

Federico Bartolozzi

*Civil Engineer & Independent Researcher, via dei Carracci, 4, 21100 Varese, Italy*  
e-mail: [ciuciuzza@iol.it](mailto:ciuciuzza@iol.it)

## Natural frequency automatic variation in seismic isolation system\*

*Received 27.10.2004, published 23.11.2004*

The proposed system is based on the following operations: 1. interruption of the continuity between the building and the foundation-soil complex; 2. self-centring of the building after an earthquake. By means of its own elastic deformation each bearing can automatically compensate the rigid deflection variation relative to support due to the horizontal component of the motion. The building remains motionless with respect to the foundation-soil complex, which moves. The vertical motion, due to the sub-undulatory shock, varies the building behaviour only partially. It remains stationary with respect to the horizontal translation, but it is subject to a low vertical translation and to a resonance possibility. In order to prevent the resonance danger, the vertical natural frequency variability takes place because of auxiliary springs which automatically increase the action of one or more main springs during an emergency, characterised by an interval of vertical seismic frequencies including the resonance one.

### INTRODUCTION

Without entering in heart of the dynamics of seismic phenomena and without any remarks about the building codes in seismic areas, it is useful to make evident that the aseismic buildings, planned to bear – besides the usual stresses – also the ones due to sub-undulatory and, especially, undulatory shocks, have a considerable limit concerning the height of the buildings. Where this limit does not occur – as in the case of buildings with a structure in either normal or prestressed reinforced concrete or in steel – an increase in the cost of the building occurs, due to an over-dimensioning of the structural elements. That being stated, the proposed aseismic system is also a system of earthquake isolation, based essentially on the simple reflection that, by breaking the continuity between the building and the foundation-soil complex, it is possible for the foundation-soil complex to move with respect to the building, which remains almost motionless under the prevailing action of the weight force.

The technique of the seismic isolation has had an impulse during the last few years, that has been more theoretical than practical. It is more widely used in the United States of America and Japan, where devices with a transverse low stiffness, such as elastomeric insulators, are still more prevalent than friction dampening devices. Although it is known in Italy, seismic isolation is hardly ever used, where aseismic buildings are constructed in accordance with the relative regulations, which impose a structural over-dimensioning. The

---

\* The work was discussed at the 2<sup>nd</sup> International Conference on Advanced Computational Methods in Engineering (ACOMEN-2002), Liège, Belgium, 28-31 May, 2002.

various systems of seismic isolation, which have been worked out by author of the paper, are all characterized by the use of friction movable bearings. A few of them are self-centering, in the sense that the building spontaneously returns to the initial position of rest after an earthquake. The others, on the other hand, permit both the centering and the locking of the building after an earthquake by means of electronically started devices.

The system, proposed in this paper, is not “new” in the unique meaning of the word, but it could be considered as such, because it originates from the merger of two original systems, of which it contains the most advantageous relative characteristics. Its operational aspect is as effective as that of systems from which it originates, but it is considerably better than both of them, because of the reduced planning aspect and the operational simplicity, which, together with the greater economic competitiveness, certainly make it more acceptable. In fact, the replacement of the complicated, electronically started, frequency converters, present in the former reference system, with naturally started easy device of the second system, is the main change of the former system, which, however, preserves unchanged both the number of bearings and their plane surface of sliding.

## 1. GEOMETRICAL AND ELASTIC CHARACTERISTICS OF THE BEARINGS

Figures 1 and 2, relative to a system with four rigid bearings, prove that, during the motion, the generic bearing is subject to the variation of rigid deflection relative to support:

$$df_i = |f - f_i| \neq 0, \quad (1)$$

with  $i = 1, 2, 3, 4$ .

In (1),  $f = a \operatorname{tg} \alpha = \text{constant}$  is the static deflection relative to support, with  $a$  is the overhang of the platform with respect to the bearing,  $\alpha$  is the inclination angle of the bearing sliding surface and  $f_i$  is the dynamic deflection relative to support. The variation of deflection relative to support changes from one bearing to the other in accordance with the following correlation:

$$df_i = (0,5L_{x/y} - c_{x/y} \pm S_{x/y}) \operatorname{tg} \beta_{x/y}, \quad (2)$$

where  $L_{x/y}$  are the distances between the bearings in the directions  $X$  and  $Y$ ,  $c_{x/y}$  are the maximum horizontal displacements of the building,  $\beta_{x/y}$  are the rotation angles of the building in the vertical planes determined by the axis of the building and the axis's  $X$  and  $Y$ .

The difference between the variations of deflection relative to supports is insignificant for opposed bearings; it is more significant for adjacent bearings. The removal of the pendulous effect, sanctioned by the correlation (1), is bound to the condition that, during an earthquake, it must be, at each instant and for any angle of the motion direction:

$$df_i - d^* f_i = 0. \quad (3)$$

That is, each bearing must be modified from a rigid one to an elastic one, equipping it with one or more springs, having established geometrical and elastic characteristics. During the motion, they must be able to compensate the variation of rigid deflection relative to supports

$df_i$ , by their elastic deformation  $d^*f_i$ . The building remains vertical and motionless with respect to the foundation-soil complex, which moves. Correlation (3) is verified both for the horizontal component of the motion and for each value of the direction angle  $\Omega$  of the earthquake. Due to the sub-undulatory shock, the vertical motion of the soil modifies the building behaviour only partially. It remains stationary with respect to the horizontal translation, but it may be subject to a low vertical translation and to the possibility of the resonance danger, defined by:

$$\phi_r = \phi_n, \quad (4)$$

where  $\phi_r$  is the vertical component of the seismic frequency and  $\phi_n$  is the vertical natural frequency of the building.

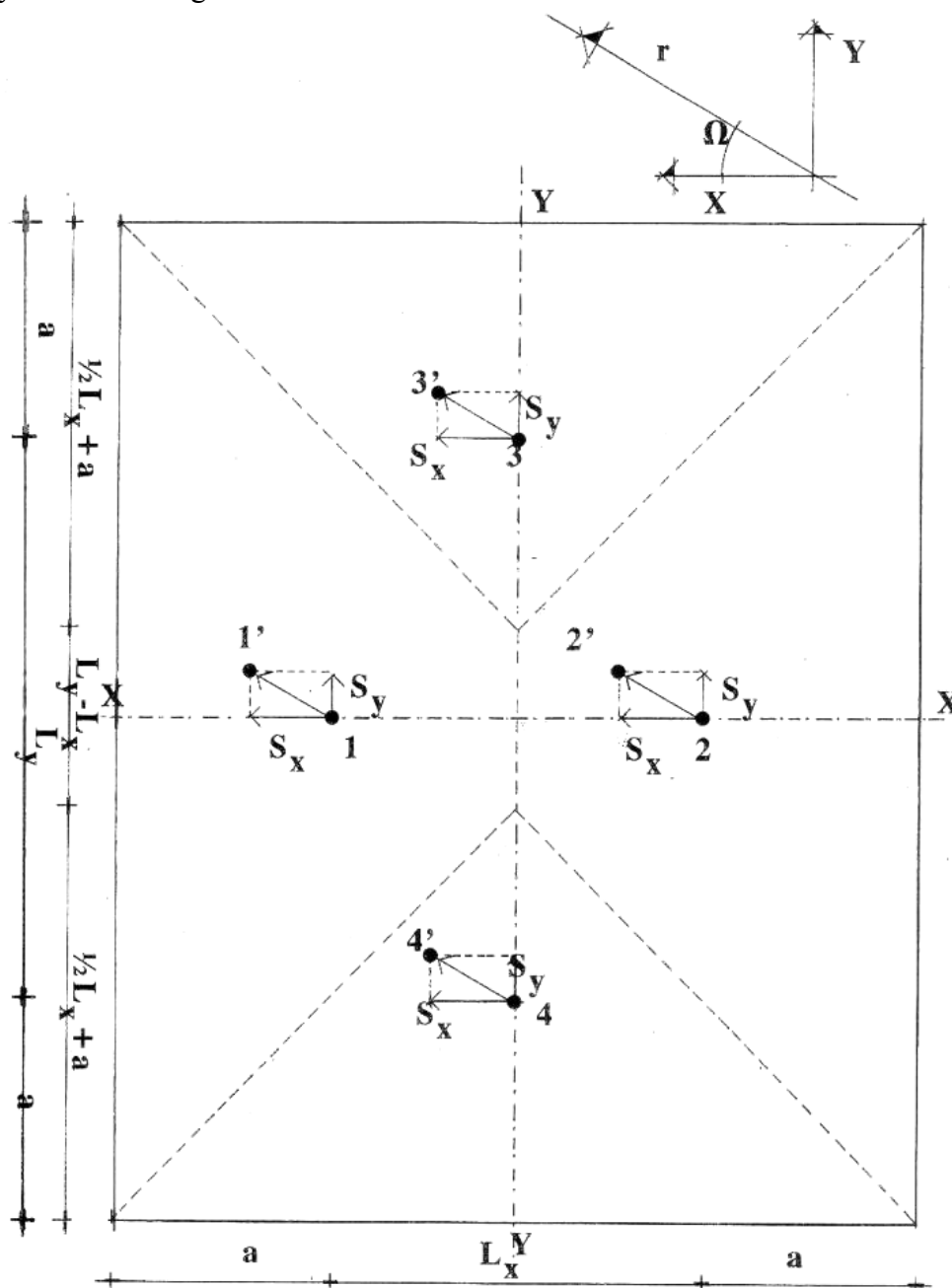


Fig. 1. Building platform and bearings



Particularly:

– for rectangular plan:

$$p_1 = p_2 = \left[ 2L_y(L_x + 2a) - (L_x^2 - 2a^2) \right] / \left[ 4(L_x + 2a)(L_y + 2a) \right], \quad (6)$$

$$p_3 = p_4 = (L_x + 2a) / 4(L_y + 2a); \quad (7)$$

– for square or circular plan:

$$p_i = 25\%. \quad (8)$$

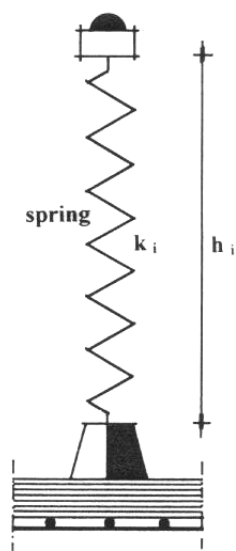


Fig. 3. Generic spring initial configuration

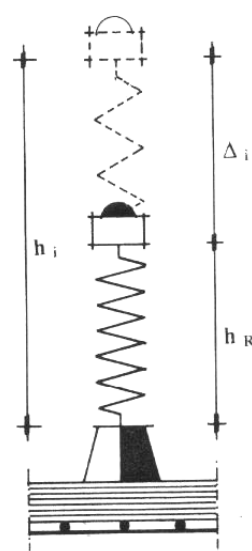


Fig. 4. Spring configuration due to the load

So that the platform remains horizontal after the deformation, it must be:

$$\Delta_i = \text{constant}. \quad (9)$$

Consequently:

– for rectangular plan it is:

$$k_1 = k_2 \text{ being } V_1 = V_2, \quad (10)$$

$$k_3 = k_4 \text{ being } V_3 = V_4;$$

– for square or circular plan it is:

$$k_i = \text{constant} \text{ being } V_i = \text{constant}. \quad (11)$$

The residual length of the spring is:

$$h_R = h_i - \Delta_i = \text{constant}. \quad (12)$$

### 3. STATE OF MOTION

#### 3.1. Horizontal translation

##### 3.1.1. Elastic deformation

Figure 5 refers to the motion component in the direction  $X$  axis. Generally, due to the motion in the direction  $r$ , individualised by the  $\Omega$  angle, the components of the displacement  $S_0$  in the directions  $X$  and  $Y$  are:

$$\begin{aligned} S_x &= S_0 \cos \Omega, \\ S_y &= S_0 \sin \Omega. \end{aligned} \quad (13)$$

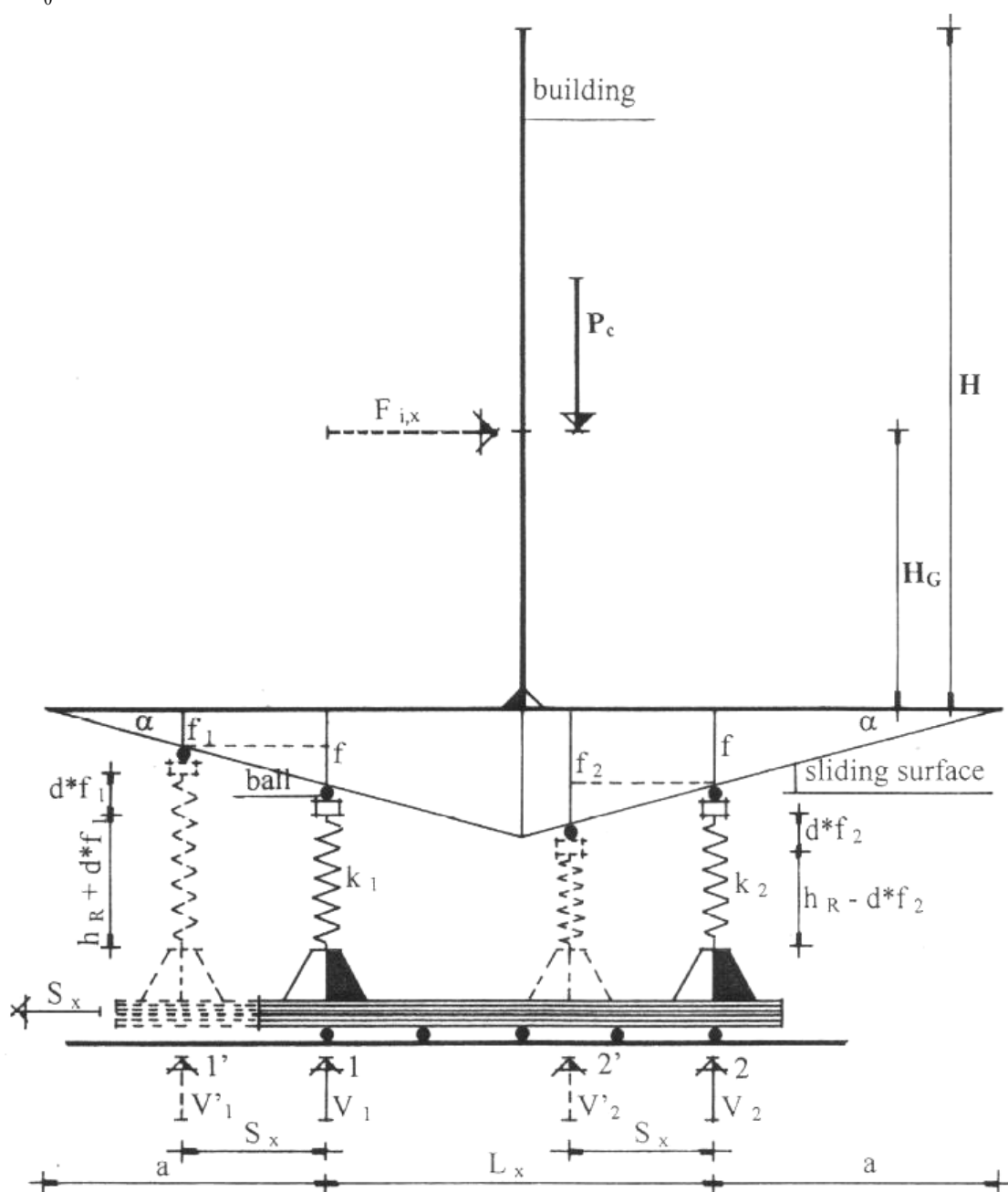


Fig. 5. Building motion state – section

The deformation of the main spring of the generic bearing is

$$d^* f_i = S_0 \operatorname{tg} \lambda_i, \quad (14)$$

where:

$$\lambda_1 = \lambda_2 = \lambda_{1,2} = \operatorname{arctg}(\cos \Omega \operatorname{tg} \alpha), \quad (15)$$

$$\lambda_3 = \lambda_4 = \lambda_{3,4} = \operatorname{arctg}(\sin \Omega \operatorname{tg} \alpha). \quad (16)$$

With reference to the hypothetical displacement of figure 5, the springs of the bearings N° 1 and 3 lengthen, the others, on the contrary, shorten. In addition, by analysis of the correlation's (15) and (16), it emerges that: 1. the elastic deformation, defined by (14), is, in absolute value, the same for opposed bearings and different for adjacent bearings; 2. the difference of elastic deformation among adjacent bearings is negligible for displacements of few millimeters and minor for displacements of few centimeters.

### 3.1.2. Reactions of the bearings and horizontal inertial force in the building

The problem concerning the dynamic equilibrium is spatial and it is statically determined by the six equations of the Rigid Systems Statics. At each instant of the motion, they express the equilibrium of all external forces (building weight, bearings reactions, friction forces and inertial force). The equations system is expressed by the following correlation:

$$\varphi_{j,1} V_1' + \varphi_{j,2} V_2' + \varphi_{j,3} V_3' + \varphi_{j,4} V_4' + \varphi_{j,5} F_{i,x} + \varphi_{j,6} F_{i,y} = \varsigma_j, \quad (17)$$

where:  $V_1', V_2', V_3', V_4'$  are the bearings reactions and  $F_{i,x}, F_{i,y}$  are the inertial force components in the directions  $X$  and  $Y$ ;  $j = 1, 2, 3, 4, 5, 6$ . The elements of the matrixes are:

– for the equilibrium to the translation in direction  $X$ :

$$\begin{aligned} \varphi_{1,1} &= \cos^2 \lambda_{1,2} \cos \Omega c_a, & \varphi_{1,2} &= a_{1,1}, & \varphi_{1,3} &= \cos^2 \lambda_{3,4} \cos \Omega c_a, \\ \varphi_{1,4} &= a_{1,3}, & \varphi_{1,5} &= -1, & \varphi_{1,6} &= 0, \end{aligned} \quad (18)$$

$$\varsigma_1 = 0;$$

– for the equilibrium to the translation in direction  $Y$ :

$$\begin{aligned} \varphi_{2,1} &= \cos^2 \lambda_{1,2} \sin \Omega c_a, & \varphi_{2,2} &= a_{2,1}, & \varphi_{2,3} &= \cos^2 \lambda_{3,4} \sin \Omega c_a, \\ \varphi_{2,4} &= a_{2,3}, & \varphi_{2,5} &= 0, & \varphi_{2,6} &= -1, \end{aligned} \quad (19)$$

$$\varsigma_2 = 0;$$

– for the equilibrium to the translation in direction  $Z$ :

$$\begin{aligned} \varphi_{3,1} &= 1 + \cos \lambda_{1,2} \sin \lambda_{1,2} c_a, & \varphi_{3,2} &= 1 - \cos \lambda_{1,2} \sin \lambda_{1,2} c_a, & \varphi_{3,3} &= 1 + \cos \lambda_{3,4} \sin \lambda_{3,4} c_a, \\ \varphi_{3,4} &= 1 - \cos \lambda_{3,4} \sin \lambda_{3,4} c_a, & \varphi_{3,5} &= 0, & \varphi_{3,6} &= 0, \end{aligned} \quad (20)$$

$$\varsigma_3 = P_c;$$

– for the equilibrium to the rotation round the direction  $X$ :

$$\begin{aligned}\varphi_{4,1} &= S_y a_{3,1}, & \varphi_{4,2} &= S_y a_{3,2}, & \varphi_{4,3} &= (0, 5L_y + S_y) a_{3,3}, \\ \varphi_{4,4} &= -(0, 5L_y - S_y) a_{3,4}, & \varphi_{4,5} &= 0, & \varphi_{4,6} &= f + H_G, \\ \zeta_4 &= 0;\end{aligned}\tag{21}$$

– for the equilibrium to the rotation round the direction  $Y$ :

$$\begin{aligned}\varphi_{5,1} &= (0, 5L_x + S_x) a_{3,1}, & \varphi_{5,2} &= -(0, 5L_x - S_x) a_{3,2}, & \varphi_{5,3} &= S_x a_{3,3}, \\ \varphi_{5,4} &= S_x a_{3,4}, & \varphi_{5,5} &= f + H_G, & \varphi_{5,6} &= 0, \\ \zeta_5 &= 0;\end{aligned}\tag{22}$$

– for the equilibrium to the rotation round the direction  $Z$ :

$$\begin{aligned}\varphi_{6,1} &= \cos^2 \lambda_{1,2} c_a [\cos \Omega S_y - \sin \Omega (0, 5L_x + S_x)], \\ \varphi_{6,2} &= \cos^2 \lambda_{1,2} c_a [\cos \Omega S_y + \sin \Omega (0, 5L_x - S_x)], \\ \varphi_{6,3} &= \cos^2 \lambda_{3,4} c_a [\cos \Omega (0, 5L_y + S_y) - \sin \Omega S_x], \\ \varphi_{6,4} &= -\cos^2 \lambda_{3,4} c_a [\cos \Omega (0, 5L_y - S_y) + \sin \Omega S_x], \\ \varphi_{6,5} &= 0, & \varphi_{6,6} &= 0, & \zeta_6 &= 0;\end{aligned}\tag{23}$$

where  $c_a$  is the friction coefficient between the building and the foundation-soil complex.

The solution of the system (17) is:

– bearings reactions:

$$V_w' = A_w / A_s, \tag{24}$$

with  $w = 1, 2, 3, 4$ ;

– inertial force in direction  $X$ :

$$F_{i,x} = A_{i,x} / A_s, \tag{25}$$

– inertial force in direction  $Y$ :

$$F_{i,y} = A_{i,y} / A_s, \tag{26}$$

– total inertial force in the generic direction  $r$ :

$$F_{i,t} = \sqrt{F_{i,x}^2 + F_{i,y}^2}, \tag{27}$$

where  $A_s$  is the main matrix of the system and  $A_w$ ,  $A_{i,x}$ ,  $A_{i,y}$  are the matrixes relative to the unknown parameters. The variation of the bearing reaction is

$$dV_i = |V_i - V_i'| \tag{28}$$

and with reference to (14), the elastic constant of the spring of the generic bearing is

$$k_i = dV_i / S_0 \operatorname{tg} \lambda_i. \tag{29}$$

Generally, if the bearing consists of  $N$  main springs, the elastic constant of the single spring is

$$k_i' = k_i / N. \tag{30}$$



### 3.1.3. Design of the main springs elastic constant

The analysis of (29) indicates that the project of the elastic constant depends on the parameters:  $\lambda_i = f(\Omega, \alpha, S_0, dV_i)$ . With respect to the sliding angle  $\lambda_i$ , the condition so that it is constant, for  $\alpha$  assigned, is  $\Omega = 45^\circ$ . In fact, due to the horizontal translation of the soil, the deformation of the spring is constant in accordance with (14). Evidently, the limit values  $\Omega = 0^\circ$  and  $\Omega = 90^\circ$  must be discarded, because they would give  $k_3 = k_4 = \infty$  and  $k_1 = k_2 = \infty$  respectively. The choice of the horizontal displacement of project is arbitrary. Discarding the limit values  $S_{0,p} = 0$  (state of rest) and  $S_{0,p} = \infty$ , owing to which the bearing would be infinitely rigid and infinitely elastic, the appropriate value of project displacement could be the maximum one registered in the area interested by the earthquake; that is to say:

$$S_{0,p} = S_{0,\max} . \quad (31)$$

Finally, the choice of  $dV = dV_{\min}$  (in which the symbol “min” means minimum) gives the spring a greater elasticity and the bearing a better fitness to the displacements. On the basis of the above considerations, the elastic constant of project has the following value:

$$k_p = dV_{\min} / S_{0,\max} \cos 45^\circ \operatorname{tg} \alpha = 1,14421 \cdot dV_{\min} / S_{0,\max} \operatorname{tg} \alpha . \quad (32)$$

In state of rest, the maximum deformation of the spring project is

$$\Delta_i = V_{\max} / k_p \quad (33)$$

and having to be for all bearings

$$\Delta_i = \Delta_p = \text{constant}, \quad (34)$$

the project elastic constant of the springs of the residual bearings is

$$k_R = V_R / \Delta_p = (V_R / V_{\max}) k_p . \quad (35)$$

## 3.2. Vertical translation

### 3.2.1. First vertical natural frequency of the building

In the hypothesis of harmonic vibration without damping, the vertical natural frequency of the building is

$$\phi_n = (1/2\pi) \sqrt{k_i g / P_c} = 0,4985 \cdot \sqrt{k_i / P_c} , \quad (36)$$

which is also the resonance frequency  $\phi_r$ . In formula (36)  $P_c$  is the building weight,  $g$  is the gravity acceleration, expressed in  $\text{m/s}^2$ ,

$$k_t = k_p + \sum k_R \text{ is the total elastic constant of the main springs.} \quad (37)$$

### 3.2.2. Emergency interval

In order to safeguard the building against the resonance danger, which takes place when – due to the sub-undulatory shock – the seismic frequency equalizes the natural frequency of the building, it is necessary to fix a frequencies interval, containing the resonance one, in which the seismic frequencies are not compatible with the building safety. In this interval it is, therefore, indispensable that a system of auxiliary springs is automatically started up in each bearing. These springs, strengthening the action of the main springs, increase the vertical natural frequency of the building, noticeably decreasing the vertical displacements of the building, which must be not greater than the fixed values of project. If  $\phi_r = \phi_n$  is the resonance frequency, the emergency lower limit frequency is

$$\phi_{e,l} = c_{e,l} \phi_r, \quad (38)$$

where:  $c_{e,l} < 1$  is the lower safety coefficient. The emergency higher limit frequency is

$$\phi_{e,h} = c_{e,h} \phi_r, \quad (39)$$

where:  $c_{e,h} = \sqrt{2 - c_{e,l}^2} > 1$  is the higher safety coefficient.

When the seismic frequency equalizes the emergency limit frequencies – see Fig. 6 – that is for  $\phi = \phi_{e,l}$  and  $\phi = \phi_{e,h}$ , the vertical displacement of the building is expressed by all of the following formulas:

$$S_c = S_v / (1 - c_{e,l}^2), \quad (40)$$

$$S_c = S_v / (1 - c_{e,h}^2) \quad (41)$$

respectively in phase and in phase opposition with the vertical displacement  $S_v$  of the soil.

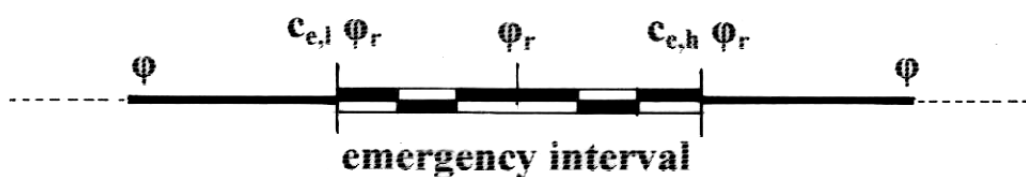


Fig. 6. Emergency interval schematic representation

With respect to the maximum vertical displacement of the soil, using (40) or (41), we design the maximum vertical displacement of the building, which is valid both for emergency limit frequencies and, generally, for the frequencies outside the emergency interval:

$$\xi_l = |S_{v,\max} / (1 - c_{e,l}^2)| = |S_{v,\max} / (1 - c_{e,h}^2)|, \quad (42)$$

where  $S_{v,\max}$  is the maximum vertical displacement of the soil.

### 3.2.3. Second vertical natural frequency of the building

When the seismic frequency is inside the emergency interval, including the resonance one, that is for  $\phi = \phi_e$ , we impose the condition that the maximum vertical displacement of the building, with reference to the resonance, is

$$\xi_{\max} = \xi_1 + \xi_2, \quad (43)$$

where  $\xi_1$  has the value of (42) and

$$\xi_2 = S_{v,\max} / \left[ 1 - (\phi_r^2 / \phi_n^{*2}) \right], \quad (44)$$

being  $\phi_r$  the first natural frequency of the building due to action of the main springs and also the resonance frequency,  $\phi_n^*$  the second natural frequency due to the combined action of the main and auxiliary springs. Developing and solving (44), we have

$$\phi_n^* = \phi_n \sqrt{\xi_2 / (\xi_2 - S_{v,\max})}, \quad (45)$$

with  $\xi_2 > S_{v,\max}$ .

The total elastic constant of the main and auxiliary springs is

$$k_t^* = 4\pi^2 \phi_n^{*2} P_c / g = 4.024 \phi_n^{*2} P_c. \quad (46)$$

The total elastic constant of the main and auxiliary springs for each bearing is

$$k_{t,i}^* = V_i k_t^* / P_c, \quad (47)$$

where  $i = 1, 2, 3, 4$  and  $V_i$  is the load on the generic bearing.

The total elastic constant of the auxiliary springs of each bearing is

$$k_{a,i} = k_{t,i}^* - k_i, \quad (48)$$

where  $k_i$  is the elastic constant of main spring of each bearing.

## 4. THE BEARING

### 4.1. Bearing with centred main spring (fig. 7)

#### 4.1.1. Constitution

It consists of: a. chamber for housing the sliding level inclined surface 3, consisting of a steel plate 1, connected to the platform by anchoring bolts, and of a steel perimetral spandrel 2; b. actual bearing 4, consisting of a movable ball (contact with rolling friction) or a fixed ball, the upper part of which is covered in Teflon (contact with sliding friction); c. chamber 5 for housing the ball 4, connected to the movable plate 6; d. coaxial chambers 7 and 8 for housing one or more main spring 13. The former chamber is movable with respect to the vertical translation and its upper part is soldered to the plate 6, the other chamber is fixed and it is connected to the base steel plate 14; e. four fixed chambers 9 for housing the auxiliary springs 12 and their respective pistons 10. The couples of chambers are arranged symmetrically with respect to the coaxial chamber 7 and 8. The central space, delimited by

each couple of chambers 9, is partially taken up by the movable mass 11, an extremity of which is inserted rigidly into the external surface of the chamber 7.

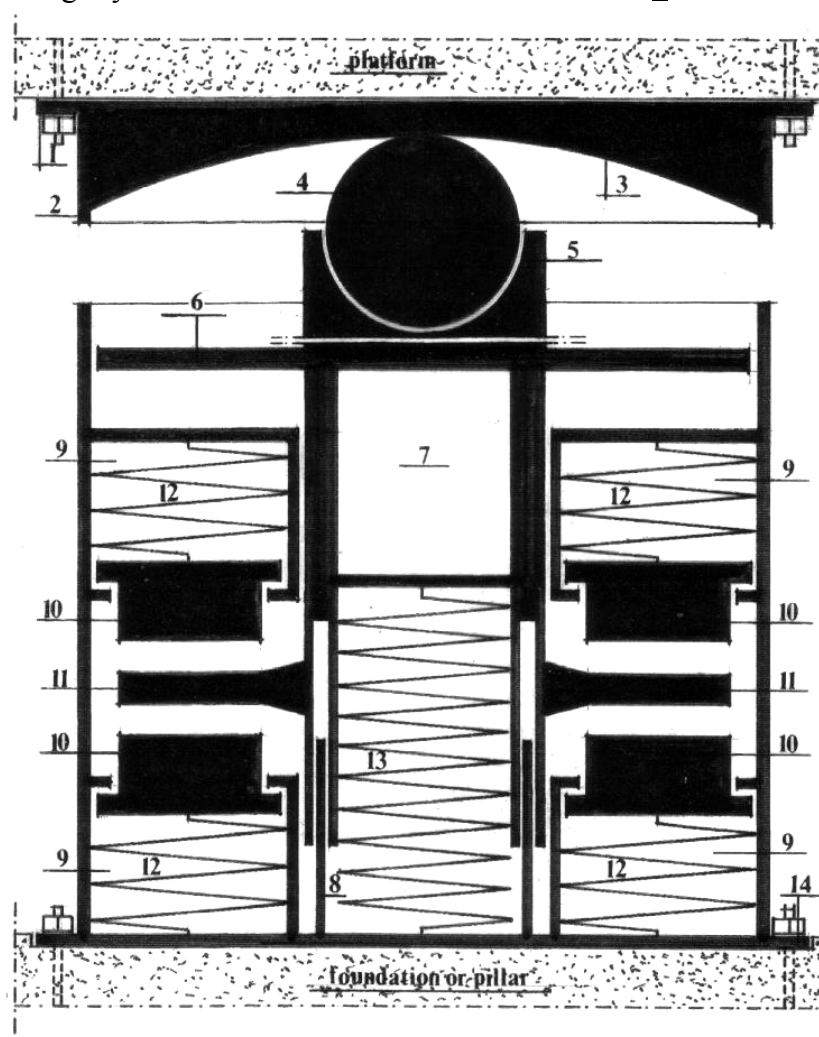


Fig. 7. Bearing with centred main spring 13

#### 4.1.2. Operating principle

When the soil is in a state of rest, the bearing is subjected to the static load transmitted from the building and the main springs 13 become shorter because of compression. The building is perfectly centred because of the level inclined surface 3 and it is motionless. In presence of an earthquake, due to the undulatory shock, the foundation-soil complex translates in a horizontal direction with respect to the building, and the variation of vertical rigid deflection due to the variation of thickness of the level inclined surface 3 is perfectly balanced by the corresponding elastic deformation of the main springs 13 in any instant and for any value of the displacement. The building remains motionless with respect to the horizontal translation of the foundation-soil complex. The horizontal inertial force does not change the static equilibrium of the building, because it has a minor value when using bearings with sliding friction and it is negligible when using bearings with rolling friction. Due to the sub-undulatory shock, the vertical motion of the soil does not modify notably the behavior of the building. In fact, it preserves the verticality, and its vertical displacements, both in phase and in phase opposition with the corresponding displacements of the soil, are

always compatible with project fixed values. When the earthquake frequency is outside to the emergency interval or coincides with the emergency limit frequencies, the mass 11 vertically translates without the participation of the auxiliary springs 12. Vice versa, due to the vertical motion of the mass 11, the participation of the auxiliary springs 12 takes place when the earthquake frequency is inside the emergency interval, including the resonance one. In this case, due to the combined action of the main and auxiliary springs, the increase of natural frequency of the building considerably decreases the vertical displacements of the building to values compatible with the safety conditions of the building.

#### 4.2. Bearing with eccentric main springs (fig. 8)

This bearing differs from the previous one because of the different position occupied by the main and auxiliary springs, which are respectively eccentric and centred. In addition to this, the mass 11 is fixed at both ends and, therefore, its static behaviour is certainly better than the bracket one of the same mass of the preceding bearing. The operating principle remains unchanged.

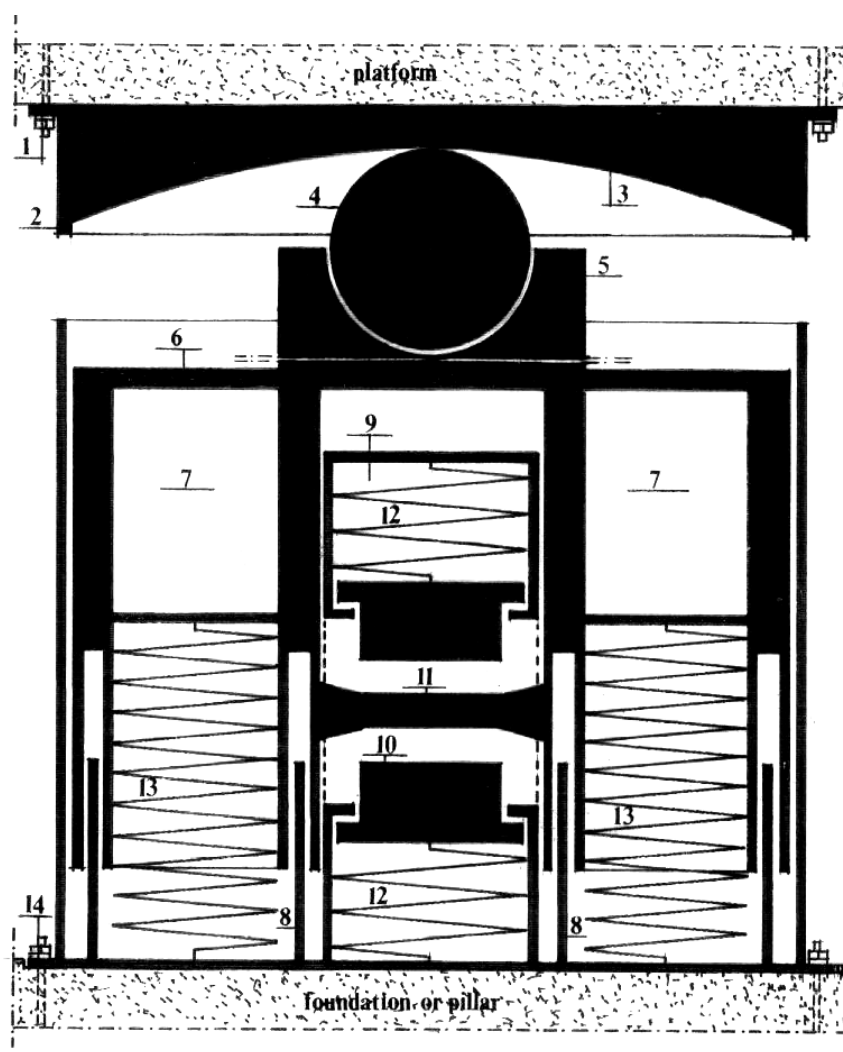


Fig. 8. Bearing with eccentric main springs 13

### 4.3. Original length of the main spring

The maximum static deformation of the spring is

$$\Delta_p = V_{\max} / k_p . \quad (49)$$

The maximum dynamic deformation, due to the horizontal component of the soil displacement, for  $\Omega = 0^\circ$  or  $\Omega = 90^\circ$ , is

$$d^* f_{0,\max} = S_{0,p} \operatorname{tg} \alpha . \quad (50)$$

The maximum dynamic deformation, due to the vertical component of the soil displacement, is

$$d^* f_{v,\max} = \left[ (2 - c_{e,l}^2) / (1 - c_{e,l}^2) \right] S_{v,\max} , \quad (51)$$

where  $S_{v,\max}$  is the vertical component of the soil maximum displacement.

Formula (45) refers to the most unfavorable situation of interaction between the foundation-soil complex and the building. It occurs because of the soil displacement downwards in phase with the building displacement, that is for  $\phi < \phi_n$ , due to the sub-undulatory shock. Therefore, the maximum length of the main spring 13 is

$$\Delta_{\max} = \Delta_p + d^* f_{0,\max} + d^* f_{v,\max} \quad (52)$$

and the original length of the spring is

$$h_i = \mu \Delta_{\max} , \quad (53)$$

where  $\mu > 1$  is the oversize coefficient for unforeseen causes.

## 5. ANALYTICAL TESTS

They consists in testing:

- for seismic frequencies no greater and no smaller than emergency frequencies, that is for  $\phi \leq \phi_{e,l}$  (phase) and  $\phi \geq \phi_{e,h}$  (phase opposition), it must be:

$$S_c = S_v / \left[ 1 - (\phi^2 / \phi_n^2) \right] \leq \xi_1 , \quad (54)$$

where  $S_v$  is the soil vertical displacement, due to the sub-undulatory shock,  $S_c$  is the building vertical displacement and  $\phi_n$  is the first natural vertical frequency of the building.

- for seismic frequencies inside the emergency interval, that is for  $\phi < \phi_e$ , it must be:

$$S_c = \xi_1 + \xi_2^* \leq \xi_{\max} , \quad (55)$$

where  $\xi_1$  is the stroke of the mass 11 – see figures – that is the maximum vertical displacement of the building concerning both the emergency limit frequencies and the seismic frequencies outside the emergency interval, and

$$\xi_2^* = S_v^* / \left[ 1 - (\phi^2 / \phi_n^{*2}) \right] \quad (56)$$

is the extra-stroke of the mass 11, being  $\phi_n^*$  the second natural vertical frequency of the building.

$$S_v'' = S_v + S_v' \quad (57)$$

with

$$S_v' = \xi_1 \left[ 1 - (\phi^2 / \phi_n^2) \right]. \quad (58)$$

Formula (58) is the soil vertical displacement concerning the vertical displacement  $\xi_l$  of the building. For  $\phi = \phi_r = \phi_n$  we have  $S_v' = 0$ ,  $S_v'' = S_v$  and, for  $S_v = S_{v,\max}$ :

$$\xi_2^* = \xi_2 = S_{v,\max} / \left[ 1 - (\phi_n^2 / \phi_n^{*2}) \right]. \quad (59)$$

## CONCLUSIONS

The theoretical results make evident the following characteristics of the proposed system:

- self-centering of the building at the end of an earthquake;
- the undulatory seismic energy absorbed by the building is constant and about 1% of the building weight, using bearings with sliding friction (pure Teflon) and it is negligible using bearings with rolling friction (steel balls). It is independent from the seismic frequency and from the degree of the motion;
- the building remains motionless with respect to the horizontal translation of the foundation-soil complex for any value of the direction angle of the earthquake, of the soil displacement and of the acceleration, both with bearings with sliding friction and bearings with rolling friction;
- the choice of the horizontal displacement for the design of the main spring elastic constant is theoretically. In practice, the choice issues from the opportuneness of using a not very elastic spring, which would cause a very high static yielding of the spring;
- the laying of the bearings with rolling friction causes, in comparison with the bearings with sliding friction, an increase of the spring original length and a decrease of the elastic constant;
- in order to prevent the resonance danger, due to the sub-undulatory shock, the natural frequency variability takes place because of auxiliary springs, which automatically increase the action of the main springs during an emergency, characterized by an vertical seismic frequencies interval, including the resonance one;
- due to the sub-undulatory shock, the total load on the bearings increases for vertical displacement upwards; on the contrary, it decreases for vertical displacement downwards, both in condition of phase and of phase opposition;
- the limited number of bearings is not an obstacle to the application of the system. For a very planimetrically extensive building, it is possible its resolution in more bodies, each of them supplied with four bearings, and connected to each other by means of flexible joints of a few centimeters thickness.

The system described above could be included among the seismic isolation systems of this paper author with “natural” functioning, in the sense that it can carry out the automatic centering of the building after an earthquake. It has the characteristics of economical competitiveness with all existing aseismic systems, due to the considerable decrease of the seismic energy in the building, to the almost total lack of the psycho-physical discomfort in the inhabitants and to the easy maintenance. In the application of the system it is necessary to consider the wind action on the building in the absence of an earthquake. In fact, this action is not dangerous for the building stability, but it is necessary that its intensity does not ever exceed the friction force between the building and the relative bearings, because otherwise, reversible horizontal displacements in the building with respect to the foundation-soil complex could occur with consequent discomfort in the inhabitants. Therefore, if because of particular climatic conditions or because of the building typology or, particularly, because of bearings with rolling friction are used, a wind intensity greater than friction one is feared, we recommend constructing the building in an area protected against wind action. Alternatively, this action could be obstructed by laying hydraulic jacks between the foundation and the building or between the retaining wall and the building, which, electronically powered, can lock the building in the absence of an earthquake and unclasp it in its presence. In this case it is not necessary to construct the building in protected area.

The proposed system needs accurate experimental testing in order to check its validity in the presence of an earthquake and, in its absence, of wind action.

#### AUTHOR'S PUBLICATIONS

- [1] Bartolozzi, F., 1995. System of Base Seismic Isolation with the Project of the Stiff Connection with Alternative Function of Elastic Anti-seismic Linkage. Proceedings of the Second International Conference on Seismology and Earthquake Engineering. Tehran, Iran, 2013-2021.
- [2] Bartolozzi, F., 1996. System of Base Seismic Isolation with the Project of the Fixed Bearing with Alternative Function of Multidirectional Movable Bearing. Proceedings of the International Conference on Earthquakes, Volcanoes and Tsunamis, Pan Pacific Hazards'96. Vancouver, Canada, Paper in CD-rom and Abstract in Book, 23.
- [3] Bartolozzi, F., 1997. Self-centring Aseismic System with Four Rigid Movable Bearings. Proceedings of the Fourth National Conference on Earthquake Engineering. Ankara, Turkey, 536-541.
- [4] Bartolozzi, F., 1998. Self-centring Aseismic System with Four Elastic Bearings and Frequency Converters. Proceedings of the Eleventh European Conference on Earthquake Engineering. Paris, France, Paper in CD-rom and Abstract in Book, 365.
- [5] Bartolozzi, F., 2000. Comparison Between Two Systems of Base Seismic Isolation. Proceedings of the International Symposium on Earthquake Engineering. Montenegro, 267-274.
- [6] Bartolozzi, F., 2000. Self-centring Aseismic System with Elastic Bearings and Hydraulic Dampers. Proceedings of the International Symposium on Earthquake Engineering. Montenegro, 275-282.
- [7] Bartolozzi, F., 2000. A Series of Friction New Bearings for Seismic Isolation. Proceedings of (UN) Coupled 2000, EGS “Stephan Mueller” Topical Conference. Amsterdam, The Netherlands, 211.
- [8] Bartolozzi, F., 2000. Centring and Locking Aseismic Bearing. Proceedings of the 2nd International Conference on Control of Oscillations and Chaos. St. Petersburg, Russia, Volume 1, 124-125.
- [9] Bartolozzi, F., 2000. Self-centring Aseismic System with Double Natural Frequency. Proceedings of the Fifth International Conference on Motion and Vibration Control 2000. Sydney, Australia, 163-166.
- [10] Bartolozzi, F., 2001. Aseismic Bearing with Partially or Totally Curved Sliding Surface and with Angular Corrector. Proceedings of TIEMS-2001, International Conference on Emergency Management. Oslo, Norway, Paper in CD-rom.
- [11] Bartolozzi, F., 2001. Aseismic System with Magnetic Insulators. Proceedings of 7th International Conference on Inspection, Appraisal, Repairs & Maintenance of Building & Structures. Nottingham, United Kingdom, 219-223.
- [12] Bartolozzi, F., 2002. Natural Frequency Automatic Variation in Seismic Isolation System. Proceedings of ACOMEN 2002, 2nd International Conference on Advanced Computational Methods in Engineering. Liège, Belgium, Paper in CD-rom.
- [13] Bartolozzi, F., 2003. Complete Calculation of Lamé Plinth at Constant Height. Proceedings of 2nd International Specialty Conference on the Conceptual Approach to Structural Design. Milan, Italy, 273-282.
- [14] Bartolozzi, F., 2004. Vertically Hinged Anti-seismic System. Proceedings of The Seventh International Conference on Computational Structures Technology. Lisbon, Portugal, Abstract Volume, 599, Paper in CD-rom.