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Using Stationary Wavelet Transform in BCH Image Coding

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This paper propose a novel technique that use efficient image compression based on stationary wavelet transform (*SWT*) decomposition and Bose-Chauhuri-Hochquenghem (*BCH*) coding for image transmission over additive white Gaussian noise (*AWGN*) channel. We know that, discrete wavelet transform (*DWT*) decomposition suffers a drawback which is the not time-invariant transform, leading to a non-redundant decomposition. This means, even with periodic signal extension, the *DWT* of a translated version of a signal X is not, in general, the translated version of the *DWT* of X . In such case, the use of an adequate wavelet transform that restore the translation invariance, appear efficient to give high-quality image reconstruction. Also, *BCH* coding is a linear block procedure that permits multiple-error bit correction over noisy channel. The idea is to combine efficient image compression given by *SWT* decomposition with an exam of the selection of the Bose-Chauhuri-Hochquenghem (*BCH*) error correction codes applied to the low pass version at different decomposition level. By using *BCH*(63,45,3) code against the channel degradation, a significal improvement is obtained.

1. INTRODUCTION

Wavelet transform [1] has been attracting attention in diverse areas such as signal processing and image processing. Wavelet decomposition scheme splits input signal into a number of frequency bands one of which is a low-resolution version of the input signal. The frequency bands are quantized and coded.

BCH code is a linear block code which permits multiple-error bit corrections [2, 3], and can achieve performance close to the Shannon capacity limit in the additive white Gaussian noise channel [4]. In this paper, we present a *BCH* image coding based on stationary wavelet transform compression scheme.

2. DISCRETE WAVELET TRANSFORM

The continuous wavelet transform is by now recognized as a fruitful technique in signal and image processing, whereas discrete wavelet transform or dyadic wavelet transform is often more appropriate for image synthesis. The continuous wavelet transform is defined by the function [5] in equation (1), with the wavelet mother $\psi(t)$

$$C(\tau, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi^* \left(\frac{t - \tau}{a} \right) dt, \quad (1)$$

where the wavelet family given by

$$\frac{1}{\sqrt{a}} \psi \left(\frac{t}{a} \right),$$

where a is a scale factor affecting the frequency and the amplitude of the signal, τ is a time factor localizing the wavelet position in the signal $x(t)$.

Knowing that the wavelet family is characterized by the following property

$$\psi(t) = \psi^*(-t)$$

and using the inverse transform, one can reconstruct the original signal by

$$x(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{a}} C(\tau, a) \psi \left(\frac{t - \tau}{a} \right) \frac{1}{a^2} d\tau da. \quad (2)$$

If we use a dyadic form, such $a = 2^j$; $\tau = k2^j$; $j \in \mathbb{N}, k \in \mathbb{Z}$, the wavelet family became

$$\psi_{j,k} = 2^{-j/2} \psi(2^{-j}t - k),$$

which form an orthonormal base of the continuous signal space. And equations (1) and (2) became

$$\begin{aligned} C_{j,k} &= \sum_{n=0}^{\infty} x_n \psi_{j,k}^*(n), \\ x_n &= \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} C_{j,k} \psi_{j,k}(n), \end{aligned} \quad (3)$$

where $\psi_{j,k}(n)$ are obtained by translates and dilates of the wavelet function $\psi(n)$, and $C_{j,k}$ represents the discrete wavelet transform coefficients which are the estimation of signal components at $(2^{-j}k, 2^j)$ in the Time-Frequency plane.

The discrete wavelet transform is the most useful technique for frequency analysis of signals that are localized in time of space. It decomposes signals into basis functions that are dilations and translations of a single prototype wavelet function, equation (3).

Actually, the discrete wavelet transform [6] corresponds to multiresolution approximation expressions. This method permits the analysis of the signal in many frequency bands or at many scales [7, 8]. In practice, multiresolution analysis is carried out using two channel filter banks composed of a low-pass (G) and a high-pass (H) filter and each filter bank is then

sampled at a half rate ($\frac{1}{2}$ down sampling) of the previous frequency. By repeating this procedure, it is possible to obtain wavelet transform of any order. The down sampling procedure keeps the scaling parameter constant ($k = \frac{1}{2}$) throughout successive wavelet transforms so that it benefits for simple computer implementation. In the case of an image, the filtering is implemented in a separable way by filtering the lines and columns.

An example can be illustrated in fig. 1. According to this procedure, the original image can be transformed into four sub-images, namely:

- *LL* sub-image: Both horizontal and vertical directions have low-frequencies.
- *LH* sub-image: the horizontal direction has low-frequencies and the vertical one has high-frequencies.
- *HL* sub-image: The horizontal direction has high-frequencies and the vertical one has low-frequencies.
- *HH* sub-image: Both horizontal and vertical directions have high-frequencies.

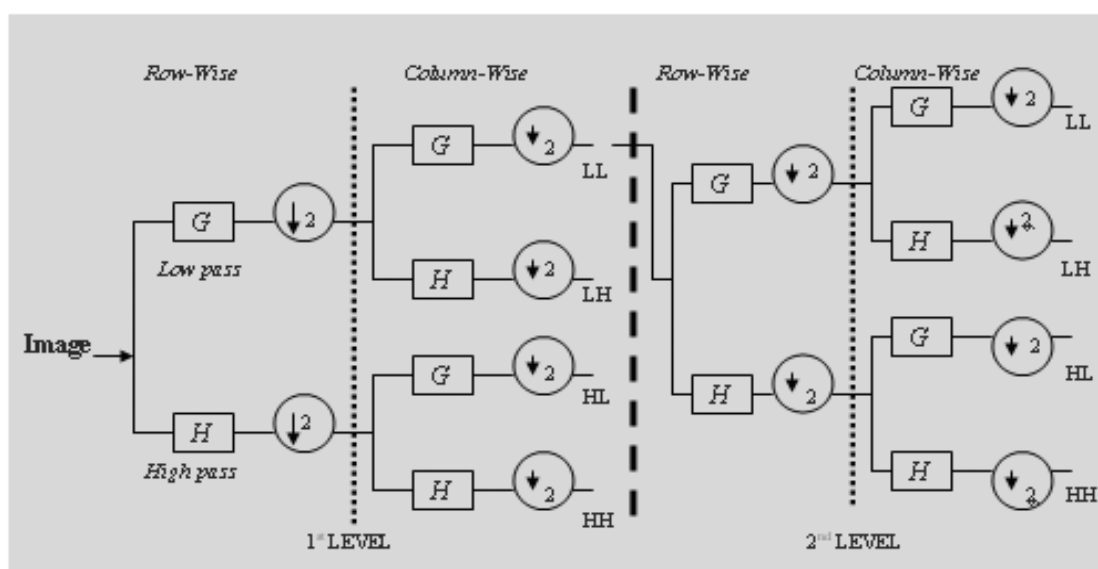


Fig. 1. Two-level discrete wavelet transform

3. STATIONARY WAVELET TRANSFORM

The discrete wavelet transform provides the information useful for texture analysis in the image. Its fast implementation is usually performed by using multiresolution analysis. The wavelet coefficients are sampled based on the Nyquist criteria. The representation is accordingly non-redundant and the total number of sample in the representation is equal to the total number of the image pixels. The major inconvenience of this representation is that it does not conserve an essential property in image processing, which is the invariance by translation. Thus pyramidal multiresolution analysis is not desirable for estimation/detection problems. In order to preserve the invariance by translation, the down sampling operation must be suppressed and the decomposition obtained is then redundant and is called a stationary wavelet transform [9]. In practice, the structure in cascade of the filter bank does not change, the only operation of down sampling are suppressed.

4. SYSTEM MODEL

Figure 2 shows the block diagram used in our study. The system consists of stationary wavelet transform compression first step, which leads to a source coding by reducing the size of the original image (256×256) to an image of size (32×32) using such daubechies wavelet, followed by optimal scalar quantifier (Lloyd algorithm) [10], and Gray coding transformation to form a block of k columns binary message and (32×32) lines.

The second step of the chain, is the *BCH* encoder, which encode our message to a matrix block using a *BCH* code (n, k) , to form a block codeword of n columns and (32×32) lines, with $n > k$. The length of each codeword is equal to n . A discrete channel is assumed with additive white Gaussian noise. At the reception, a *BCH* decoder decodes the received coded message followed by Gray decoding and some inverse transformations (inverse quantization and inverse stationary wavelet reconstruction) to form reconstructed image.

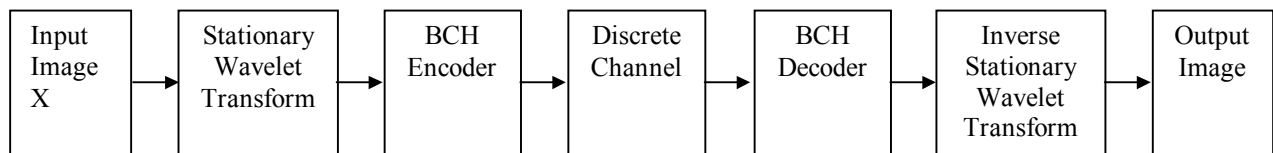


Fig. 2. Block diagram of the BCH image coding

We investigate the performance of our system in a discrete channel model with *AWGN*. Here, we use the bit error rate (*BER*) as the performance measure for *BCH* coder; and an image Peak-signal to noise ratio (*Peak-SNR*) [11] as the system performance measure for quality image reconstruction, equation (4), which is the signal to noise ratio based on pixel difference between the original image and reconstructed image:

$$PSNR \equiv 10 \log_{10} \left(\frac{255^2}{(1/L_x L_y) \sum_x \sum_y (S_{x,y} - \hat{S}_{x,y})^2} \right), \quad (4)$$

where L_x and L_y are horizontal and vertical sizes of the image, and $S_{x,y}$ and $\hat{S}_{x,y}$ represent the intensities of the original and reconstructed pixels, respectively.

5. SIMULATION AND RESULTS

In our simulations, in order to analyze the performance of using stationary wavelet transform at different level decomposition ' N ', we first suppose that channel degradation affects more the first bits of the binary sequence of the coded message, and with a fixed *BCH*(15,5,3) code applied to the low pass version of the decomposition. The sub-images at the second level are the low-pass filtered version of the *LL* image from first level, bearing low-frequency information on neighbouring pixels. These characteristics contributed to an efficient performance.

After decoding, there is good correction of channel errors using *BCH* coding. The noise present in the reconstructed images, figures 3-b and 3-c, is due the step of inverse quantization. Using the Lloyd algorithm [11], with arbitrary initial codebook of length equal to 2^q (q quantization parameter), the quantization index of the input signal is based on this initial codebook and on the decision point partition (partition is a strict ascending ordered $q-1$ vector that specifies the boundaries).



(a) Original Image



(b) Reconstructed Image:
 $N=3$ and $BCH(15,5,3)$, with
quantization parameter $q=2$



(c) Reconstructed Image:
 $N=4$ and $BCH(15,5,3)$, with
quantization parameter $q=2$



(d) Reconstructed Image:
 $N=3$ and $BCH(63,45,3)$, with
quantization parameter $q=3$



(e) Reconstructed Image:
 $N=3$ and $BCH(63,45,3)$ with
quantization parameter $q=4$

Fig. 3. Reconstructed image after *BCH* decoding with different stationary wavelet decomposition level and quantization parameter over noisy channel

We see that, more is the length of the initial book, more efficient inverse quantization is; and using $BCH(63,45,3)$, a good image quality is obtained fig. 3-d and fig. 3-e. To analyse more these results, tables 1 and 2 gives error rate received code (BER) with correlation parameter (between reconstructed and original image) at different decomposition level for $BCH(15,5,3)$ and $BCH(63,45,3)$ respectively. We can see a close relation between the decomposition level N and the used BCH code $(63,45,3)$.

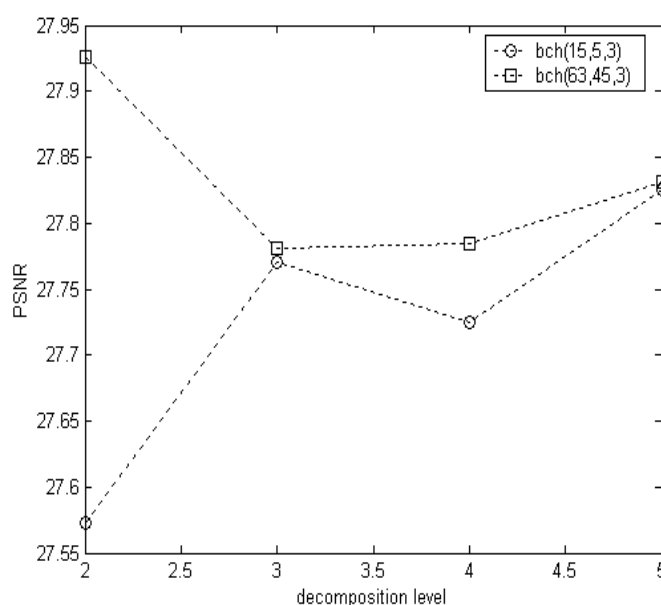
Table 1. $BCH(15,5,3)$

Level N	Error rate received code	Error rate after decode	Corr
2	0.1867	0	0.9580
3	0.1874	0	0.9581
4	0.1859	0	0.9587
5	0.1863	0	0.9604

Table 2. $BCH(63,45,3)$

Level N	Error rate received code	Error rate after decode	Corr
2	0.1868	0	0.9588
3	0.1855	0	0.9608
4	0.1867	0	0.9611
5	0.1854	0	0.9645

In fig. 4, the variation of the $PSNR$ at different decomposition level shows that the use of $BCH(63,45,3)$ code, provides a visible performance than $BCH(15,5,3)$ code without necessary further decompositions level.

Fig. 4. Received image $PSNR$ against decomposition level

We can see, in fig. 5, a significant improvement in the error rate received code at level $N=3$ which is less in the case of using $BCH(63,45,3)$ code.

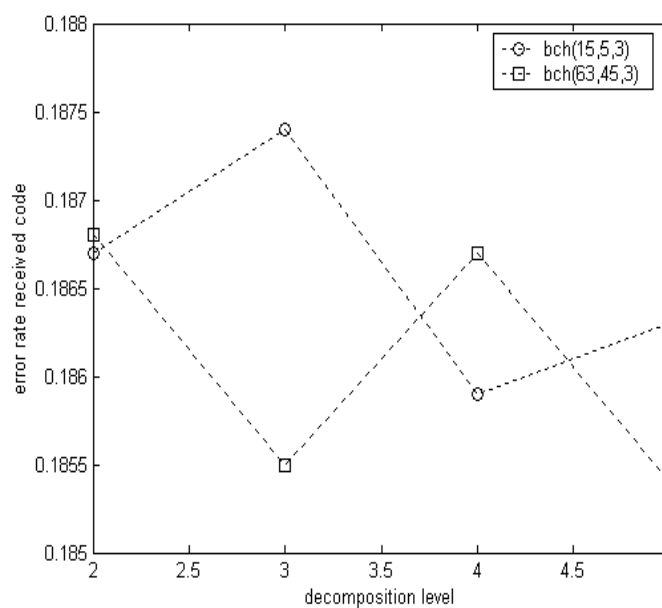


Fig. 5. Error rate received code against decomposition level

The received image correlation, given in fig. 6, shows good correlation parameter when the $BCH(63,45,3)$ code is used at different stationary wavelet decomposition level.

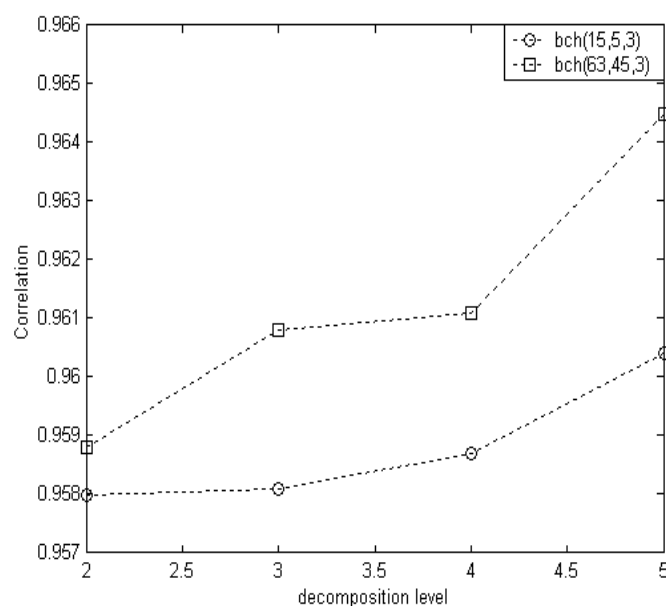


Fig. 6. Image correlation against decomposition level

6. CONCLUSION

In this paper, we have investigated the characteristics of the stationary wavelet transform which provides the texture information in images to source coding with BCH codes for channel correction. By preserving invariance by translation in stationary wavelet compression, we have obtained significant improvement in reconstructed images at different level of decomposition. There is a close relation between the number of level and the type of $BCH(n,k,t)$ code. By assuming that channel affects the first bits of the binary sequence message (which are more significant), the selection of the BCH code provides significant improvement when ($N=4$, $BCH(15,5,3)$ and $N=3$, $BCH(63,45,3)$).

In perspective of this work, the effect of joint channel estimation by stationary wavelet filtering will be studied.

REFERENCES

- [1] C. S. Burrus, R. A. Gopinath, H. Guo. Introduction to wavelets and Wavelet Transform. *Prentice-hall International, Inc*, New Jersey, 1998.
- [2] N. Matoba, S. Yoshida. Still image transmission using un-equal error protection coding in mobile radio channel. *Electron. Comm. Japan 1, Commun.*, 79 (4), 75–85, April 1996.
- [3] C. W. Yap, K. N. Ngan, R. Liyanapathirana. Error protection scheme for the transmission of H.263 coded video over mobile radio channels. *Proc. SPIE*, vol. 3024, part 2, 1997, pp. 1241–1249.
- [4] C. Berrou, A. Glavieux, and P. Thitimajshima. Nera Shanon limit error correcting coding and decoding: Turbo codes”. In *IEEE Int. Conf. on Communications*, pp. 1064–1070, 1993.
- [5] P. Abry. Ondelettes et turbulence. Multiresolutions, algorithmes de décomposition, invariance d’échelles. *Diderot Editeur*, Paris, 1997.
- [6] R. K. Young. Wavelets Theory and its Applications. *Kluwer Academic Publishers*, Boston, 1993.
- [7] I. Daubechies. Orthonormal Bases of Compactly Supported Wavelets. *Comm. Pure and Applied Mathematics*, vol. 41, no. 7, pp. 909–996, 1988.
- [8] S. Mallat. A Theory of Multiresolution Signal Decomposition: The Wavelet Representation. *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 11, pp. 674–693, 1989.
- [9] S. Mallat. A Wavelet Tour of Signal Processing. *Academic press*, San Diego, 1988.
- [10] N. Moreau. Technique de compression des signaux. *Masson*, Paris, Milan, Barcelone, 1995.
- [11] R. Stedman, H. Gharavi, L. Hanzo, R. Steele. Transmission of subband-coded images via mobile channels. *IEEE Trans. Circuits Systems for Video Tech.*, 3 (1), Feb., 1993.