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Structural energy flows in water filled pipes: implementation of measurement technique based on PVDF reusable strain gauges*

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The purpose of this work is an implementation the newly developed measurement technique for structural energy estimation in-situ. The technique based on reusable PVDF strain gauges in measurement of structural energy flows transferred by longitudinal and shear forces, bending and torsional moments in water filled pipes are presented and discussed. Developed technique was implemented for determination of the efficiency of noise control means in the pipeline with the test rig «Flow». The technique also applied in order to determine the components of vibrational energy flows generated by pump incorporated into the rig «Impedance». Measurements were performed when real pumps have been run. The relationships between four structural components in pipe walls and water ones transferred with plane waves in the pipe channel were determined. Degree of uncertainty in proposed technique was revealed using error analyses taking into account the real data about coherence function and measured phase angle between force/moment and linear/angular velocity. The reliability of preformed measurements at tonal components of structural energy spectra proofed employing an error analyses and energy balance calculation in pipe cross section. The possibilities and limitations of the technique were addressed.

1. INTRODUCTION

Experimental determination of vibrational energy flows is a powerful tool in the modern applied acoustics. All energy transfer paths are forming the total vibrational or sound field in transportation (car, ship aircraft) compartments and therefore the lack of data about just single energy transfer channel makes the situation uncertain. User should has at hand entire information concerning all transfer paths such as machine resilient supports, pipeline hangers and pipe walls, air medium and plane waves propagation in liquid filled pipes. The appropriate experimental technique should be applied at each individual case. Energy flow approach opens a number of additional options for designer during development of noise control strategy. Having at hand data about energy flows it is possible to judge in straightforward manner, which transfer path dominants when vibrational energy transmits from machine (source) to foundation (receiving structure).

J. Verheij [1], R. Pinnington and R. White [2] and others proposed the technique for estimation of energy flows propagated via machinery supports and pipe hangers. Acoustic

*This paper had no linguistic support from English speaking expert. Editorial staff is not responsible for lingual mistakes in the article power radiated in the form of air borne noise can be also measured by means of commercially available measurement equipment and appropriate software. Double or triple hydrophone methods which are resemble to Fahy [3] and Chung [4] technique for air borne noise can be also successfully applied in measurement of energy component propagated throughout the liquid channel of energy transmission in water filled pipes.

Methods for measurement of energy flow in pipe wall structure were proposed by J. Verheij [1] and G. Pavic [5]. The methods were based on conventional transducers array (accelerometers) and finite difference technique. The methods could be implemented in specific cases when structure (beam or rod) straight enough because the measurement of longitudinal component of energy flow requires a long measurement distance due to high speed of longitudinal waves. A measurement of bending wave component using 4-point technique can be performed in narrow frequency range due to its high sensitivity to phase mismatch between measuring chains. The chain implies here a measurement channel including transducer, preamplifier and FFT analyzer. The two or three point technique are disregarded the effect of evanescent waves whose contribution could add 3 or 6 dB into the total measurement uncertainty.

Developing a new generation of electromechanical transducer — reusable PVDF strain gauges — reveals the number of advantages for measuring the vibrational energy flows, vibration induced strains, forces and moments in pipe and shaft structures by measuring surface strain. The PVDF strain gauges produce the electrical charge which is directly proportional to dynamic strain and therefore it is possible to use them jointly with traditional measurement equipment, namely, charge preamplifiers and modern FFT analyzer if their channels are equipped according to ICP standard. The PVDF strain gauges also make possible the measurement of extremely small magnitudes of strain about 10^{-5} με (μm/m according to denotation accepted by Russian standard). In fact it can be explained that PVDF film more elastic than wire and quartz. Therefore the film is able to produce more powerful signal. Despite lower piezoelectric modulii than ceramic and quartz the cross coupling mechanic to electric efficiency of PVDF film is more than traditional peizoelectrically active material. This phenomenon makes film very useful in designing the transducers, actuators and other practical applications.

The reusable PVDF strain gauges allow to measure strain and force directly which make possible to reduce the order of derivative approximated by FD and give an opportunity to alleviate drawbacks of parametric methods with conventional wire strain gauges. The latter become poor for measurement of small dynamic (vibrational) strain. The advantage of reusable PVDF strain gauges is their high sensitivity to strain and possibility to calibrate and use them after having at hand the sensitivity vs. frequency as for usual vibration transducer. The formulae for measurement of individual components in total energy flow become simpler in comparison with traditional FD approach. This circumstance gives an additional advantage for overcoming the specific error inherent to FD technique for example narrow frequency range and long measurement distance. All techniques discussed earlier (see for example [6] and [7]) were based on single used PVDF gauges and they had implemented in so-called «academic structures» – free-free beams and beams with anechoic ends. This paper contains experimental results and their error analyses obtained with real pipeline configurations and running pumps. This configuration allows revealing the advantages and drawbacks of proposed technique.

2. METHODS FORMULATION AND TRANSDUCER ARRAYS

Measurement technique for vibrational energy propagated via pipe wall structure based on mathematics model, which describes a structure oscillations by use of Euler-Bernoully model. In other words the beam layer become plane and parallel each other due to shear force action (there is no shear strain dependence on beam thickness and rotational inertia of beam element also negligible). The used symbols and notations are given in Appendix A.

Error of the technique at upper cut-off frequency due to implementation of above model can be calculated by formula (1) obtained by Nikolay A. Kuznetzov (Krylov Institute).

$$\delta = \left| 1 - \left\{ \left[\pi^2 f_{high}{}^2 \frac{\rho\, I_{bend}}{ES} \left(1 - \frac{E}{\phi G}\right)^2 + 1 \right]^{\frac{1}{2}} + \pi\, f_{high} \sqrt{\frac{\rho\, I_{bend}}{ES}} \left(1 + \frac{E}{\phi G}\right) \right\}^{\frac{1}{2}} \right|, \quad (1)$$

where $\phi = \dfrac{6(1+\nu)(1+n^2)^2}{(7+6\nu)(1+n^2)^2 + (20+12\nu)n^2}$ – form quotient for annular cross-section of the pipe;

$n = \dfrac{D_{inner}}{D_{outer}}$ – diameter ratio; ν – Poisson ratio of pipe material.

Actually, the equation (1) obtained as a difference between bending wave number derived according to rigorous solution for equation of beam motion taking into account rotational inertia and thickness shearing of cross section and Euler-Bernoully's model.

The plots of the error due to implementation of the theory as a function of pipe geometry and frequency are shown in Fig. 1.

Another assumption is that the measurement part of the pipe should be straight and uniform. The interaction between liquid medium and pipe structure supposed to be negligible employing the mathematics model in measurement technique. Expression (1) demonstrates that technique based on PVDF application has also high frequency limitation due to model involved in the technique.

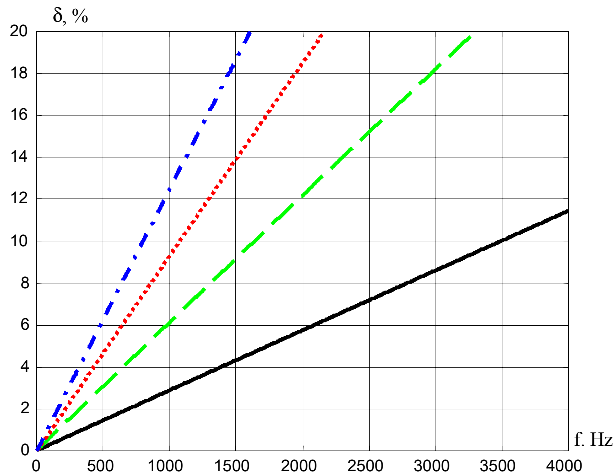

Fig. 1. Error due to technical model vs. frequency and parameters of pipe cross section
(—) – inner diameter 50 mm, wall thickness 5 mm; (- - - -) – inner diameter 100 mm, wall thickness 5 mm; (……) – inner diameter 150 mm, wall thickness 5 mm; (-.-.-.) – inner diameter 200 mm, wall thickness 5 mm

Forces and moments acting in a beam cross section can be expressed by well-known equations:

$$F_x = E\,S\frac{\partial \xi_x}{\partial x};\tag{2}$$

$$M_\varphi = G\,J_{tors}\frac{\partial \varphi}{\partial x};\tag{3}$$

$$F_z = -\frac{\partial M_\psi}{\partial x};\tag{4}$$

$$M_\psi = -E\,I_{bend}\frac{\partial^2 \xi_z}{\partial x^2}.\tag{5}$$

The partial derivatives in the equations (2)–(5) can be extracted using data about surface strains $\varepsilon_{1x}(f)$, $\varepsilon_{2x}(f)$, $\varepsilon_{3x}(f)$, $\varepsilon_{4x}(f)$ and displacements of beam structure $\xi_{1y}(f)$, $\xi_{2y}(f)$, $\xi_{3y}(f)$, $\xi_{4y}(f)$:

$$F_x(f) = E\,S\left(\frac{\varepsilon_{1x}(f)+\varepsilon_{2x}(f)}{2}\right);\tag{6}$$

$$M_\psi(f) = -\frac{E\,I_{bend}}{D_{outer}}\left(\varepsilon_{1x}(f)-\varepsilon_{2x}(f)\right);\tag{7}$$

$$F_z(f) = -\frac{E\,I_{bend}}{\Delta_{bend}D_{outer}}\left(\varepsilon_{1x}(f)-\varepsilon_{2x}(f)-\varepsilon_{3x}(f)+\varepsilon_{4x}(f)\right);\tag{8}$$

$$M_\varphi(f) = \frac{G_{tors}\,J_{tors}}{\Delta_{tors}\,D_{outer}}\left(\xi_{1y}(f)-\xi_{2y}(f)-\xi_{3y}(f)+\xi_{4y}(f)\right).\tag{9}$$

For linear and rotational vibration velocities the formulae are:

$$v_x(f) = \frac{\ddot{\xi}_{1x}(f)+\ddot{\xi}_{2x}(f)}{2\,j\omega};\tag{10}$$

$$v_z(f) = \frac{\ddot{\xi}_{1z}(f)}{j\omega};\tag{11}$$

$$\psi(f) = \frac{\ddot{\xi}_{1x}(f)-\ddot{\xi}_{2x}(f)}{D_{outer}};\tag{12}$$

$$\dot{\varphi}_1(f) = \frac{\ddot{\xi}_{1y}(f)-\ddot{\xi}_{2y}(f)}{j\omega\,D_{outer}};\ \dot{\varphi}_2(f) = \frac{\ddot{\xi}_{3y}(f)-\ddot{\xi}_{4y}(f)}{j\omega\,D_{outer}};\tag{13}$$

$$\frac{\partial \dot{\varphi}}{\partial x} \approx \frac{\dot{\varphi}_1-\dot{\varphi}_2}{\Delta_{tors}};\ \frac{\partial \varphi}{\partial x} = \frac{\left(\xi_{1y}(f)-\xi_{2y}(f)-\xi_{3y}(f)+\xi_{4y}(f)\right)}{\Delta_{tors}D_{outer}}.\tag{14}$$

Arrangements of the strain gauges for measurement of the strain $\varepsilon_{1x}(f)$, $\varepsilon_{2x}(f)$, $\varepsilon_{3x}(f)$, $\varepsilon_{4x}(f)$ and accelerometers with double integration their signals for measuring the displacements $\xi_{1y}(f)$, $\xi_{2y}(f)$, $\xi_{3y}(f)$, $\xi_{4y}(f)$ are shown from Fig. 2 to Fig. 4 including block diagrams of measurement set-ups.

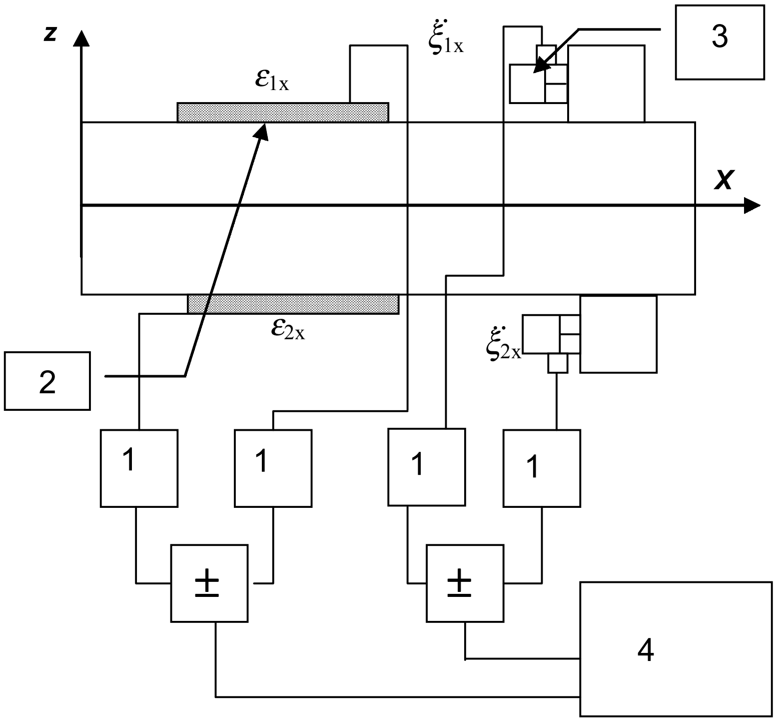

Fig. 2. Arrangement of PVDF gauges at pipeline for measurement of flow components due to longitudinal force and bending moment

1 – charge preamplifiers; 2 – PVDF strain gauges; 3 – accelerometers; 4 – FFT analyzer

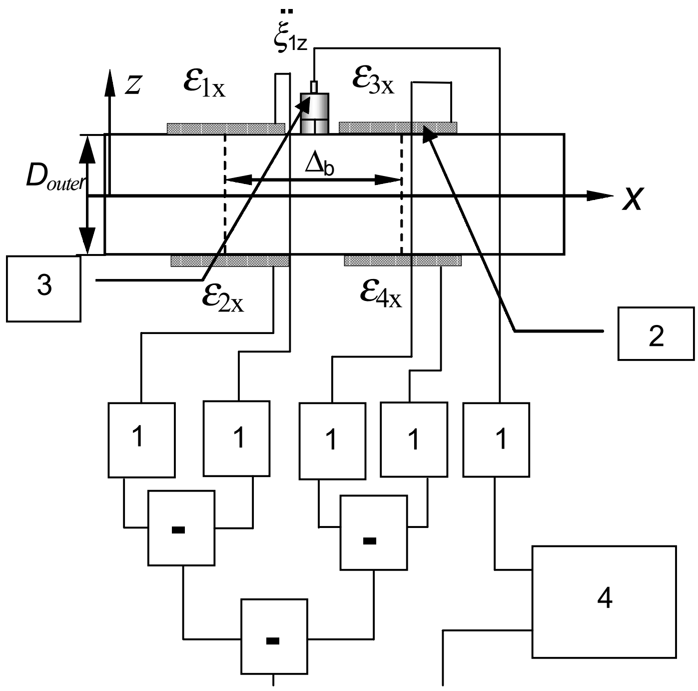

Fig. 3. Arrangement of PVDF gauges at pipeline for measurement of flow components due to shear force

1 – charge preamplifiers; 2 – PVDF strain gauges; 3 – accelerometers; 4 – FFT analyzer

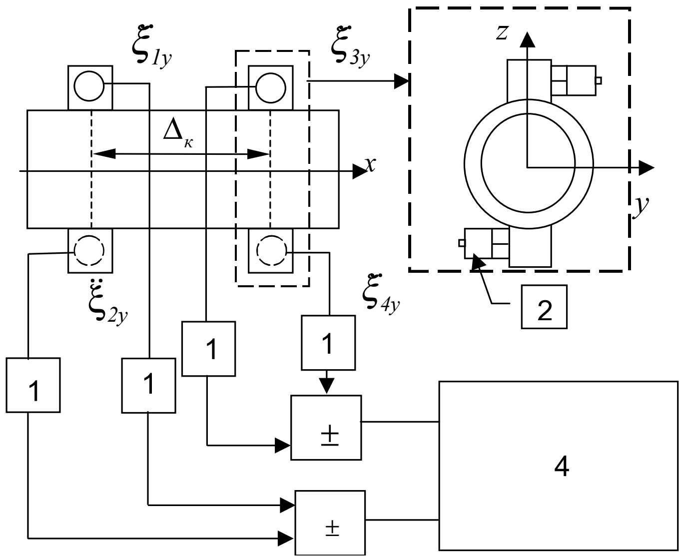

Fig. 4. Arrangement of accelerometers at pipeline for measurement of flow components due to torsional moment (Verheij's method)

1 – charge preamplifiers; 2 – accelerometers; 3 – active summation and subtracting device; 4 – FFT analyzer

Expressions for determination of the vibrational energy radiated due to harmonic forces and moments are:

$$W_{F_x}(f) = \frac{1}{2}\mathrm{Re}\left\{F_x(f)\cdot v_x^{\,*}(f)\right\}; \tag{15}$$

$$W_{F_z}(f) = \frac{1}{2}\mathrm{Re}\left\{F_z(f)\cdot v_z^{*}(f)\right\}; \tag{16}$$

$$W_{M_\psi}(f) = \frac{1}{2}\mathrm{Re}\left\{M_\psi(f)\cdot \dot{\psi}^{*}(f)\right\}; \tag{17}$$

$$W_{bending}(f) = W_{F_z}(f) + W_{M_\psi}(f); \tag{18}$$

$$W_{M_\varphi}(f) = \frac{1}{2}\mathrm{Re}\left\{M_\varphi(f)\cdot \dot{\varphi}^{*}(f)\right\}. \tag{19}$$

Substituting the forces/moments and linear/angular velocities to Eq. (15)–(19) and applying Fourier transform it is possible to derive the formulae for determination of the energy flows in terms of cross spectra as it given further on:

For component of longitudinal force

$$W_{Q_x}(f) = \frac{E\,S}{4\omega}\left\{\mathrm{Im}\,G_{\Delta\varepsilon_x\ddot{\xi}_x}(f)\right\}, \tag{20}$$

where $\Delta\varepsilon_x = \varepsilon_{1x} + \varepsilon_{2x}$ and $\xi_x = \xi_{1x} + \xi_{2x}$.

For component of bending moment

$$W_{M_\psi}(f) = \frac{E\,I_{bend}}{D_{outer}^2\,\omega}\left\{\mathrm{Im}\,G_{\Delta\varepsilon_\psi\,\ddot{\psi}}(f)\right\}, \tag{21}$$

where $\Delta\varepsilon_\psi = \varepsilon_{1x} - \varepsilon_{2x}$ and $\ddot{\psi} = \ddot{\xi}_{1x} - \ddot{\xi}_{2x}$.

There are two advantages of technique employing the equation (9) and (10). One of them an opportunity to make measurement both longitudinal and bending moment components simultaneously. Another one the lack of lower and upper frequency limitation due to finite difference method because of direct measurement of longitudinal force and bending moment.

For shear force component

$$W_{Q_z}(f)=\frac{E\,I_{bend}}{D_{outer}\;\Delta_{bend}\;\omega}\left\{\mathrm{Im}\,G_{\Delta\varepsilon_z\,\ddot{\xi}_z}(f)\right\}, \qquad (22)$$

where $\Delta\varepsilon_z=\varepsilon_{1x}-\varepsilon_{2x}-\varepsilon_{3x}+\varepsilon_{4x}$.

For component of torsional moment

$$W_{M_\varphi}(f)=\frac{G\,J_{tor}}{D^2_{outer}\;\Delta_{tor}\;\omega^3}\mathrm{Im}\,G_{\Sigma_1\Sigma_2}(f), \qquad (23)$$

where $\Sigma_1=\ddot{\xi}_{1y}+\ddot{\xi}_{1y}$ – instantaneous sum of vibrational acceleration from opposite positioned accelerometers at the points 1 and 2, $\Sigma_2=\xi_{3y}+\xi_{3y}$ – instantaneous sum of vibrational acceleration from opposite positioned accelerometers at the points 3 and 4, respectively.

The analog devices have to be used for measurement performing with PVDF strain gauges and accelerometers. The devices provide more precise result vs. post processing of averaged spectral functions as it shown in [8]. The analog device for simultaneous summing and subtracting of time domain signal was designed and tested by author at Chalmers University of Technology [9]. The spectra of vibrational and hydrodynamic noise generated by pumps comprise powerful tonal components which usually concentrated in frequency band from 5 to 400 Hz with 90% of their energy. Therefore a frequency analysis of vibrational power is limited here by upper frequency 400 Hz in this particular case. Such upper frequency limit has to be chosen also due to limitation of Euler-Bernoully model at high frequencies.

3. ENERGY FLOWS ALONG RIG PIPELINES

3.1 MEASUREMENT OF ENERGY FLOWS WITH «FLOW» RIG

The purpose of this experiment to determine an efficiency of applied noise control means for pipelines using parameter of vibrational energy. A scheme of the measurement part on the rig is depicted in Fig. 5.

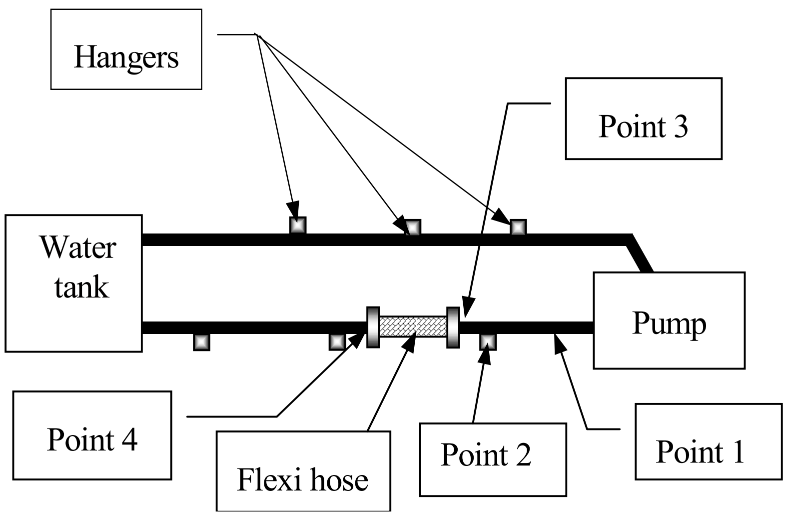

Fig. 5. Arrangement of measurement point for monitoring of structural energy flows

Four cross sections of the pipeline were measured during experiments with the running pump. Vibrational energy transferred by plane waves in water medium was also measured using Fahy's technique with two hydrophones inserted into water feeding and intake pipes [3].

The measurement results are shown from Fig. 6 to Fig. 8. Energy was measured at point 1, 3 and 4. The energy emitted into the resilient mount at point 2 was estimated using complex transmissibility technique [2] taking into account all six degree of freedom, which means that rotational stiffness values of isolator were measured and substituted into equations of energy estimation. The energy balance was calculated using equation (24) and final result is shown in Fig 7.

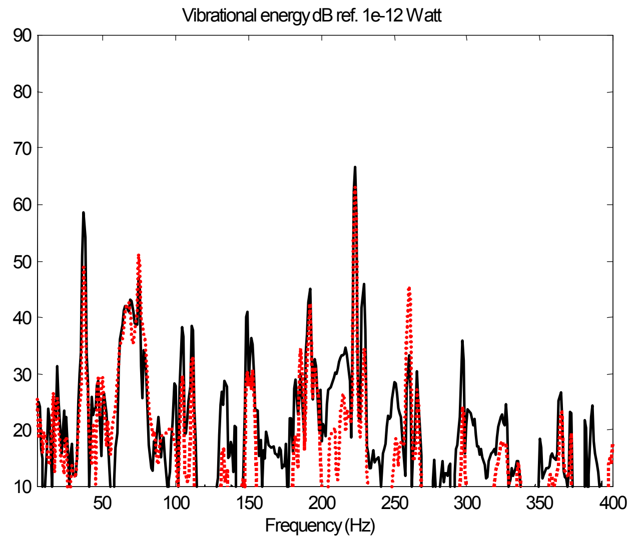

Fig. 6. Energy balance in pipe cross section Eq. (24) (---) – energy input (----) – energy transferred

Here we assumed that water borne energy losses is negligible (there were no any special means applied) thus only structural components were considered.

$$W_{long}^{input} + W_{bend}^{input} + W_{tors}^{input} \geq W_{long}^{transt} + W_{bend}^{transt} + W_{tors}^{transt} + W_{hanger}^{total}, \qquad (24)$$

We can see that inequality (24) becomes true over all frequency range however it is impossible to get the exact energy balance due to transformation of structural component of the energy to the air-borne noise and bending waves with polarization out-of-plane. The air borne noise and bending waves in another plane were not monitored during this experiment.

The apparent violation of energy conservation law at frequencies about of 70 and 260 Hz (see Fig. 6) is due to the back flow from water tank. Hanger substantially reduces the forward waves and flow due to the injection branch of the pipe becomes dominant.

As an example how to implement the proposed technique for estimating the efficiency of structural vibration reduction by flexible hanger and hose in energy terms is depicted in Fig. 7 and Fig. 8. Figures 7 and 8 show that hanger more efficient here comparing to the flexible hose.

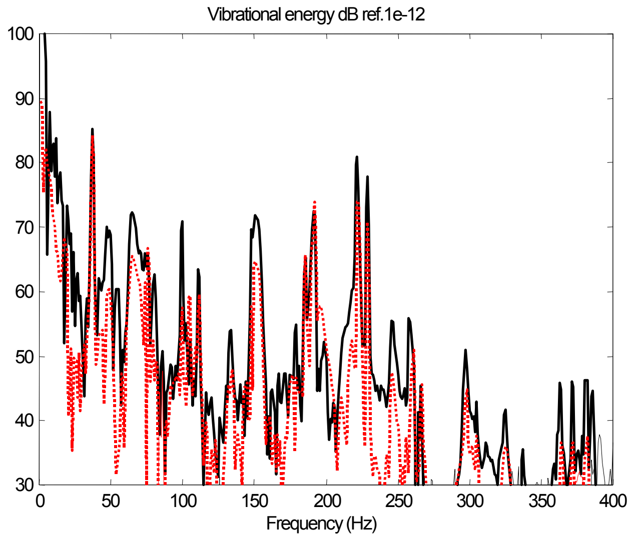

Fig. 7. Structural energy reduction by flexible hose,

(—) – power input, (- - -) – power transfer

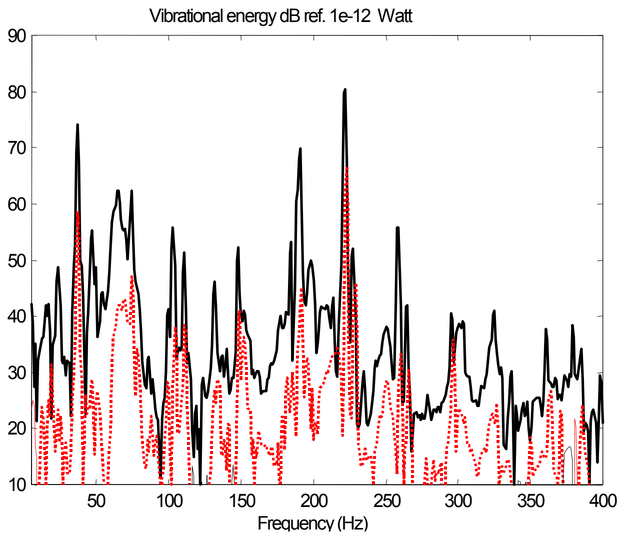

Fig. 8. Structural energy reduction by hanger, *(—) – power input, (- - -) – power transfer*

3.2 MEASUREMENT OF ENERGY FLOWS WITH «IMPEDANCE» RIG

This experiment was made to reveal the relationship between different components of energy flow generated by pump *in situ* in order to describe it as a source of water and structure borne sound in energy terms. A transducer arrangement is depicted in Fig. 9.

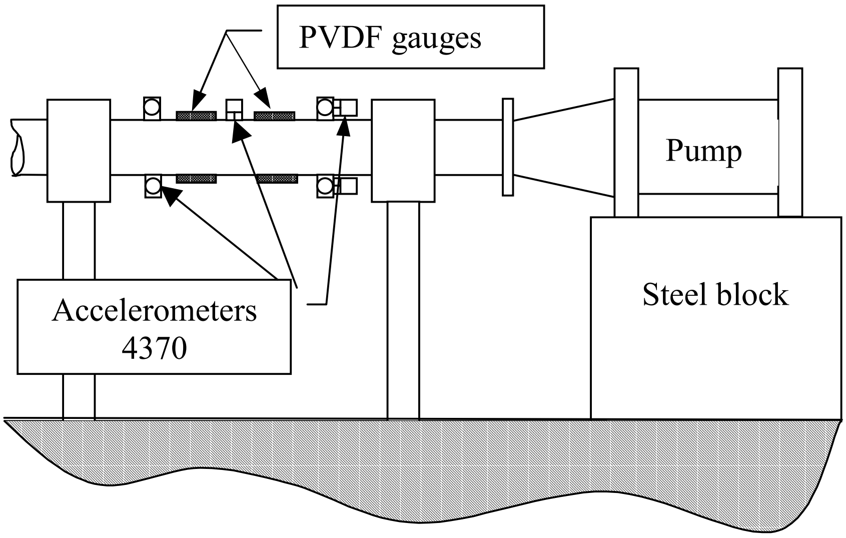

Fig. 9. Transducers arrangement at the pipe of "Impedance" rig

In this specific case the components of shear and bending moment are almost equal each other in a wide frequency range. Hence this fact makes possible the usage of far field approximation when total bending energy flow can be represented in form of double component of the bending moment (see Fig. 10)

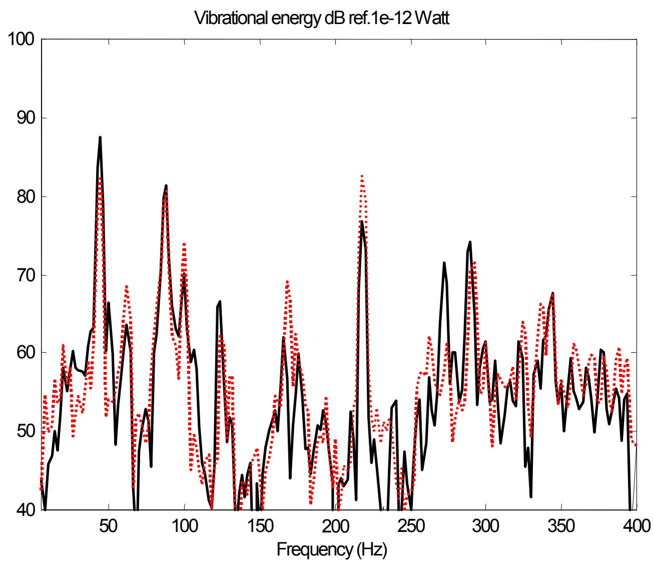

Fig. 10. Shearing force (- - -) and bending moment (—) components

This circumstance could make measurement procedure easier and there is no reason to use the shearing force measurement arrangement. Figure 11 demonstrates that energy transferred via water of 10 dB higher than total structural component at frequency which proportional to product of two values, namely, the pump rotor number of revolutions and number of propeller blades.

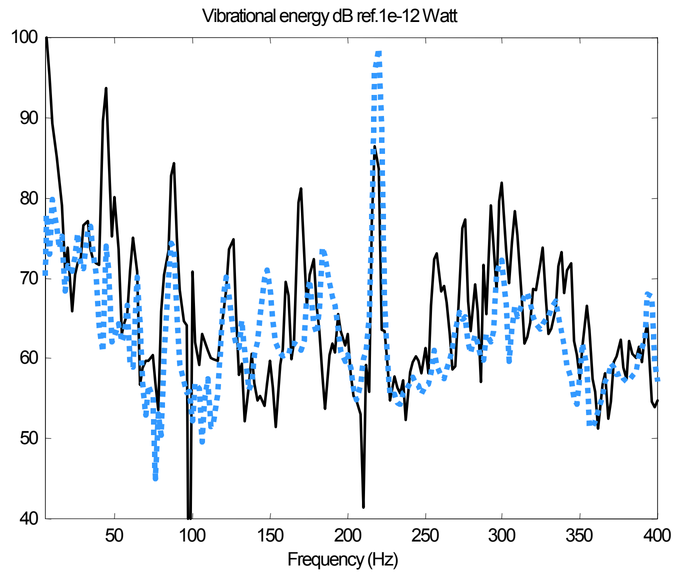

Fig. 11. Structure (—) and water borne (- - -) components

At other frequencies vibrational energy transferred by structural components only. The longitudinal component dominates in total structural energy flow due to the rigid connection of the pump and attached pipelines. (see. Fig. 12).

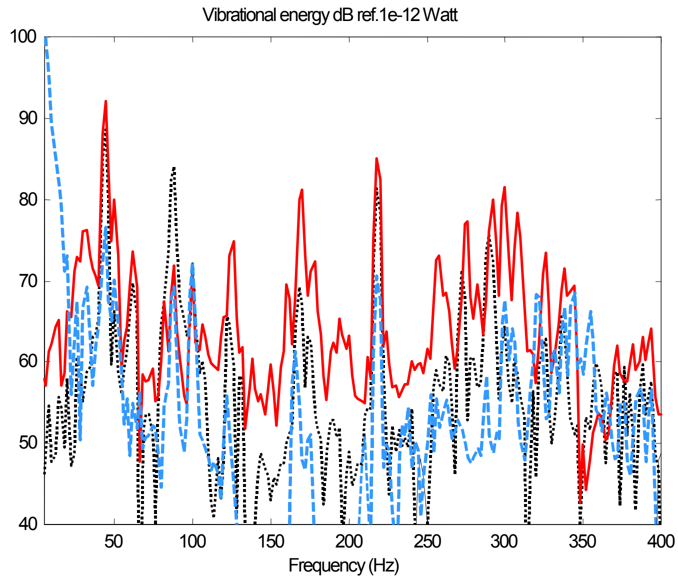

Fig. 12. Three structural components: (—) – longitudinal, (······) – bending and (- - - -) – torsional

4. ERROR ANALYSES USING ACTUAL MEASURED DATA

Here we shall consider two types of error: one due to low signal to noise ratio or low value of coherence function between force/moment and linear/angular velocities. The second type is error due to ratio of phase mismatch between measurement chains and measured phase angle between force and linear velocity (moment and angular velocity). If the measurement error can exceed more than 100% let us put into consideration the term – "measurement uncertainty".

The uncertainty of first type has a random nature and depends on the coherence function and phase angle. Seybert derived the expression for the random error [10] in the case when two microphones method is implemented when sound intensity measurement technique proposed by Fahy [3] is used. Seybert's approach is universal and expression for random error can be applied for all cases where imaginary part of cross spectrum is used as a primary data in vibrational energy estimation. The formulae for bias (instrumentation) error are the function of phase mismatch between force/moment and linear/rotational acceleration channels. Here we assumed that real measured phase angle or its smoothed estimation has to be taken into account. The error can be written in the form

$$\theta_{bias} = \frac{\Delta\alpha}{\tan\alpha}. \tag{25}$$

Another type of error is random one, which can be expressed by Seybert's equation:

$$\theta_{random} = \left(\frac{1}{n_d} \left\{ \frac{1}{\gamma_{Fv}^2} + \left(\frac{1-\gamma_{Fv}^2}{2\gamma_{Fv}^2} \right) \frac{1}{(\tan\alpha)^2} \right\} \right)^{1/2}. \tag{26}$$

Here and after the α angle is actual one obtained during measurement as a function of frequency. Usually there are numerous singular frequency points where phase angle becomes small as well as coherence function. It means that energy transferred by corresponding waves is small due to lightly dampening of the structure or high reflections occur. The low coherence function shows that components of the wave motion namely, wave magnitudes, are also small.

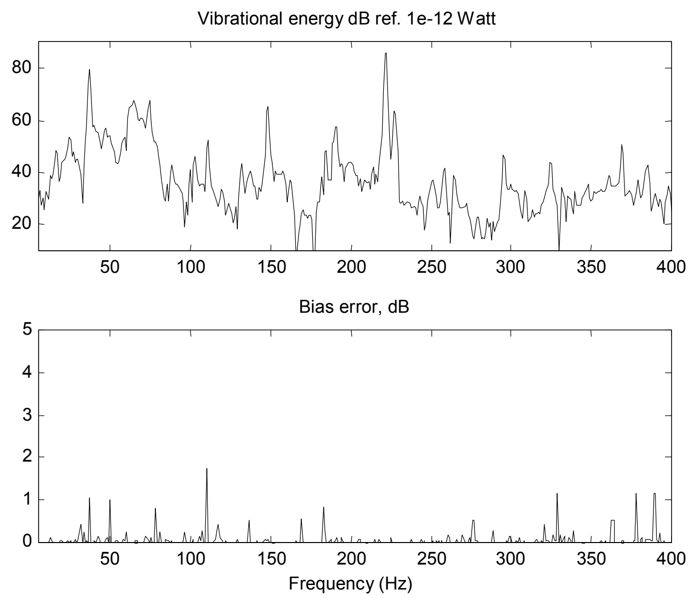

Fig. 13. Structural energy spectrum and its bias error

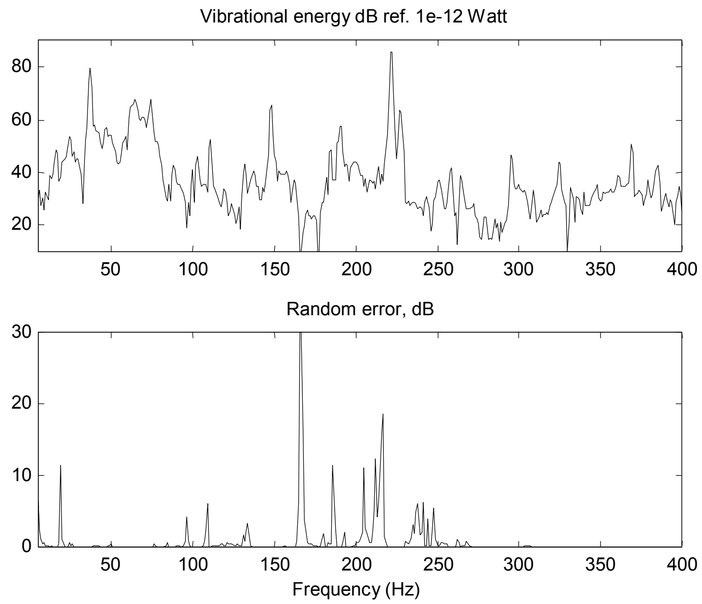

Fig. 14. Structural energy spectrum and its random error
(500 independent samples)

If the structure is lightly damped the uncertainty of measurement will rapidly increase. The results of the estimation are depicted in Fig. 13 and Fig. 14. It can be seen that bias error does not exceed ±2 dB. The uncertainty of measurement rapidly increases at frequencies where signal to noise ratio is small (see Fig. 14) In other words the random error dominates.

The uncertainty can reach values of 10–30 dB at frequencies where energy values smaller than 30 dB referring to the threshold level 10^{-12} Watt. It should be noted that real values of vibrational energy that can be estimated correctly (with acceptable uncertainty ±3 dB) have to be more than 10^{-9} Watt (30 dB) when PVDF strain gauges sensitivity are of 30 mV per micro strain. There is only one way for error reduction. It is increasing a number of samples, which limited by observation time and time interval where vibration of machine can be considered as steady state random process. A logical connection between Seybert's principle in intensity measurements and uncertainty principle in quantum mechanics could be made. Both of them do not allow getting the desirable accuracy by measurer will.

DISCUSSION

Technique involved reusable PVDF strain gauges was developed and discussed in the paper. In contrast to FD technique the proposed one has number of advantages that allow to make correct estimation of structure borne vibrational energy *in situ* with running pumps and real pipe configurations. High frequency limitation due to involved mathematical model has calculated in each individual case. The error analyses with actual measured data about coherence function and phase angle of cross spectra allow to determine the low tolerable limit of energy estimation. It is approximately 10^{-9} Watt involving this measurement technique and attainable PVDF gauge sensitivity. The sensitivity can be increased by use of ICP electronic circuit. The preliminary experiments have shown the attainable sensitivity with ICP about of 300–500 mV per micro strain.

It is shown that pump as a source of vibrational activity can be described by use of water-borne and structure-borne components and sometimes it is not correct to consider both transfer channels separately or skip one of them from transfer path analyses. This phenomenon was investigated with the «Impedance» rig. The relationships between four structural components and water ones transferred by plane waves in the pipe channel were determined and discussed. In the case with «Impedance» rig a structural component transferred mostly by longitudinal waves in the pipe walls whilst water borne sound dominates at several frequency. Using the developed technique an efficiency of noise control means applied to «Flow» rig in energy terms was estimated. The efficiency of resilient hanger becomes more than efficiency of flexible hose *in situ*. The energy balance in pipe cross section and error analyses shows a high confidence level of obtained data. Moreover it can be declared that total error in vibrational energy estimation at the tonal components of energy spectrum does not exceed ±3 dB where coherence and phase angle between force/velocity, moment/angular velocity are high enough.

ACKNOWLEDGEMENTS

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APPENDIX A: SYMBOLS AND NOTATION

E	Young modulus
G	Shearing modulus
ν	Poisson ratio
ρ	Material density of pipeline or shaft
ξ , v , $\ddot{\xi}$	Vibrational displacement velocity and acceleration
F_x	Longitudinal force
F_z	Shear force
M_ψ	Bending moment
M_φ	Torsional moment
ε_{mi}	Dynamic strain at point m along axis i
D_{outer}	Outer pipe diameter
D_{inner}	Inner pipe diameter
I_{bend}	Moment of inertia for ring cross section refer to axes y and z
J_{tors}	Polar moment of inertia of annular cross section
Δ_{tors}	Distance between accelerometers for measuring a component of torsional moment
$\Delta_{bending}$	Distance between PVDF strain gauges for measuring a component of shear force
γ^2_{Fv}	Coherence function between force and velocity
$б_{Fv}$	Phase angle between force and velocity
$\Delta\alpha$	Channels phase mismatch
n_d	Number of samples
$G_{AB}(f)$	Complex valued cross spectrum between processes $A(t)$ и $B(t)$
θ_{bias}	Bias measurement error
θ_{rand}	Random error
W	Vibrational power
L_W	Vibrational power level
Im, Re	Real and imaginary part of complex value
S	Annular cross section area
ω	Angular frequency
f	Frequency