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I. Grushetsky, A. Smol'nikov

Krylov Shipbuilding Research Institute
Russia, 196158, Saint Petersburg, Moskovskoe shosse, 44, eeaa@online.ru

FEM application for calculation of coupling loss factors used in SEA, L-shaped beams case

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The way for coupling loss factors (CLF) determination consisting in finite element method (FEM) simulating of coupled subsystems is presented. The feature of the way is the energy in subsystem remote from energy source is fully dissipated in this subsystem. CLF determination was performed for two beams coupled at right angle. Calculating results agree with well known analytical solution for such a junction of semi-infinite beams. Applying CLF derived from FEM simulating, vibration energy of structure like "stairs" composed of four beams was calculated by statistical energy analysis (SEA), used here under the name of energy method (EM), (approximate method) and by FEM (exact method). Exact and approximate results agree well in octave frequency bands containing two or more resonance frequencies of the structure.

INTRODUCTION

The most common methods for computing of vibration and sound radiation of complex structures are finite element method (FEM) and energy method (EM). EM is often designated as statistical energy analysis (SEA), taking into account some assumptions when applying the method. Commercial packages that implement these two methods are designed: ANSYS, SYSNOISE, ABACUS (FEM), AutoSEA, SEAM, SEADS (EM) and others.

FEM is exact and multi-purpose method. It bases on fundamental theory of elasticity. But using the method numerical problems arise when total number of elements becomes too large (for complex structures and when frequency increases). Thus FEM is usually applied at low frequencies, EM — at middle and high frequencies.

In EM, stationary mode of mechanical system is considered and balance of energies which is injected, outputted and dissipated in each subsystem is made up. These energies are averaged over time, subsystem space (volume, area or length), and frequency (summed in frequency ranges). The most advantages of the method are robustness for inaccuracy of input data and lack of necessity in detailed description of a system. Thus EM is attractive for computing sound and vibration in complex structures.

However EM is approximate method in principle. Moreover coupling loss factors (CLF), which are used in EM (SEA), are approximate. Commonly used CLF were derived from analytical solution for simplified physical models and on some assumptions. Such CLF can be possibly applied in restricted frequency range where these assumptions are valid, although energy conservation law is valid at any frequencies. Besides, analytical solutions do not take into account specific features of some real structures. Therefore applying analytical CLF for practical computations can yield very approximate results or can not be proved at all.

Mentioned limitations do not prevent applying EM since the engineering approaches are always approximate and some ways for increasing of accuracy of the calculations are available.

One of promising methods on EM improving is using FEM for CLF derivation. For simulation of several coupled subsystems many fewer finite elements are required than for simulation of complete system. Therefore FEM solution for CLF can be obtained employing ordinary computer up to quite high frequencies where EM is applied for complete complex system.

Some studies on using FEM for CLF derivation were carried out for plate junctions. In the present work we shall consider CLF derivation by an example of beams and the procedure for CLF determination will differ from previous studies.

1. BASIS OF ENERGY METHOD

The main stage when EM applying is forming mathematical model of a system — set of energy balance equations. Left part of energy balance equations is matrix describing losses in subsystems and mechanical (acoustical) coupling of subsystems. Input power is in the right part. Unknowns are the acoustical or vibration energy in subsystems. Noise or vibration levels for practical purpose are determined from the energy. Well-known energy balance equations are the following:

$$\omega\begin{pmatrix} \eta_1+\sum\limits_{\substack{i=1\\ i\neq 1}}^{n}\eta_{1i} & -\eta_{21} & \cdots & -\eta_{n1} \\ -\eta_{12} & \eta_2+\sum\limits_{\substack{i=1\\ i\neq 2}}^{n}\eta_{2i} & \cdots & -\eta_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ -\eta_{1n} & -\eta_{2n} & \cdots & \eta_n+\sum\limits_{\substack{i=1\\ i\neq n}}^{n}\eta_{ni} \end{pmatrix}\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix}=\begin{pmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{pmatrix}, \tag{1}$$

where η_1, $\eta_2,\ldots\eta_n$ are the internal loss factors (ILF); η_{12}, $\eta_{21},\ldots\eta_{ni}$ are the coupling loss factors (CLF); E_1, $E_2,\ldots E_n$ are the unknown energies of subsystems; W_1, $W_2,\ldots W_n$ are the input energies; ω is the circular frequency.

Coupling loss factors, as mentioned before, are usually analytically calculated. However analytical formulas are available for limited types of subsystem junctions and can not be always applied in practice with confidence in accuracy of results. Lately some attempts are made to calculate CLF using FEM [1, 2].

2. FEM FOR CLF DEFINING. GENERAL APPROACH

General procedure for CLF definition using FEM is the following. FEM model of subsystems in junction is constructed. External forces are applied. Input energy and displacements (pressure) in nodes are defined. Using displacements (pressure) oscillation energies in subsystems are defined. Using energies of subsystems and input energy, CLF can be determined and be used then in energy balance equations.

When determining in this way CLF, in [1] junctions of finite plates with ILF corresponding to plates material were considered. Set of equations for CLF definition was often singular and results for CLF were significantly depended from small changes of subsystem properties (geometrical parameters).

We shall consider and verify on L-shaped beams junction the way for CLF determination that consists in modelling of attached subsystem (beam which is not affected force) as semi-infinite. Such a model is often used when vibration propagation is studied. For example, in physical experiments the subsystems, distant from excited one, are damped by immersing in sand. When simulating by FEM, semi-infinite subsystem will be consider as finite one but with high ILF.

According to energy conservation law, input energy is equal to dissipated energy in closed system.

$$\eta E = W / \omega . \qquad (2)$$

If the excited beam is a part of a structure (as in fig. 1) then the energy losses in beam are caused by internal losses and by energy outflow into attached beam. Loss factor defining from (2) is the sum of ILF and the loss factor describing energy outflow into attached beam i.e. CLF. Energy balance equation for subsystem where energy is injected (beam 1 in fig. 1) is the following:

$$(\eta + \eta_c) E = W / \omega . \qquad (3)$$

In this equation, reverse inflow from beam 2 is not taken into account; it is considered that the energy is fully dissipated in attached beam.

CLF can be determined from (3):

$$\eta_c = W / (\omega E) - \eta . \qquad (4)$$

3. COMPUTING OF CLF FOR L-SHAPED BEAMS JUNCTION

Let us consider the procedure of CLF determination by the example of L-shaped beams simply supported at junction and ends presented in fig. 1. Length of beam 1 — 1.1 m, beam 2 — 0.9 m, cross section — 1×5 cm. Beams are made of the same material (steel), but ILF of beam 2 is high (0.9). Energy is entered in beam 1 by transverse force in figure plane. Simple support in junction is applied to simplify calculations and analysis, because only transverse (bending) vibrations arise in such system.

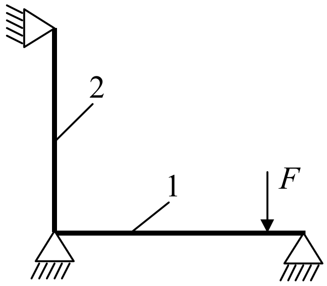

Fig. 1. L-shaped beams junction under consideration

When FEM simulating, the beams are divided into standard beam elements of 1 cm length. Vibration energy and input energy were calculated from complex displacements in nodes. Energy of beam transverse vibration (E) is calculated by the formula:

$$E = \frac{1}{2}\omega^2 \sum_{n=1}^{N} M_n \xi_n^2 , \tag{5}$$

where ξ_n is the displacement amplitudes, M_n is the mass per node, N is the number of nodes in mesh.

As the beam is homogeneous and is modelled by the same elements (masses of elements are the same) then $E = \omega^2 M \overline{\xi^2}/2$, M is the beam mass, $\overline{\xi^2}$ is the squared absolute value of displacement averaged over nodes.

Input power is

$$W = \frac{1}{2}\mathrm{Re}\left(F \cdot v^*\right), \tag{6}$$

where F is the affecting point force (the real number which is specified under FEM simulating, 1 N), v^* is the complex conjugate velocity at the node where the force acts.

As $v = i\omega\xi$, then $W/\omega = -(F \cdot \mathrm{Im}\xi)/2$. Imaginary part of displacement at the node where the force acts is always negative and input power is always positive.

Calculations for separate beam excited by transverse force are shown that the loss factor determined from (2) is practically equal to the loss factor entered as input data under FEM simulating, when this loss factor less than 0.05. When loss factor increases the difference between entered and calculated loss factors increases. It occurs because of travelling waves, which become appreciable in comparison with standing waves (reflected, reverberate field). For the following calculations ILF in beam 1 is equalled 0.01.

Computing of energies and input power according to formulas (5) and (6) were carried out at resonance frequencies of the system composed of two beams (fig. 1). Obtained results were summed in octave frequency bands. As a result of modal analysis (automatic procedure in FEM package), among resonance frequencies there were transverse and longitudinal resonance frequencies. At longitudinal resonance frequencies, resonant vibration does not arise in considered system; both energies and input power are small. Thus their contribution in total values in octave frequency bands is insignificant.

Both input power and vibration energy generally depend on force position. Our calculations were performed for twelve random force positions and results were averaged. Calculation results (formula (4)) are presented in fig. 2, number of resonance frequencies in octave bands is in table 1. One can see from fig. 2, that calculated CLF does not depend on force position if three or more resonance frequencies are in octave bands (250 and 500 Hz).

Coupling loss factor can be expressed through energy transmission coefficient (τ):

$$\eta_c = \frac{c_g \tau}{\omega L}, \tag{7}$$

where c_g is the group speed of bending waves, L is the length of excited beam.

For L-shaped infinite beams of the same cross section energy transmission coefficient is well known [3]: $\tau = 0.5$ (in this case vibration isolation of junction is 3 dB). CLF calculated by (7) for $\tau = 0.5$ is also presented in fig. 2. One can see that numerical and analytical results agree well.

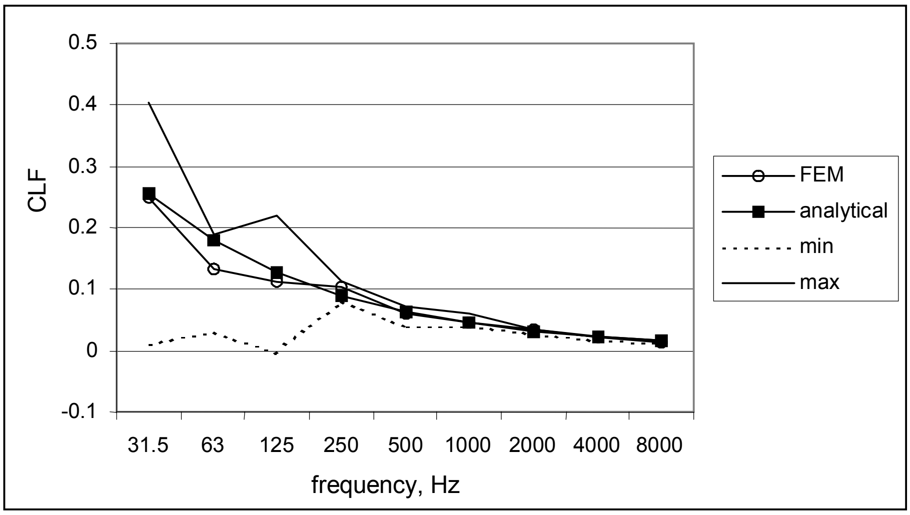

Fig. 2. Coupling loss factors (CLF) of two beams coupled at right angle (fig. 1) under condition that the energy which goes via junction into beam 2 is totally absorbed (η_2=0.9). FEM calculation (averaged over 12 force position, max and min values); analytical results for $\tau = 0.5$

Table 1. Number of resonant frequencies of two coupled beams (fig. 1) in octave frequency bands

31.5	63	125	250	500	1000	2000	4000	8000
1	1	1	3	3	5	9	11	19

4. EVALUATION OF EM APPLICABILITY BY AN EXAMPLE OF BEAM STRUCTURE

For evaluation of EM applicability, let us compute vibration energy of beams that compose structure like “stairs” with simply supported junctions (fig. 3). Calculations are carried out using EM and FEM, which we consider as exact method substituting physical experiment.

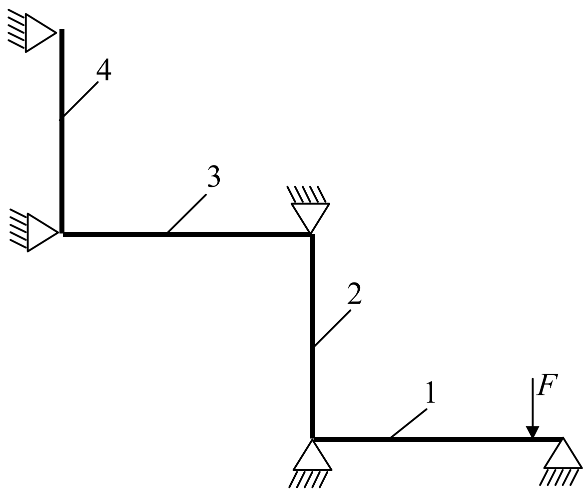

Рис. 3. Structure (“stairs”) composed of consequently coupled beams in which only transverse vibration arises

Set of energy balance equations for the structure in fig. 3 is the following

$$\omega\begin{pmatrix} \eta_1+\eta_{12} & -\eta_{21} & 0 & 0 \\ -\eta_{12} & \eta_2+\eta_{21}+\eta_{23} & -\eta_{32} & 0 \\ 0 & -\eta_{23} & \eta_4+\eta_{32}+\eta_{34} & -\eta_{43} \\ 0 & 0 & -\eta_{34} & \eta_4+\eta_{43} \end{pmatrix}\begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{pmatrix}=\begin{pmatrix} W_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (8)$$

It was found, that FEM results significantly depend on small changes of beams length. For example, calculations were made for three modifications of “stairs” presented in table. 2 The energies (dB ref. 10^{-12} Wt) of beams 1 and 4 for these three modifications of the “stairs” are in fig. 4 and 5. The difference amounts up to 10 dB.

Table 2. Length (m) of beams composing “stairs”

	beam 1	beam 2	beam 3	beam 4
modification 1	1.1	1.05	1.0	0.95
modification 2	1.1	1.1	1.1	1.1
modification 3	1.1	0.9	1.1	0.9

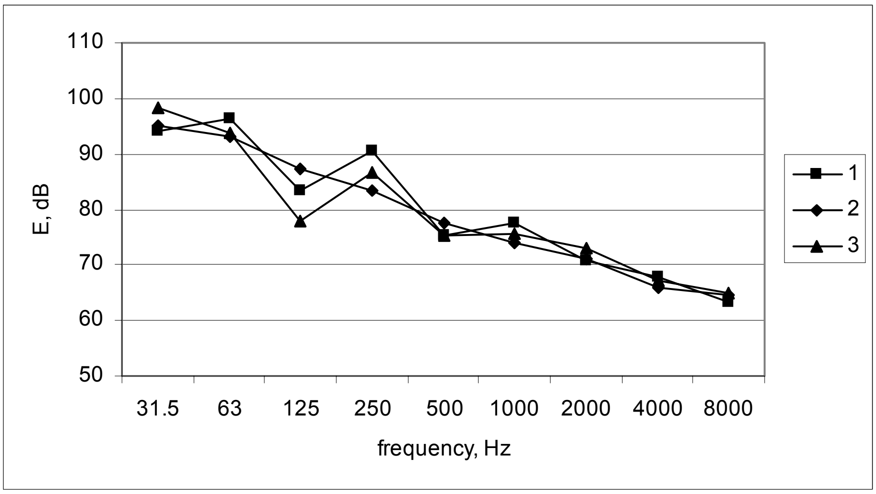

Fig. 4. Vibration energy of beam 1 for three structure modifications (FEM computing)

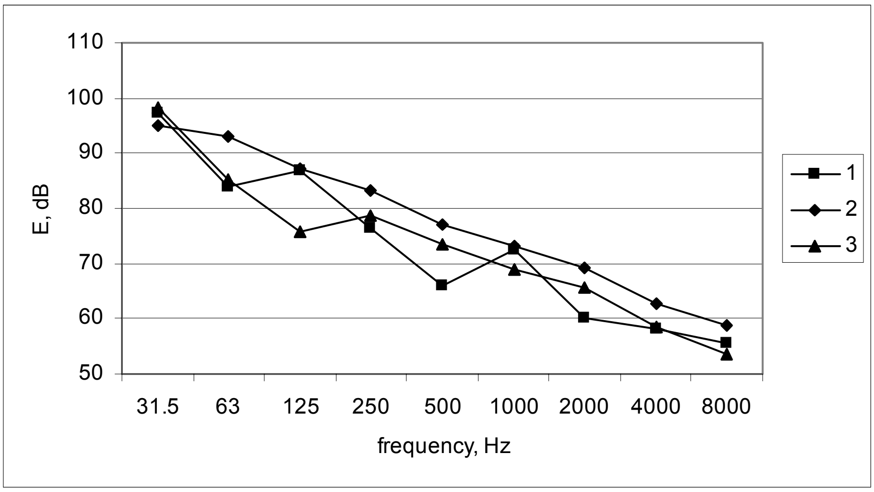

Fig. 5. Vibration energy of beam 4 for three structure modifications (FEM computing)

At the same time the energies computed by EM very little depend on specific dimensions of beams. For the following comparison of results, we shall use FEM results which are averaged over the three modifications.

This approach bases on the following. In real structures dimensions are not known exactly. Some inaccuracy, when structures and their elements are manufactured, always exists. Since any computing yields good results if simulation is adequate, then structure model should take into account the possible deviation of dimensions and other properties of the structure from their formal, expected values. In other words, any formally exact structure model does not correspond exactly to its real condition. Thus the results of single calculation can not be exact for real structure. One of the ways for accuracy increasing is the FEM computing for some ensemble of the same structure, which properties (dimensions, material properties) differ a little within some range. Then the results are averaged and possible variations of the results are included in a report.

CLF for EM calculating was determined from FEM calculation of two beams as shown before; input energy was calculated by FEM as well.

The differences between vibration energies of four beams composing "stairs" which were calculated using exact method (FEM; averaging over three modifications of the structure) and EM (i.e. the difference between exact and approximate solutions) are presented in fig. 6.

On the basis of the obtained results one can say about acceptable for practice accuracy of the approximate computing in a wide frequency range. There is a tendency of accuracy increasing when frequency increases (when the number of resonance frequencies in octave bands increases; see table 3). Absolute values of the difference between exact and approximate solutions at frequencies up to 1000 Hz (from 2 to 6 resonance frequencies are in octave bands) are less than 3 dB. At frequency 1000 Hz and higher (more than 9 resonance frequencies are in octave bands) the difference is less than 1 dB.

If larger amount of FEM data (more than for three modifications of the structure) will be used for analysis, accuracy and limits of EM application can be evaluated more accurate.

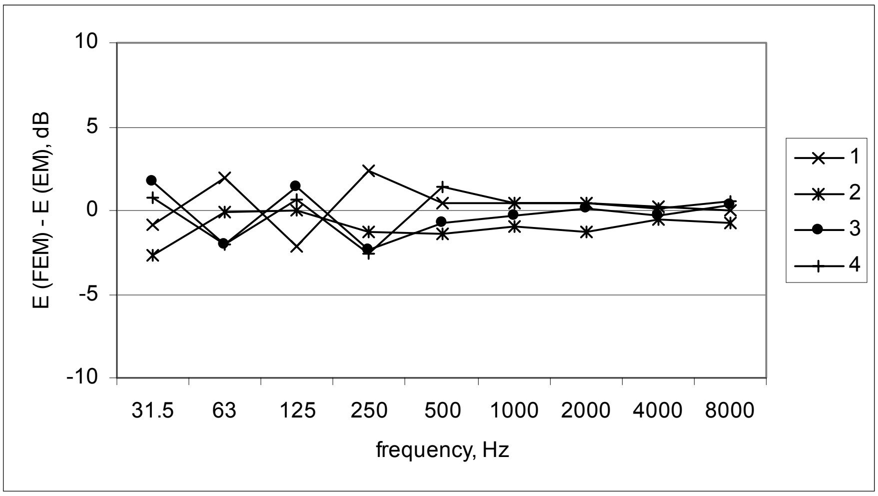

Fig. 6. Difference between exact (FEM) and approximate (EM) solutions for each beam composed structure in fig. 3

Table 3. Number of resonant frequencies in octave frequency bands

	31.5	63	125	250	500	1000	2000	4000	8000
one beam	1	–	1	2	1	4	4	7	10
"stairs" (fig. 3)	2	2	3	6	6	9	16	23	33

CONCLUSIONS

The way for CLF determination consisting in FEM simulating of coupled subsystems is presented. The feature of the way is the energy in subsystem remote from energy source is fully dissipated in this subsystem. CLF determination was performed for two simply supported beams coupled at right angle (L-shaped beams). Calculating results agree with well known analytical solution for such a junction of semiinfinite beams.

Using CLF derived from FEM simulating, vibration energy of structure like "stairs" composed of four beams was calculated by EM (approximate method). The structure was also computed by FEM (exact method). It was found out that the FEM results are significantly influenced by small deviation of beams dimensions (length). Therefore averaged FEM results were used for comparing with EM results. Averaging was carried out over three modification of structure in which the beams with the same number differ a little in length. Exact (FEM) and approximate (EM) results agree well in octave frequency bands containing two or more resonance frequencies of the structure ("stairs"). Absolute values of the difference between exact and approximate solutions are less than 3 dB. When number of resonance frequencies in frequency band increases, accuracy of approximate solution increases.

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