figure 7 demonstrate the effect of branching angles on ω for various RL_2. It is clear that θ has negligible effect in the range between 0 and 1 radiance (57^o). However, θ has stiffening effect for certain values of RL_2 while it has a softening effect for other values of RL_2. Consequently the vibration frequency increases or decreases accordingly.

The effect of flow velocity on the lowest three modes when $\theta = 30°$ is demonstrated in figure 8. The branching angle actually increase the critical flow velocity as demonstrated by comparing figure 7 to figures 3, 4, and 5. By carefully examining figure 8 one can conclude that the Coriolis effects are negligible for the range of flow velocities demonstrated. The effect of U on ω for various mass ratios (β) are demonstrated figure 8 also when $\theta = 30°$. It is clear that as the mass ratio increases so the frequency and also the critical flow velocity. This can be explained by noting that while U acts as centrifugal force opposing the stiffness restoring forces, increasing β would result in changing the sign of Coriolis force along the span of the vibrating tube which acts as a counter effect against the centrifugal forces, thus increasing the system frequency. It should be noted that similar behavior is also found in straight pipes.

Finally, the effect of flow velocity on the mode shapes is investigated. Figure 9 demonstrate the effect of flow velocity on the second mode shape for $\beta = 0.2$ for the no flow condition and when $U = 2$. It is clear that the effect of U on distorting the mode shape is not considerable since neutral stability is maintained hence no energy exchange between the fluid and the tube exists. Similar things can be said about other modes of vibration. Figure 10 shows the variation of β on ω at $\theta = 0$ to compare with straight pipes. Furthermore, Fig. 11 shows the lowest three mode shapes.

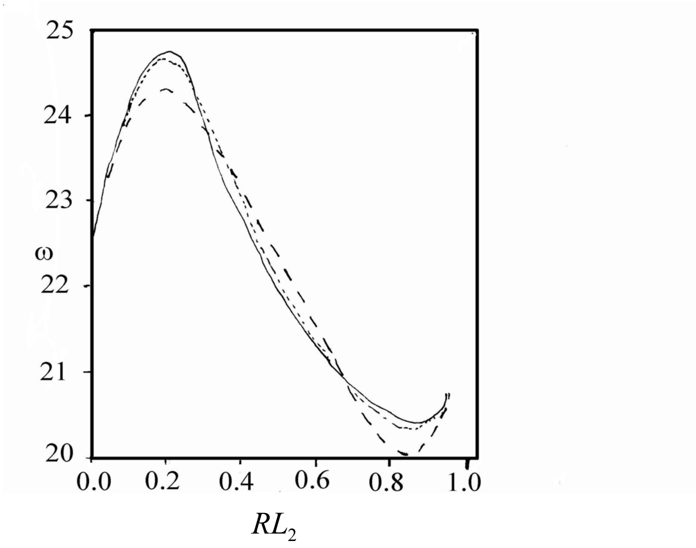

Figure 6. Effect length ratio on non dimensional frequency

—— $\theta = 30°$

… $\theta = 45°$

– – – $\theta = 60°$

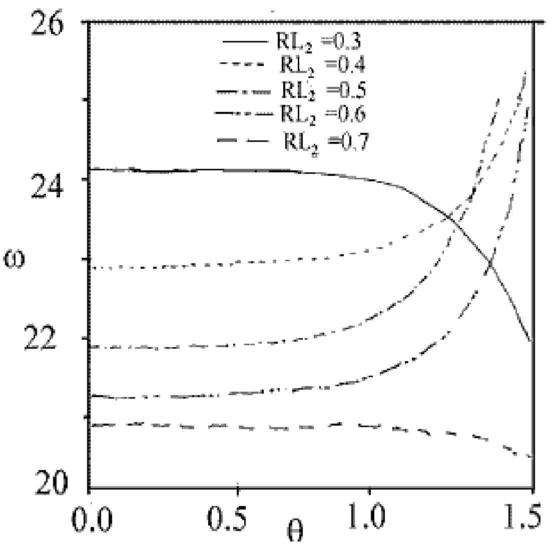

Figure **7**. Effect of the branching angle on non-dimensional frequency of the first modes

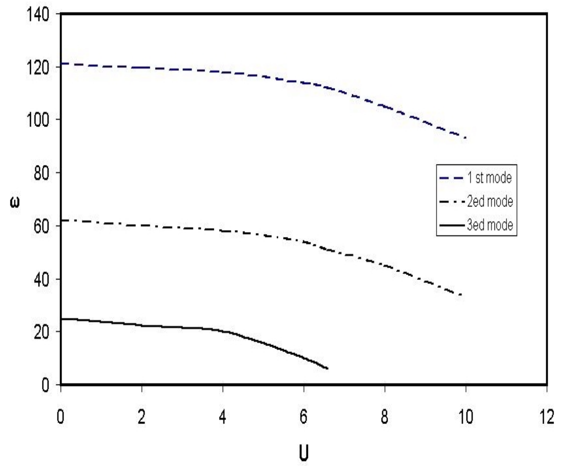

Figure 8. Effect of non-dimensional flow velocity on non-dimensional frequency of the lowest three modes.

$RL_2 = 0.4$, $\theta = 30°$

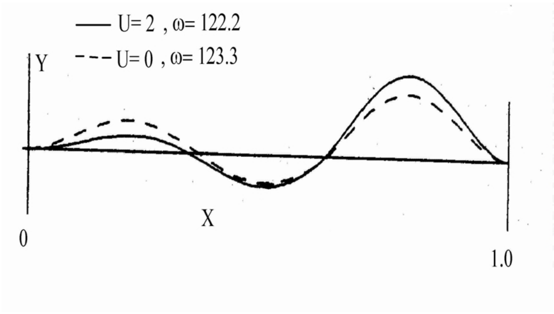

Figure 9. Effect of non-dimensional flow velocity on third mode shape

$\beta = 0.4$, $RL_2 = 0.4$, $\theta = 30°$

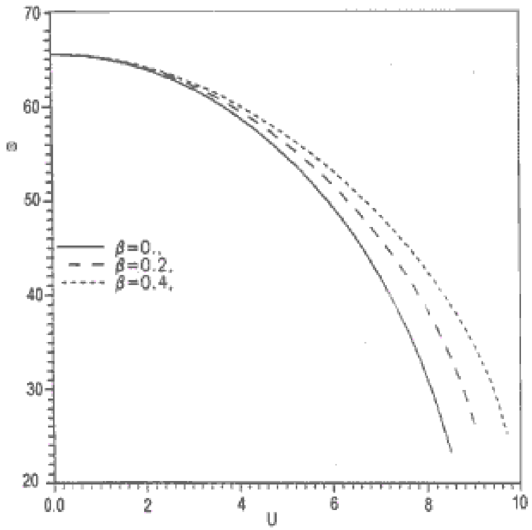

Figure 10. Effect of U on ω for various β when $\theta = 0°$ and $RL_2 = 0.1$ (second mode)

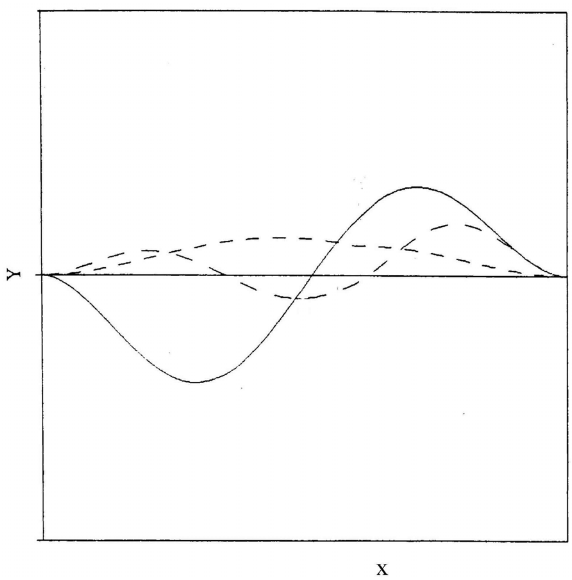

Figure 11. The lowest mode shapes for no flow conditions, $RL_2 = 0.4$, $\theta = 30°$

- - - - - *first mode*

– – – – *second mode*

——— *third mode*

NOMENCLATURE

A	– cross-sectional area of tube	T	– axial tension in the tube
L	– total length of tube including branch	X	– distance along tube
E	– modulus of elasticity of tube material	Y	– transverse deflection of the tube from equilibrium
I	– moment of inertia	V	– constant velocity of fluid in tube
M	– momentum	m_f	– mass per unit length of fluid
P	– fluid pressure in tube	m_p	– mass per unit length of tube
Q	– shear forces in the external surface	t	– time

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