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Design sensitivity analysis of Statistical Energy Analysis models using Transfer Path Approach

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Statistical Energy Analysis (SEA) is an attractive tool to predict noise and vibration at high frequencies. It provides an ideal framework to carry out optimisation and sensitivity analysis. Design Sensitivity Analysis (DSA) using SEA helps the designer in making changes, by indicating the model parameters that cause the greatest benefit in either narrow or broadband noise and/or vibration reduction. This paper describes a simple and effective way of getting design sensitivities of noise and/or vibration response (or objective function formed using total energy response) to subsystem damping in SEA contest. Final form of design sensitivity vector, derived using Transfer Path Approach (TPA), suggests that the method is equally applicable for analytical and experimental DSA. A general proof for convergence of sensitivity vector thus derived is given to support the approach. Example of an automobile model, formed from 2 *acoustic spaces* and 24 *plates*, is given to validate and demonstrate the method.

INTRODUCTION

Design optimization techniques can be used to improve any non-optimal design. Optimization methods use search algorithms to minimize an objective (or cost) function. The algorithms must verify design parameters and use the resulting response trends to determine an optimal design. Integration of optimization techniques and numerical structural and/or acoustic response predictions tools like Finite Element Method (FEM) and Boundary Element Method (BEM) would help investigate a large range of design alternatives. However, the above tools are useful at low frequencies and do not give good estimates of response at high frequencies. In addition, the displacement frequency response function found using these methods has sharp peaks at resonance frequencies; the differentiation is not stable at these resonance frequencies and hence introduces instability in optimization algorithms.

A Statistical Energy Analysis (SEA) model of a system design is useful for the optimization of acoustic/vibration control parameters in an efficient manner. Frequency-averaged energy or power is well suited for broadband frequency optimization. The SEA energy and power variables are generally smooth functions of frequency and, therefore, give more stable convergence in optimization algorithms [1]. It is stated that the total energy amplitude response determined in SEA is more robust than the displacement amplitude response determined in deterministic techniques like FEM. This concept makes SEA formulation particularly well suited for optimization. The key benefits of a general

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methodology for optimization using SEA will be its applicability to a broad class of problems, including quiet machinery designs in industrial environments, reducing far-field radiated noise signatures of submerged military vehicles, and reducing interior noise levels in automobiles and aircrafts.

Finding design variables that will cause maximum reduction in the objective function has been an important aspect of optimization. It is too laborious and/or practically impossible to calculate first derivatives experimentally. In addition, to date, no method is available for experimental DSA of SEA models. Therefore, the objective of present paper is to find a general method applicable for **analytical** and **experimental** DSA with *damping loss factor* of *subsystems* as design variables. Transfer Path Approach (TPA) is used to derive the sensitivity vector. The sensitivity vector thus found will be useful to rank the design variables according to their importance in reducing the cost function. Final form of vector suggests that known Damping Loss Factor (DLF) values of the subsystems can form this vector. This simplicity allows designer to implement the DSA experimentally.

1. THEORY

To find the sensitivity of the objective function to Damping Loss Factors (DLFs) of the subsystems, we will make use of TPA. The objective function is ratio of total energy of receiving subsystem to excited subsystem. For m parallel paths, the objective function F will take the form [2]

$$F = \frac{E_{n+1}}{E_1} = \eta_p^1 + \eta_p^2 + \cdots + \eta_p^i + \cdots + \eta_p^m, \tag{1}$$

where E_{n+1} and E_1 are total energies of $n+1^{th}$ and *first* subsystem respectively. The η_p^i is the Path Loss Factor (PLF) of i^{th} transmission path, subscript p stands to distinguish the DLF from PLF; superscript i indicates the transmission path number.

The average PLF between excitation and receiving subsystem can be written as η_{ap}^F, where subscript ap is to show that the PLF is found by taking arithmetic averaged of ‘m’ Path Loss Factors (PLFs), and superscript F indicates the objective function for which the paths are averaged. Here, F can also be written as $m \times \eta_{ap}^F$.

The path loss factor of i^{th} transmission path can be written as [2]

$$\eta_p^i = \frac{\eta_{12}\eta_{23}\cdots\eta_{pq}\eta_{qr}\eta_{rs}\cdots\eta_{n,n+1}}{\eta_2\eta_3\cdots\eta_q\eta_r\eta_s\cdots\eta_{n+1}}, \tag{2}$$

where η_{pq} is the Coupling Loss Factor (CLF) between p^{th} and q^{th} subsystem of the SEA model, η_q is DLF of the q^{th} subsystem.

Let there are k number of transmission paths that include r^{th} subsystem as one of the intermediate subsystem (*see* Figure 1).

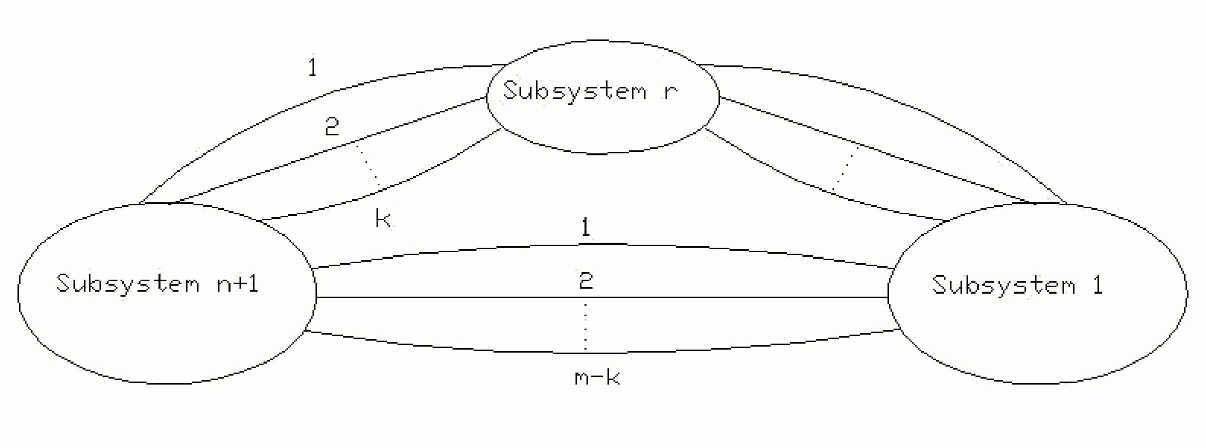

Fig. 1. Transmission paths through r^{th} subsystem

As CLF between subsystems are assumed independent on DLFs of corresponding subsystems (wave approach), the first order partial derivative of objective function, *F*, with respect to η_r gives

$$\frac{\partial F}{\partial \eta_r} = -\left(\frac{c_1}{\eta_r^2} + \frac{c_2}{\eta_r^2} + \cdots + \frac{c_i}{\eta_r^2} + \cdots + \frac{c_k}{\eta_r^2} \right), \tag{3}$$

where $c_i = \frac{\eta_{12}\eta_{23}\cdots\eta_{pq}\eta_{qr}\eta_{rs}\cdots\eta_{n,n+1}}{\eta_2\eta_3\cdots\eta_q\eta_s\cdots\eta_{n+1}}$, similar expressions can be written for c_1, c_2 and c_k coefficients.

The equation (3) can be rearranged as

$$\frac{\partial F}{\partial \eta_r} = -\frac{1}{\eta_r}\left(\frac{c_1}{\eta_r} + \frac{c_2}{\eta_r} + \cdots + \frac{c_i}{\eta_r} + \cdots + \frac{c_k}{\eta_r} \right), \tag{4}$$

where the term in brackets is part of objective function expression. Let us denote this part by F_r, as every transmission path in this part ensures the presence of r^{th} subsystem.

The sensitivity of objective function to DLFs of other subsystems can be similarly found. Finally, the gradient vector of the objective function can be written as

$$\nabla F = \left\{ -\frac{F_1}{\eta_1} \quad -\frac{F_2}{\eta_2} \quad \cdots \quad -\frac{F_r}{\eta_r} \quad \cdots \quad -\frac{F_{n+1}}{\eta_{n+1}} \right\}^T . \tag{5}$$

Though equation (5) looks simple, it does not give a simple way of evaluating the values of *F* functions. Because, every *F* function includes product of different Coupling Loss Factors in its definition. Evaluating this product for each *F* function is easy for analytical SEA, but little tedious and time consuming for experimental SEA. To simplify the gradient vector calculation process, we assume that the average of PLFs used to compose objective function *F* is approximately equal to average of PLFs of each part of objective functions like F_1, F_2 and F_{n+1}. This assumption will allow us to write

$$\frac{F}{m} \approx \frac{F_1}{a} \approx \frac{F_2}{b} \approx \cdots \approx \frac{F_r}{k} \approx \cdots \approx \frac{F_{n+1}}{z}, \tag{6}$$

where *a*, *b*,…, *k*,…, *z* are the number of transmission paths passing through 1^{st}, 2^{nd} ,…, r^{th} ,…, $n+1^{th}$ subsystems respectively.

Substituting for F_1 to F_{n+1} from equation (6), equation (5) reduces to

$$\nabla F_{approx} \approx -\frac{F}{m}\begin{Bmatrix} \frac{a}{\eta_1} \\ \frac{b}{\eta_2} \\ \cdots \\ \frac{k}{\eta_r} \\ \cdots \\ \frac{z}{\eta_{n+1}} \end{Bmatrix} = -\frac{F}{m}\begin{Bmatrix} \frac{a}{\eta_1} & \frac{b}{\eta_2} & \cdots & \frac{k}{\eta_r} & \cdots & \frac{z}{\eta_{n+1}} \end{Bmatrix}^T . \tag{7}$$

As all design variables are of same type (reciprocal of DLFs), scaling of approximate gradient vector in equation (7) will not distort the objective function shape. Scaling gradient vector by –(*m/F*) will give the desired search direction as

$$\mathbf{d} = -\frac{m}{F}\nabla F_{approx} = \begin{Bmatrix} \frac{a}{\eta_1} & \frac{b}{\eta_2} & \cdots & \frac{k}{\eta_r} & \cdots & \frac{z}{\eta_{n+1}} \end{Bmatrix}^T , \tag{8}$$

where **d** is the search direction to be used in optimization algorithm.

Once structure is modeled in SEA, the number of transmission paths passing through a particular subsystem can easily be determined and will remain constant throughout the optimization process. Then the search direction in equation (8) can be easily composed by multiplying these constants, *a*–*z*, with reciprocals of design variable values. By looking at direction vector, it is clear that the function will decrease rapidly in the co-ordinate direction that has highest modulus of element of vector. As values of parameters *a*–*z* and DLFs are ensured to be positive, the function has maximum descent in the co-ordinate direction that has maximum value of element in direction vector. Arranging the above direction vector in descending order of modulus of elements will give the required sequence, and rating of subsystems.

2. CONVERGANCE PROOF

To ensure that the search direction **d** in equation (8) points towards the minimum of objective function at all possible values of parameters *a*–*z*, the convergence proof is needed. For any direction to point in a descent direction, from any point in a feasible domain of the objective function

$$(\nabla F)^T \mathbf{d} \le 0,$$

$$\begin{Bmatrix} \dfrac{-F_1}{\eta_1} & \dfrac{-F_2}{\eta_2} & \cdots & \dfrac{-F_r}{\eta_r} & \cdots & \dfrac{-F_{n+1}}{\eta_{n+1}} \end{Bmatrix} \begin{Bmatrix} \dfrac{a}{\eta_1} \\ \dfrac{b}{\eta_2} \\ \cdots \\ \dfrac{k}{\eta_r} \\ \cdots \\ \dfrac{z}{\eta_{n+1}} \end{Bmatrix} \le 0, \tag{9}$$

$$-\left(\frac{aF_1}{\eta_1^2} + \frac{bF_2}{\eta_2^2} + \cdots + \frac{kF_r}{\eta_r^2} + \cdots + \frac{zF_{n+1}}{\eta_{n+1}^2} \right) \le 0.$$

As objective function values F_1 to F_{n+1} and parameters *a*–*z* are ensured to be positive, it is clear that the search direction **d** will point in the descent direction.

3. EXAMPLE: AUTO MODEL

A simplified passenger vehicle model that can be used to identify the system characteristics, prior to a practical engineering model analysis of a vehicle system, is shown in Fig. 2. This model considers the simplified body of a car. This car body is assumed to be made from plates that are welded together (this is not a very realistic construction). A real car would have frames, pillars and stiffeners. This simple car model can be used to demonstrate some of the system characteristics that do not change much from the real model. Once validated with experiment, they can be used to predict the response with desired accuracy. The material and subsystem data are summarized in Table 1 and first three columns of Table 2, respectively.

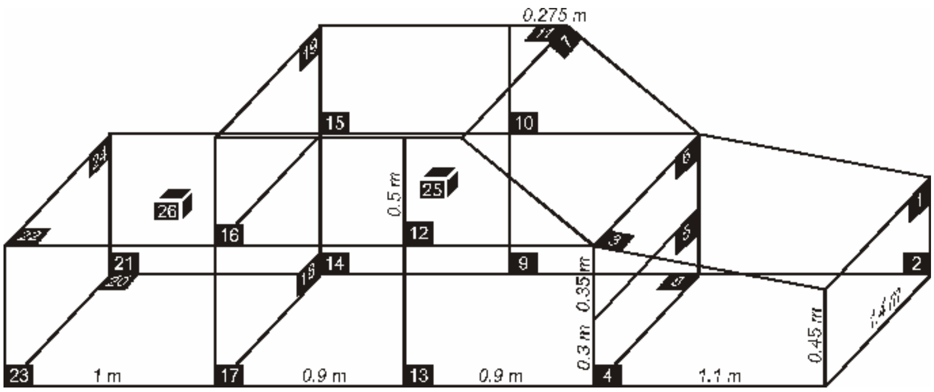

Fig. 2. Auto model (after Sarradj [3])

Table 1. Material properties for auto model

Sr. No.	Name	Young's Modulus / Speed of sound	Density (kg/m^3)	Poisson
1	Steel	210 GPa	7800	0.3
2	Glass	60 GPa	2500	0.2
3	Air	344 m/s	1.19	0

Table 2. Initial design for auto model

Sr. No.	Name	Material Name	Initial Design $\mathbf{x}_0$ (m)
1	Front	Steel	0.0006
2	Fender left front	Steel	0.0006
3	Hood	Steel	0.0006
4	Fender right front	Steel	0.0006
5	Firewall	Steel	0.0006
6	Dashboard	Steel	0.0006
7	Windshield	Glass	0.006
8	Floor	Steel	0.0006
9	Door left front	Steel	0.0006
10	Window left front	Glass	0.004
11	Roof	Steel	0.0006
12	Window right front	Glass	0.004
13	Door right front	Steel	0.0006
14	Door left back	Steel	0.0006
15	Window left back	Glass	0.004
16	Window right back	Glass	0.004
17	Door right back	Steel	0.0006
18	Div wall	Steel	0.0006
19	Back window	Glass	0.004
20	Trunk floor	Steel	0.0006
21	Fender left back	Steel	0.0006
22	Trunk top	Steel	0.0006
23	Fender right back	Steel	0.0006
24	Back	Steel	0.0006
25	Passenger compartment	Air	--
26	Trunk	Air	--

A damping loss factor of 1% was assumed for all plate type subsystems. Acoustic subsystems were assumed to have a frequency independent reverberation time of 200 *ms*. The frequency dependent damping loss factor for both of these room type subsystems, η_{room}, were calculated from

$$\eta_{room} = \frac{2.2 f}{T}, \tag{10}$$

where f is the frequency in *Hz* and T is time in seconds, *s*.

Engine is the source of vibration supports on two bearings. In reality there will be a total of three or four supports on left and right fenders. The frequency independent source strength of 0.5 watts is assumed in each fender. With this information in hand, an equivalent SEA model of the automobile was constructed and shown in Table 3.

Table 3. SEA model for auto

Subsystem Number	Connectivity	Subsystem Number	Connectivity
1	2, 3, 4	14	8, 9, 15, 18, 25
2	1, 3, 5, 6, 9	15	10, 11, 14, 19, 25
3	1, 2, 4, 6, 7	16	11, 12, 17, 19, 25
4	1, 3, 5, 6, 13	17	8, 13, 16, 18, 23, 25
5	2, 4, 6, 8, 9, 13, 25	18	8, 14, 17, 19, 20, 21, 22, 23, 25, 26
6	2, 3, 4, 5, 7, 9, 13, 25	19	11, 15, 16, 18, 22, 25
7	3, 6, 10, 11, 12, 25	20	8, 18, 21, 23, 24, 26
8	5, 9, 13, 14, 17, 18, 25	21	14, 18, 20, 22, 24, 26
9	2, 5, 6, 8, 10, 14, 25	22	18, 19, 21, 22, 24, 26
10	7, 9, 11, 15, 25	23	17, 18, 20, 22, 24, 26
11	7, 10, 12, 15, 16, 19, 25	24	20, 21, 22, 23, 26
12	7, 11, 13, 16, 25	25	5 – 19
13	4, 5, 6, 8, 12, 17, 25	26	18, 20 – 24

Two groups of transfer paths, 2-step and 3-step, from source to receiving subsystem are considered. Tables 4 and 5 list the six 2-step and twenty-two 3-step transmission paths from *fender left* and *fender right* to *passenger compartment*.

Table 4. Two-step transmission paths

Sr. No.	Transmission path (Subsystem numbers)	Sr. No.	Transmission path (Subsystem numbers)
1	2-5-25	4	4-5-25
2	2-6-25	5	4-6-25
3	2-9-25	6	4-13-25

Table 5. Three-step transmission paths

Sr. No.	Transmission path (Subsystem numbers)	Sr. No.	Transmission path (Subsystem numbers)
1	2-5-8-25	12	4-5-8-25
2	2-5-13-25	13	4-5-9-25
3	2-5-6-25	14	4-5-6-25
4	2-6-5-25	15	4-6-5-25
5	2-6-7-25	16	4-6-7-25
6	2-6-13-25	17	4-6-9-25
7	2-3-6-25	18	4-3-6-25
8	2-3-7-25	19	4-3-7-25
9	2-9-10-25	20	4-13-12-25
10	2-9-14-25	21	4-13-17-25
11	2-9-8-25	22	4-13-8-25

4. RESULTS AND DISCUSSION

Consider the ratio of passenger compartment total energy to either left or right fender total energy as an objective function F. Note that, due to symmetry of auto model and input power at current design point, the total energies of left and right fenders are equal. Therefore, to maintain the simplicity and clarity in presentation, this equivalent form of objective function is taken. For a general case, one need to take sum of ratios between total energy of passenger compartment and left and right fenders as the objective function. The energy of passenger compartment in dB, FdB, can then be found making use of equation (1) as

$$FdB = 10\log_{10}\left(\frac{F \times E_1}{10^{-12}}\right), \tag{11}$$

where E_1 is the total energy of either left or right fender.

Differentiating the above equation with respect to DLF of passenger compartment gives the following equation:

$$\frac{\partial FdB}{\partial \eta_{room}} = \frac{10}{F}\frac{\partial F}{\partial \eta_{room}} + \frac{10}{E_1}\frac{\partial E_1}{\partial \eta_{room}}. \tag{12}$$

Note that, the effect of change in damping loss factor of passenger compartment on total energy of either left or right fender, E_1 is assumed as very less in comparison with the effect of same change in damping loss factor on $F = E_{n+1}/E_1$. Neglecting the second term in equation (12) and substituting for sensitivities of objective function to DLFs of subsystems from equation (7) gives

$$\frac{\partial FdB}{\partial \eta_{room}} = -\frac{10}{m}\mathbf{d}_{room}, \tag{13}$$

where $\mathbf{d}_{room}$ is the subset of $\mathbf{d}$ corresponding to η_{room}.

Sensitivities of objective function, in dB, to DLFs of any subsystem in the model can also be given by

$$\frac{\partial FdB}{\partial \eta} = -10^7 \left[\log_{10}\left(\frac{E_{n+1}}{10^{-12}}\right) - \log_{10}\left(\frac{E'_{n+1}}{10^{-12}}\right)\right], \tag{14}$$

where E'_{n+1} is the total energy of n+1th subsystem calculated after small increment in DLF from η to η'. Here, the increment in η is taken as 10^{-6}. The negative sign in equation (14) indicates that the total energy decreases as η increases.

The above equation needs two solutions of SEA model. One to find the total energy of n+1th subsystem at the current design and the other at perturbed design. This way of getting sensitivity of objective function is called traditional approach in this paper.

We will compose the objective function by using 2-step and 3-step transmission paths. To investigate the effect of number of transmission paths on calculated sensitivity vector, we will consider two cases: Case-I and Case-II. In Case-I, only 2-step transmission paths are considered, while in Case-II, 2-step and 3-step transmission paths are considered. The intention is to address the improvement (if any) achieved by adding 3-step transmission paths to Case-I. Tables 6 and 7 list the parameters (a, b, …, z in equation (6)) required for sensitivity calculations for Case-I and Case-II respectively.

Table 6. Number of transmission paths through subsystems in Case-I

Subsystem Numbers	Number of transmission paths
25	6
5 and 6	2
9 or 13	1

Table 7. Number of transmission paths through subsystems in Case-II

Subsystem Numbers	Number of transmission paths
25	28
6	12
5	10
9 or 13	6

Table 8 compares the sensitivity of total energy of passenger compartment, calculated by present approach using equation (13) with sensitivity calculated by traditional approach using equation (14), to its frequency dependent DLF. It can be seen that the sensitivity values found by present approach are reasonably close and gives the same trend (over frequency) as traditional values.

Table 8. Sensitivity comparison of Case-I and Case-II with traditional approach

Frequency (Hz)	η_{n+1}	Present Approach		Traditional Approach
		Case-I	Case-II	
100	0.11	–49.59	–45.46	–23.45
200	0.055	–99.17	–90.91	–70.70
400	0.0275	–198.35	–181.82	–147.20
800	0.01375	–396.69	–363.64	–315.20
1600	6.875×10^{-3}	–793.39	–727.27	–619.70
3150	3.4375×10^{-3}	–1586.80	–1454.50	–1233.60

To arrange the subsystems according to importance of their damping values in descending order, the above analysis is repeated at 1000 Hz 1/3 octave band frequency for subsystems tabulated in Tables 6 and 7. The traditional approach gives the sequence as $25 \Rightarrow 6 \Rightarrow 9$ or $13 \Rightarrow 5$. The present approach in Case-I suggests that the sequence is $25 \Rightarrow$ (5 or 6) $\Rightarrow 9$, while in Case-II it suggests the sequence as $25 \Rightarrow 6 \Rightarrow 5 \Rightarrow$(9 or 13). It is clear that, as you consider more number of transmission paths, the approximation to sequence is better and will give more insight into the model. Due to the restrictive assumption and less number of transmission paths considered in the present analysis, present approach fails to order the subsystem 9 or 13 in its correct place. However, it should be noted that addition of few more paths in the analysis will resolve this problem and will help the designer to arrange the subsystems according to their correct sequence to required depth. In this paper, no guidelines are provided towards consideration of minimum number of transmission paths required to give the correct sequence (up to required depth) and authors want to keep the topic open for further research.

The major advantages of present approach over traditional approach are:

1. Needs information of DLFs of subsystems only
2. Method is logical, simple and computationally more efficient
3. Can be verified **experimentally** very easily

CONCLUSIONS

A new method to calculate the design sensitivity vector for damping loss factors of subsystems in SEA context is proposed. The transmission path approach is used to derive the DSA vector. A general proof for convergence of design sensitivity vector thus derived is given to support the approach. Derived final form of DSA vector shows that the method can be used experimentally with very little computational efforts. An example of auto model formed from 2 *acoustic spaces* and 24 *plates* is used to validate and demonstrate the method. Results show that the method is more accurate with more number of transmission paths. Its advantages and further research required to make the method more powerful are also listed.

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