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Strain sensing of interior structural radiation modes on a simply supported panel

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Radiation modes have shown great promise as a suitable measure for estimating the power radiated from structures into both the free field and enclosures. These modes may be sensed with either discrete out-of-plane transducers or continuous strain transducers. Here appropriate sensor shape equations are derived to accurately quantify the interior radiation modes from a simply supported panel radiating into a cavity using piezo-electric strain transducers.

INTRODUCTION

The active control of sound transmission into cavities falls into two main categories: Active Noise Control (ANC) and Active Structural Acoustic Control (ASAC). ANC systems attempt to reduce the interior sound levels by directly controlling the acoustic field with loudspeakers, whereas ASAC systems act by directly modifying the response of the structure. When using structural actuators it is also common to employ structural sensors since this results in a non-intrusive physical control system. It has been shown that purely structural systems often lead to increased sound pressure levels if the cost function is also structural, i.e. structural kinetic energy [1, 2]. A more appropriate structural cost function can be obtained via radiation modes of the structural/cavity system, which are orthogonal in terms of their contribution to the acoustic response of the cavity [3]. Therefore, by minimising the amplitudes of one or more radiation modes, one is guaranteed a reduction in the radiated acoustic sound field.

Radiation mode shapes can be considered to be independent of frequency over a limited bandwidth [4]. This allows radiation modes to be sensed with fixed gain transducers such as meta-sensors comprised of several discrete sensors (with appropriately weighted outputs) or continuous strain sensors. The measurement of radiation modes using out-of-plane transducers is straightforward since the mode shapes of radiation modes are independent of structural boundary conditions. However, the same does not hold for strain sensors because the in-plane surface strain induced by a particular out-of-plane vibration profile is dependent on the structural boundary conditions. Subsequently, the strain radiation mode shapes are also a function of the structural boundary conditions.

Several papers [5–7] have investigated the use of a quadratically-weighted strain integrating sensor (QWSIS) to measure the volume velocity from rectangular panels which incidentally corresponds to the first radiation mode of a rectangular enclosure [8]. In this paper the sensor shape equations are extended to include all radiation modes for a simply

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supported rectangular panel radiating into a rectangular cavity. The theory for using shaped PVDF film sensors is initially reviewed. The quadratically-weighted strain integrating sensor for measuring the volume velocity of a simply supported rectangular panel is discussed and then a universal sensor shape equation is presented which can be used to calculate the sensor shape required for each radiation mode. The work presented here is a summary of the procedure, the derivations of which can be found elsewhere [9].

It should be noted that although radiation mode shapes are independent of the structural boundary conditions, the sensor equations derived below are only applicable to a simply supported rectangular panel. This is because the surface strain is a function of the boundary conditions and therefore, the surface strain mode shape of a radiation mode for a simply supported rectangular panel will be quite different to a rectangular panel with differing boundary conditions (for example clamped-clamped).

The internal radiation mode shapes used for the following example are those of a rectangular cavity with two of the three dimensions the same as the rectangular panel. However, the sensor equations derived here are applicable for any cavity, provided that the acoustic mode shapes on the surface of the panel are identical to that of the rectangular cavity which is often the case at low frequencies for similar shaped cavities. If the acoustic mode shapes of the cavity differ from the ones used here, then the same approach may still be employed to derive analytical expressions for the strain sensor shapes.

1. BACKGROUND

For shaped sensors to be of any practical use it is essential that the quantity which is to be measured is independent of frequency. If the quantity does not meet this restriction then the sensor is only suitable at a single frequency. It has been shown that radiation mode shapes can be considered independent of frequency over a narrow frequency band when the acoustic modal density is low [4, 10] and consequently strain sensing of internal radiation modes is practical.

1.1. Discrete Transducers

Creating meta sensors by summing the weighted output from several discrete transducers is the simplest method of creating a distributed sensor. The sensor equations can be derived directly by using the least squares expression for the modal filter [11]. Maillard and Fuller [12] successfully used a set of discrete accelerometers to measure and control the volume acceleration (the most efficient free-field radiation mode) of a plate radiating into free space. Such discrete sensing systems are not without their problems, the biggest being aliasing of higher order structural modes and modal spillover.

Morgan [13] and Snyder et al. [14] discussed the issues of discrete modal sensing for real systems and found that a non-trivial number of sensors may need to be employed. Judicious placement of the discrete sensors can improve the performance of the meta sensors. For example, when the physical system is of regular geometry, it is often possible to use symmetry to eliminate the measurement of unwanted modes [14]. This was suggested by Elliott and Johnson [15] to aid in the sensing of particular clusters of structural modes of a rectangular panel.

1.2. Continuous Piezoelectric Transducers

It has been shown that continuous sensors have many advantages over discrete sensors, the most important being the enhancement in observability and reduction in spillover [16]. The following section considers radiation mode sensing using PVDF film. PVDF film is an extremely flexible piezo-electric polymer which develops a charge between the two electrodes when subjected to an applied strain. The film has been used successfully as a distributed sensor by many researchers [17–19].

1.2.1. Sensor equations

Lee [17] showed that the total charge generated by a piezoelectric lamina is a function of the integral of strain over the surface of the lamina and is expressed as

$$q(t)=\int_S \Gamma(\vec{\mathbf{x}})\left[\begin{array}{c} e_{31}\dfrac{\partial u(\vec{\mathbf{x}},t)}{\partial x}+e_{32}\dfrac{\partial v(\vec{\mathbf{x}},t)}{\partial y}+e_{31}\left(\dfrac{\partial u(\vec{\mathbf{x}},t)}{\partial y}+\dfrac{\partial v(\vec{\mathbf{x}},t)}{\partial x}\right) \\ -\dfrac{t_s+t_f}{2}\left(e_{31}\dfrac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial x^2}+e_{32}\dfrac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial y^2}+2e_{36}\dfrac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial x\partial y}\right)\end{array}\right]dS(\vec{\mathbf{x}})\,, \quad (1)$$

where $u(\vec{\mathbf{x}},t)$, $v(\vec{\mathbf{x}},t)$ and $w(\vec{\mathbf{x}},t)$ are the displacements in the local coordinates x, y and z at a location $\vec{\mathbf{x}}=[x,y,z]$ at time t; the *z*-axis defines the normal to the surface of the lamina; $\Gamma(\vec{\mathbf{x}})$ is the shape function of the PVDF sensor, t_s and t_f are the thickness of the shell and film respectively, and e_{31}, e_{32} and e_{36} are the directional piezoelectric [(charge/area)/strain] field intensity constants given by

$$\begin{bmatrix} e_{31} \\ e_{32} \\ e_{36}\end{bmatrix}=\begin{bmatrix} E/(1-\nu^2) & E\nu/(1-\nu^2) & 0 \\ E\nu/(1-\nu^2) & E/(1-\nu^2) & 0 \\ 0 & 0 & E/2(1+\nu)\end{bmatrix}\begin{bmatrix} d_{31} \\ d_{32} \\ d_{36}\end{bmatrix}, \quad (2)$$

where E and ν are the Young's Modulus and Poisson's Ratio of the PVDF respectively, and d_{31}, d_{32} and d_{36} are the piezoelectric strain constants. The first four terms in Equation (1) represent the strain arising from in-plane motion ($\dfrac{\partial u(\vec{\mathbf{x}},t)}{\partial x},\dfrac{\partial v(\vec{\mathbf{x}},t)}{\partial y},\dfrac{\partial u(\vec{\mathbf{x}},t)}{\partial y}$ and $\dfrac{\partial v(\vec{\mathbf{x}},t)}{\partial x}$), the second three terms ($\dfrac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial x^2},\dfrac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial y^2}$ and $\dfrac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial x\partial y}$) are the out-of-plane terms arising from bending in the shell. In general, for thin shells excited by out-of-plane forces, the in-plane terms are negligible compared to the out-of-plane terms, and therefore may be neglected. Thus

$$q(t)=\int_S -\left(\frac{t_s+t_f}{2}\right)\Gamma(\vec{\mathbf{x}})\left[e_{31}\frac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial x^2}+e_{32}\frac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial y^2}+2e_{36}\frac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial x\partial y}\right]dS(\vec{\mathbf{x}})\,. \quad (3)$$

The subscript 6 takes into account the possibility that the principal axes of the PVDF film are not coincident with the principal axes of the structure to which it is bonded [20]. Subsequently, when the lamina is placed on the surface of the shell with no skew angle as shown in Figure 1, then the piezoelectric constant e_{36} is zero. Therefore the charge equation reduces to

$$q(t)=\int_S -\frac{t_s+t_f}{2}\Gamma(\vec{\mathbf{x}})\left[e_{31}\frac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial x^2}+e_{32}\frac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial y^2}\right]dS(\vec{\mathbf{x}})\,. \quad (4)$$

For typical PVDF film, the thickness is negligible when compared to the thickness of a typical shell and is thus neglected here:

$$q(t)=\int_S -\frac{t_s}{2}\Gamma(\vec{\mathbf{x}})\left[e_{31}\frac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial x^2}+e_{32}\frac{\partial^2 w(\vec{\mathbf{x}},t)}{\partial y^2}\right]dS(\vec{\mathbf{x}})\,. \quad (5)$$

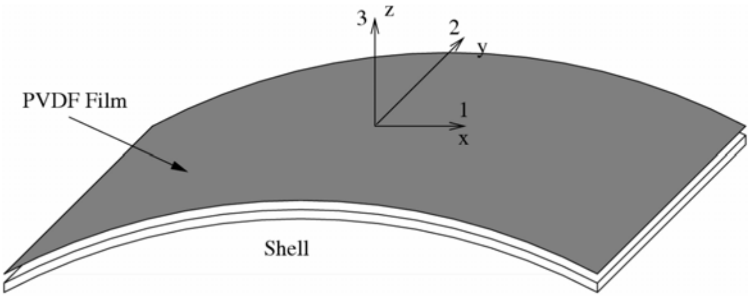

Figure 1. Shell and PVDF film orientation where the z-axis is the poling direction of the film

1.2.2. Sensor shapes for modal sensing of normal structural modes

Lee and Moon [18] showed that for a simply supported plate with length *a*, width *b* and mode shape functions of the form

$$\psi_{i,j}(\vec{\mathbf{x}})=\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right) \quad (6)$$

with the velocity normal to the surface of the structure given by

$$v(\vec{\mathbf{x}})=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}A_{i,j}\psi_{i,j}(\vec{\mathbf{x}}) \quad (7)$$

and using the condition of orthogonality, Equation (5) can be written as

$$q(t)=-\frac{t_s}{2}\int_S \Gamma(\vec{\mathbf{x}})\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}A_{i,j}\left[\begin{array}{l} e_{31}\left(-\frac{i^2\pi^2}{a^2}\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)\right) \\ +\,e_{32}\left(-\frac{j^2\pi^2}{b^2}\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)\right)\end{array}\right]dS(\vec{\mathbf{x}})\,, \quad (8)$$

where $A_{i,j}$ is the modal amplitude of the i,j^{th} structural mode. It should be noted that Equation (8) was given incorrectly by Lee and Moon [18]. Setting the sensor shape function to

$$\Gamma(\vec{\mathbf{x}})=\sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right) \quad (9)$$

the charge developed by the sensor (Equation 8) is given by

$$q(t) = -\frac{t_s}{2}\int_S \sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} A_{i,j}\begin{bmatrix} e_{31}\left(-\frac{i^2\pi^2}{a^2}\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)\right) \\ +e_{32}\left(-\frac{j^2\pi^2}{b^2}\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)\right)\end{bmatrix} dS(\vec{\mathbf{x}})\,. \tag{10}$$

Evaluating the integral over the surface and using orthogonality, this can be correctly reduced to

$$q(t) = \delta_{il}\delta_{jm}A_{i,j}\frac{t_s}{2}\frac{1}{4}\left[e_{31}\frac{ib}{ja} + e_{32}\frac{ja}{ib}\right], \tag{11}$$

where δ_{il} and δ_{jm} are the Kronecker delta functions corresponding to the x and y directions. Therefore, this implies that for a simply supported panel, the sensor shape function to measure the structural modes is proportional to the normal structural mode shape displacement functions. This does not hold true for all structural mode shapes but only for systems with mode shapes proportional to the double spatial derivative of the mode shape function.

2. SENSING RADIATION MODES

The following derivation is based on the assumption that the radiation mode shapes are identical to the dominant acoustic mode shapes at the structural boundaries. This assumption holds at low frequencies for any shaped enclosure when the acoustic modal density is low and the flexible structure forms a large part of the bounding surface [8]. If the latter two conditions do not hold, then at high frequencies the internal radiation modes of the panel degenerate to approximately the free field radiation mode shapes [7] which form an entirely different set of mode shapes than the ones considered here.

2.1. Sensing the bulk compression acoustic mode/volume velocity radiation mode

Rex and Elliott [5] showed that if a sensor had a sensitivity that varied quadratically over the surface of a beam, then the output from the sensor was proportional to the total transverse displacement over the length of the beam. The theory of a Quadratically-Weighted Strain-Integrating Sensor (QWSIS) was extended to rectangular plates by Johnson and Elliott [6] and Johnson [7]. The authors show that for a clamped plate the appropriate sensor shape function to measure the volume velocity from the plate is given by

$$\Gamma(x,y) = \alpha(ax - x^2)\,, \tag{12}$$

where α is a constant. The total charge output from such a sensor was given by

$$q = t_s e_{31}\alpha\frac{U}{j\omega}, \tag{13}$$

where U is the volume velocity of the plate. However, Johnson [7] showed that for a simply supported plate the situation is a little more complicated and the solution requires two layers of PVDF on top of one another as shown in Figure 2. Here the piezoelectric axis of the second film is rotated by 90 degrees and in doing so Johnson [7] found a novel way to remove the sensitivity of the sensor to bending in the *y* direction. Alternatively, rather than stacking the two sensors on top of each other a sensor could be placed on either side of the panel [21].

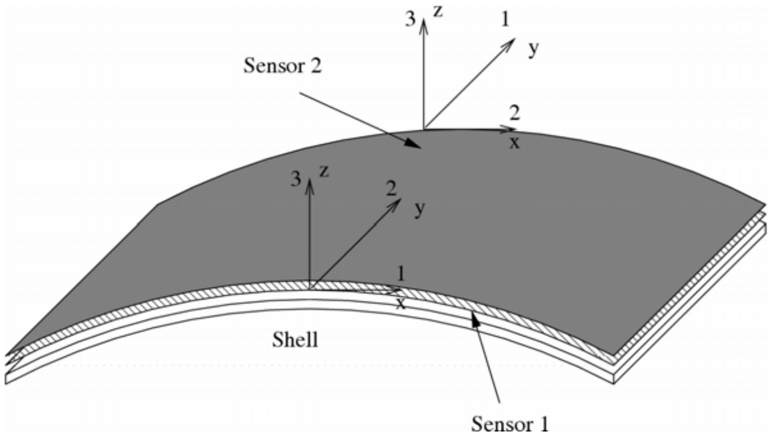

Figure 2. The orientation and axes for the sensor configuration used by Johnson [7] to eliminate the q_y component of the induced charge and hence measure the volume velocity of a plate

Assuming that the two piezoelectric constants e_{31} and e_{32} are known, using the quadratic sensor Equation (12) and combining the outputs from the two layers using the appropriate weighting ($\frac{e_{32}}{e_{31}}$), the volume velocity of a simply supported panel is given by [7]

$$q_0 = q_1 - \frac{e_{32}}{e_{31}} q_2 = \left(1 - \left(\frac{e_{32}}{e_{31}}\right)^2\right) t_s e_{31} \alpha \frac{U}{j\omega}, \tag{14}$$

where q_0 is the combined sensor output, and q_1 and q_2 are the charge outputs from the first and second sensor respectively.

Johnson [7] points out that for such an approach to work the piezoelectric constants must be accurately known and the pair of sensors must be fixed in a manner such that they are equally sensitive to any strain across the surface of the plate. In practice this may be difficult to achieve, particularly if only a single surface is available for the lamina since any sensor, apart from the one bonded directly to the surface of the structure, will often have a slightly uneven surface available for mounting. This is especially common along the edges of the lower sensors which lie beneath the upper sensors and the structure.

Several other authors have derived the sensor shape function to measure the volume velocity radiated from a panel into free space from measured experimental modal analysis data with some success [22, 23]. This approach could also be used numerically. However, both techniques are time consuming and require considerable measurement effort. An alternative approach is to use the fact that it is the odd-odd structural modes which form the first (volume velocity) radiation mode, just like it is the odd sine terms in the Fourier series which form a square wave. It can also be shown that a sheet of PVDF film with unit sensitivity across the whole surface of the panel only responds to the odd-odd modes. Therefore, although not entirely accurate it is possible to get a measure of the volume velocity from uniformly weighted surface sensors.

If the sensor shape function is set equal to unity across the panel surface (which is the same surface weighting as the bulk compression acoustic mode), i.e.

$$\Gamma(\vec{\mathbf{x}}) = 1, \tag{15}$$

it can be shown that the charge developed by a single layer sensor is given by [9]

$$q = \frac{t_s}{2} e_{31} \frac{4}{ab} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{A_{i,j}}{ij} \left(i^2 b^2 + \frac{e_{32}}{e_{31}} a^2 j^2 \right) (1-(-1)^i)(1-(-1)^j), \tag{16}$$

and the charge developed by a double layer sensor is given by [9]

$$q = \frac{t_s}{2} e_{31} \left(1 - \left(\frac{e_{32}}{e_{31}} \right)^2 \right) \frac{4b}{a} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{A_{i,j}}{ij} i^2 (1-(-1)^i)(1-(-1)^j). \tag{17}$$

Comparing this to the expression developed by Johnson [7] for the charge developed by using a double layer quadratic sensor shape on a simply supported panel which accurately senses the volume velocity of the panel,

$$q = \frac{t_s}{2} e_{31} \left(1 - \left(\frac{e_{32}}{e_{31}} \right)^2 \right) \frac{4\alpha}{\pi^2} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{A_{i,j}}{ij} (1-(-1)^i)(1-(-1)^j), \tag{18}$$

shows that both of the double layer expressions, given by Equations (17) and (18) are identical apart from the additional i^2 term in Equation (17). This means that the structural modes with larger indices along the *x*-axis are biased when using a uniform sensor. For systems which are long and narrow, i.e. the j index increases much more quickly than the i index, the uniform sensor is certainly a suitable approximation at low frequencies.

Clark and Fuller [24] experienced exactly this problem when using uniform strip sensors on a rectangular simply supported plate to control the efficiently radiating modes from the panel. They noted that as the excitation frequency increased the sensor became increasingly responsive to higher order modes. In concluding, they noted that almost optimal levels of control were achieved on resonance (when a single mode dominates the response and the control mechanism is modal control). However, the control achieved off-resonance using the strip sensors was slightly less than that offered by several microphones (when several structural modes contribute to the response and the control mechanism is modal rearrangement).

It will be seen in Section 2.2.1 that these problems persist when sensing the higher order modes using sensor shapes proportional to the acoustic pressure on the surface of the structure. In Section 2.2.2 a technique for overcoming such limitations will be introduced.

2.2. Sensing the higher order radiation modes

The sensor shape and charge equations derived above are only for the first (volume velocity) radiation mode. For higher order modes an alternative sensor shape is required. As mentioned earlier, Cazzolato and Hansen [8] showed that for certain conditions (low acoustic modal densities and the flexible structure covers a high percentage of the cavity boundary) the radiation mode shapes are identical to the acoustic mode shapes of the enclosed space across the surface of the structure. In the case of a rigid rectangular cavity, the acoustic mode shapes across the surface of a single side of the cavity are given by

$$\phi_{l,m}(x,y) = \cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right), \tag{19}$$

where a and b are the dimensions of the sides, l and m are the modal indices and x and y are the physical coordinates. It can also be shown that the Fourier series (for $0 \le \theta \le \pi$) of a cosine in terms of sines is given by

$$\cos(n\theta) = \sum_{k=1}^{\infty} a_{k,n} \sin(k\theta), \tag{20}$$

where

$$a_{k,n} = \frac{2}{\pi}\left(1-(-1)^{k+n}\right)\frac{k}{(k-n)(k+n)}. \tag{21}$$

Therefore, by inserting Equation (20) into Equation (19) it is possible to derive an expression for the radiation mode shapes in terms of the normal structural mode shapes (Equation 6). The above relationship will now be used to derive the expression for the sensor shape to sense the higher order radiation modes.

2.2.1. Simple Approach

Using the Fourier Series defined by Equation (20), it can be shown that if the sensor is to measure the higher order radiation modes (which at low frequencies resemble Equation 23), then the charge developed should be [9]

$$q(t) = \frac{t_s}{2}\beta\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} A_{i,j}\left(1-(-1)^{i+l}\right)\left(1-(-1)^{j+m}\right)\left(\frac{i}{(i-l)(i+l)}\right)\left(\frac{j}{(j-m)(j+m)}\right), \tag{22}$$

where β is some constant. One possible solution to the sensor shape function is to use the mode shape of the desired radiation mode (which at low frequencies resemble the acoustic mode shapes), i.e.

$$\Gamma_{l,m}(\vec{\mathbf{x}}) = \cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right). \tag{23}$$

Using the sensor shape Equation (23), it can be shown that the charge developed by a single layer sensor is [9]

$$q(t) = \frac{t_s}{2}\frac{e_{31}}{ab}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} A_{i,j}\left(i^2b^2 + \frac{e_{32}}{e_{31}} j^2a^2\right)\left(1-(-1)^{i+l}\right)\left(1-(-1)^{j+m}\right) \times\left(\frac{i}{(i-l)(i+l)}\right)\left(\frac{j}{(j-m)(j+m)}\right) \tag{24}$$

and the charge developed by a two layer sensor sensitive to bending in the *x*-direction is [9]

$$q(t) = \frac{t_s}{2}e_{31}\left(1-\frac{e_{32}^2}{e_{31}^2}\right)\frac{1}{ab}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} A_{i,j} i^2\left(1-(-1)^{i+l}\right)\left(1-(-1)^{j+m}\right) \times\left(\frac{i}{(i-l)(i+l)}\right)\left(\frac{j}{(j-m)(j+m)}\right). \tag{25}$$

Although Equations (24) and (25) will always guarantee that the appropriate modes are sensed, they suffer from the same problem inherent in Equations (16) and (17), where the sensors are over sensitive to the higher order modes in two or one direction respectively. However, as a first order approximation it can be quite useful since no effort is required to calculate the sensor shape and the procedure does not require prior knowledge of the structural mode shapes. Therefore, the procedure is suitable for any geometric arrangement of structure and cavity.

2.2.2. Accurate Approach

If an accurate estimate of the higher order radiation modes is required, then the following formulation of the shape function is necessary. Intuitively, a sensor equation with a $1/i^2$ bias will remove the bias in the charge sensitivity inherent in Equation (25). It can be shown that for a simply supported rectangular panel with a sensor shape function given by [9]

$$\Gamma_{l,m}(\vec{\mathbf{x}}) = \sum_{k=1}^{\infty}\overline{A}_{k,l}\sin\left(\frac{k\pi x}{a}\right)\sum_{k'=1}^{\infty}\overline{A}_{k',m}\sin\left(\frac{k'\pi y}{b}\right), \tag{26}$$

where

$$\overline{A}_{k,l} = \frac{2}{\pi}\left(1-(-1)^{k+l}\right)\frac{k}{(k-l)(k+l)}\frac{1}{k^2}, \tag{27}$$

$$\overline{A}_{k',m} = \frac{2}{\pi}\left(1-(-1)^{k'+m}\right)\frac{k'}{(k'-m)(k'+m)} \tag{28}$$

and l and m are the modal indices of the desired radiation mode. The term $1/k^2$ in (27) is to remove the i^2 bias in the *x*-direction in Equation (25). Then the charge developed by a single sensor is [9]

$$q(t) = -\frac{t_s}{2}\frac{e_{31}}{a^2}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} A_{i,j}\left(1+\frac{e_{32}}{e_{31}}\frac{j^2a^2}{i^2b^2}\right)\left(1-(-1)^{i+l}\right)\left(1-(-1)^{j+m}\right) \times\left(\frac{i}{(i-l)(i+l)}\right)\left(\frac{j}{(j-m)(j+m)}\right) \tag{29}$$

and the charge developed by a two layer sensor sensitive to bending in the *x*-direction is [9]

$$q(t) = -\frac{t_s}{2} e_{31}\left(1-\frac{e_{32}^2}{e_{31}^2}\right)\frac{1}{a^2}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} A_{i,j}\left(1-(-1)^{i+l}\right)\left(1-(-1)^{j+m}\right) \times\left(\frac{i}{(i-l)(i+l)}\right)\left(\frac{j}{(j-m)(j+m)}\right). \tag{30}$$

The notable difference between the double layer expression of Equations (25) and (30) is the absence of the i^2 term in Equation (30). It can be seen that Equation (30) is identical to desired expression given by Equation (22) where

$$\beta = -e_{31}\left(1-\frac{e_{32}^2}{e_{31}^2}\right)\frac{1}{a^2}. \tag{31}$$

Therefore using Equation (26) for the sensor shape it is possible to measure accurately the modal amplitudes of the radiation modes. It can be shown that if $l = 0$ and $m = 0$, Equation (26) successfully collapses to the quadratic shape function (Equation (12)) proposed by Johnson [7], and the charge output of the dual layer sensor given by Equation (30) collapses to the charge output for the dual layer volume velocity sensor (Equation (18)) calculated by Johnson [7]. Therefore the sensor shape function given by Equation (26) is a universal sensor shape function for a simply supported panel since it is capable of being used to derive the sensor shape for each of the interior radiation modes, including the volume velocity radiation mode, for a rectangular cavity. The sensor shapes for sensing the first four radiation modes derived from Equation (30) with respect to the *x*-axis are shown in Figure 3. These curves have been normalised so that the maximum absolute value is unity.

2.2.3. Polynomial Approximations

In an attempt to simplify Equation (26) for use in practice, a low order polynomial of the form

$$\Gamma_l(\vec{\mathbf{x}}) = \sum_{i=0}^{\infty} a_i \left(\frac{x}{a}\right)^i, \tag{32}$$

where a_i is the polynomial coefficient and x/a is the non-dimensional coordinate, was fitted to the first half of the expression (i.e. the *x* direction) and the results have been plotted in Figure 3 with the polynomial coefficients shown in Table 1. The polynomial expression for the first (volume velocity) radiation mode, $l = 0$, is identical to Equation (12) derived by Johnson and Elliott [6]. It is also interesting to note that the second radiation mode shape ($l = 1$) is very similar to the second order structural mode shape ($\sin(2\pi x/a)$), which indicates that shaped sensors designed to measure certain radiation modes could be very sensitive to leak through of undesired structural modes. Note that the polynomial coefficient for the first radiation mode along the *y*-axis ($m = 0$) is unity as expected. The overall sensor shape equation is therefore given by the product of Equations (32) and (33), i.e. $\Gamma_{l,m}(\vec{\mathbf{x}}) = \Gamma_l(\vec{\mathbf{x}})\Gamma_m(\vec{\mathbf{x}})$.

Table 1. Sensor equation polynomial coefficients for the first 4 radiation modes bounded by $0 \le \frac{x}{a} \le \pi$

Radiation Mode Index, l	Coefficient, a_i							
	$\left(\frac{x}{a}\right)^0$	$\left(\frac{x}{a}\right)^1$	$\left(\frac{x}{a}\right)^2$	$\left(\frac{x}{a}\right)^3$	$\left(\frac{x}{a}\right)^4$	$\left(\frac{x}{a}\right)^5$	$\left(\frac{x}{a}\right)^6$	$\left(\frac{x}{a}\right)^7$
0	0	4	–4	-	-	-	-	-
1	0	$\frac{10\pi}{3}$	-10π	$\frac{20\pi}{3}$	-	-	-	-
2	0	0	-5π	10π	-5π	-	-	-
3	0	0	0	$\frac{-600\pi}{7}$	$\frac{3000\pi}{7}$	$\frac{-5400\pi}{7}$	$\frac{4200\pi}{7}$	$\frac{-1200\pi}{7}$

It should be noted that each strain sensor is only capable of sensing a single radiation mode. At low frequencies a single sensor is often adequate to control the majority of sound transmission over a significant frequency range [4, 10]. However, at higher frequencies where several acoustic modes can dominate the response, then an equivalent number of sensors are required. The application of several strain sensors over a surface may prove to be impractical.

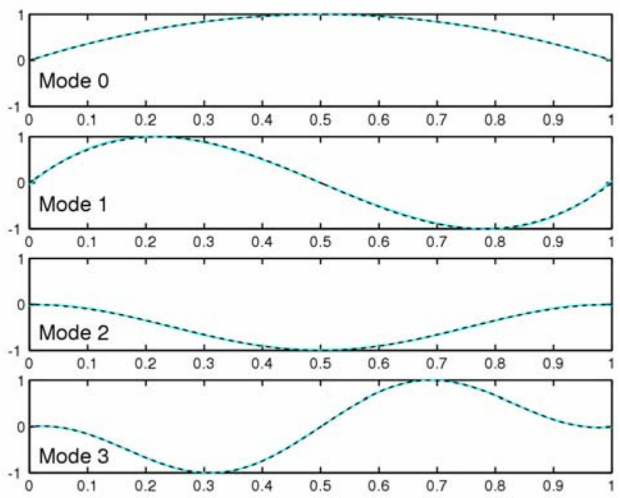

Figure 3. Normalised sensor shapes in the *x* direction to measure the first four radiation modes on a simply supported panel. Sensor shape (solid) and low order polynomial (dashed)

CONCLUSIONS

The equations which define the sensor shapes to measure the interior radiation modes for a simply supported panel have been derived. Expressions for the volume velocity radiation mode have been validated. The performance of a strain sensor with a shape proportional to the pressure of an acoustic mode across the surface of the panel was investigated with the advantages and limitations of the approach highlighted. Finally, the QWSIS approach was used to arrive at an expression for the sensor shapes to measure all interior radiation modes of a simply supported panel.

The strain sensor equations derived using the simple approach, where the sensor shape is proportional to the acoustic mode shape at the enclosure boundary, have been used in experiments to actively control low frequency sound transmission from a simply supported rectangular panel into a rectangular cavity [9, 25]. Limited success was achieved, with the radiation mode sensors suffering from leak-through of undesired structural modes as predicted.

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