Electronic Journal «Technical Acoustics»
http://webcenter.ru/~eeaa/ejta/

2005, 9

I. Grushetsky, A. Smol'nikov

Krylov Shipbuilding Research Institute
Russia, 196158, Saint Petersburg, Moskovskoe shosse, 44, e-mail: eeaa@online.ru

Comparison of FEM and SEA for vibration computing in periodic structures

Received 15.03.2005, published 31.03.2005

Some examples of vibration energy computing in beams structure using Finite Element Method (FEM) and Statistical Energy Analysis (SEA) are presented. Calculations are carried out for periodic structure, that consists of equal beams and of beams whose lengths are random variables with equal mean value and some deviation. The later case is real one, when the dimensions of structure elements are not known exactly but are specified as a mean value and some tolerance. It is shown that the beams energies, calculated by FEM, in structure consisting of equal beams and beams, whose lengths are random variables with equal mean value, are different. Thus FEM computation which uses solely mean characteristics can be inaccurate. Difference between average FEM and SEA results is not significant. It is concluded that SEA – less expensive method – is quite suitable for practical engineering computing.

INTRODUCTION

The most common methods for computing of vibration and sound radiation of complex structures are Finite Element Method (FEM) and Energy Method (EM). EM is often designated as Statistical Energy Analysis (SEA), taking into account some assumptions when applying the method. Commercial packages that implement these two methods are designed: ANSYS, NASTRAN, ABACUS (FEM), AutoSEA, SEAM, SEADS (SEA) and others.

FEM is exact and multi-purpose method. It bases on fundamental theory of elasticity. But using the method numerical problems arise when total number of elements becomes too large (for complex structures and when frequency increases). Thus FEM is usually applied at low frequencies, SEA — at middle and high frequencies.

The intrinsic property of any real structure and composing elements is uncertainty of characteristics. For example, dimensions are specified with some tolerance. Material characteristics (elasticity moduli, loss factors, etc.), applied forces (loading points or areas, phases), etc. are not known exactly. Mean (most probable) values are usually used as input data for engineering calculations.

The example of FEM calculation of vibration energy in periodic beam structure is presented in this paper. Both determinate and random values are used as input data. It is shown that using determinate (mean) values solely can give erroneous results. Comparison of FEM and SEA results is also carried out.

1. EXAMPLE OF BEAM STRUCTURE

The structure consisting of four consecutively joined at right angle beams, presented in fig. 1, is used for calculations. Stiff junctions are simply supported. In such a structure only transverse (bending) vibrations arise under the action of transverse force. This circumstance simplifies calculations and analysis but does not confine conclusions.

Beams cross-section is 5×1 cm. The beams are joined along long side of the cross-section. Material is steel, internal loss factor is 0.01, affecting harmonic force (F) at the free end of beam 1 is 1 N. Energy of transverse vibration is computed by FEM and SEA.

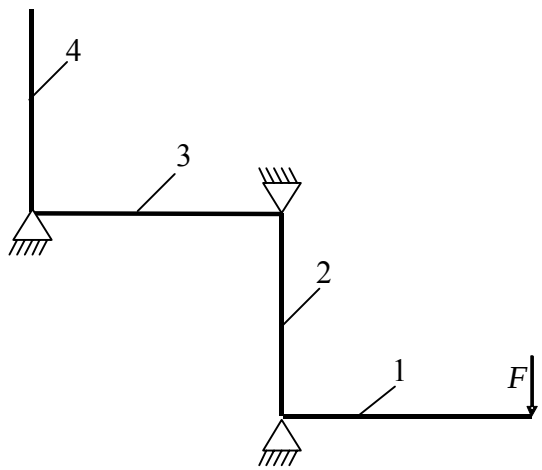

Fig. 1.
The structure, consisting of consecutively joined beams, that is used for calculations

Calculations are carried out for periodic structure consisting of equal beams. Periodic structures are widely used. Some examples are ribbed structures, standard construction. It was detected [1] that vibration decrease under propagation in periodic structure is less than in non-periodic one. Therefore periodic structures case is most important for noise control.

Sizes of real structures and their elements are given as mean value and tolerance as a rule. Mean values are generally used in engineering computing. In this study we calculate vibration energy of beam structure using two types of input data. The first type is solely mean lengths of beams, which are equal 1 m. The second one is random samples of beams lengths from normal distribution with mean value 1 m and dispersion 0.05 m.

The difference between randomly generated beams lengths can be 10 cm and even more. Such a large difference of formally equal elements is not typical for mechanical structures. However, not directly length, but relation between resonant frequencies of adjacent beams influences on energy propagation in structure. This relation depends on both lengths and a lot of other factors: initial curvature, residual welding and static stresses etc. Finally, additive effect of these factors leads to decreasing of probability of resonant frequencies coincidence. We approximately take this phenomenon into account using larger dispersion of beams lengths.

Calculations were carried out for ten modifications of beam structure, which differ in lengths of beams composing structure. Modifications are presented in Table 1.

Table 1. Beams lengths (m), composing structure in fig. 1
(random samples from normal distribution with mean value 1 m and dispersion 0.05 m)

modification	beam 1	beam 2	beam 3	beam 4
1	0.9892	1.0071	1.0235	1.0501
2	1.0493	1.0350	0.9771	1.0060
3	1.0505	1.0574	0.9558	1.0926
4	0.9786	1.0182	0.9459	1.0120
5	1.0587	0.9244	1.0271	1.0177
6	1.0285	0.9319	0.9700	0.9944
7	0.9656	1.0308	0.9913	1.0109
8	0.9420	1.0415	0.9268	1.0102
9	0.9697	0.8558	1.0357	1.0653
10	1.0476	0.9948	0.9756	0.9647

2. FEM CALCULATION

At FEM modeling, the beams were divided into standard beam elements. Length of each element is about 1 cm. Beam's vibration energy (E_b) was calculated from complex displacements in nodes of a mesh by formula

$$E_b = \frac{1}{2}\omega^2 \sum_{n=1}^{N} M_n \xi_n^2 , \qquad (1)$$

where ξ_n — displacement amplitude in node, M_n — beam's mass per a node, N — number of nodes in mesh.

Calculations were carried out at resonant frequencies of structure and the results were summed in octave bands. Results for beams vibration energy (individual results for ten modifications of the structure and average values) are presented in fig. 2. Individual results are characterized by a significant dispersion. Fig. 3 shows that the energy dispersion mainly decreases when frequency increases and the energy dispersion is larger for beams which farther from the excitation point (beam 1).

Fig. 2 also shows vibration energy of beams composing structure, when the beams are equal (1 м long). In this case, energy decrease along the structure is less in comparison with average decrease for ten random modifications (fig. 4). For example, difference of energies of beams 1 and 4 less than 5 dB at high frequencies if beams are equal, but the average difference for ten random modifications is about 10 dB. The reason is the coincidence of resonant frequencies for equal beams intensifies energy exchange.

Note (fig. 4б) that vibration energies of equal beams 2, 3 and 4 are nearly the same. This result agrees with theoretical data, presented, for example, in [1]: vibration decrease in structure with periodical obstacles like ribs on plate can be seen at initial (one or two) obstacles. Farther there is no vibration reduction. However in-situ measurements of vibration

in deck plating and ship hull (these structures are considered as periodically ribbed structures) show that vibration decrease does not depend on distance from excitation point [1]. It may mean that real structures should not be considered as strictly periodical.

Thus, usage of equal beams lengths as input data, leads to overestimated energy for distant beams. Possibility of sizes (and resonant frequencies) coincidence for real structures is low. Formally periodical structure is not periodical in fact. For correct calculation it is rightly to consider the length as random variable. In this case the most probable (average) result is lower than the result derived for equal beams. It seems the latter result more correctly conforms to vibration propagation in complex real structures. However it is necessary to carry out a lot of calculations using a set of random samples from population of random variables, which characterize, in the general case, geometry, materials properties, applied forces etc. Computing becomes more expensive.

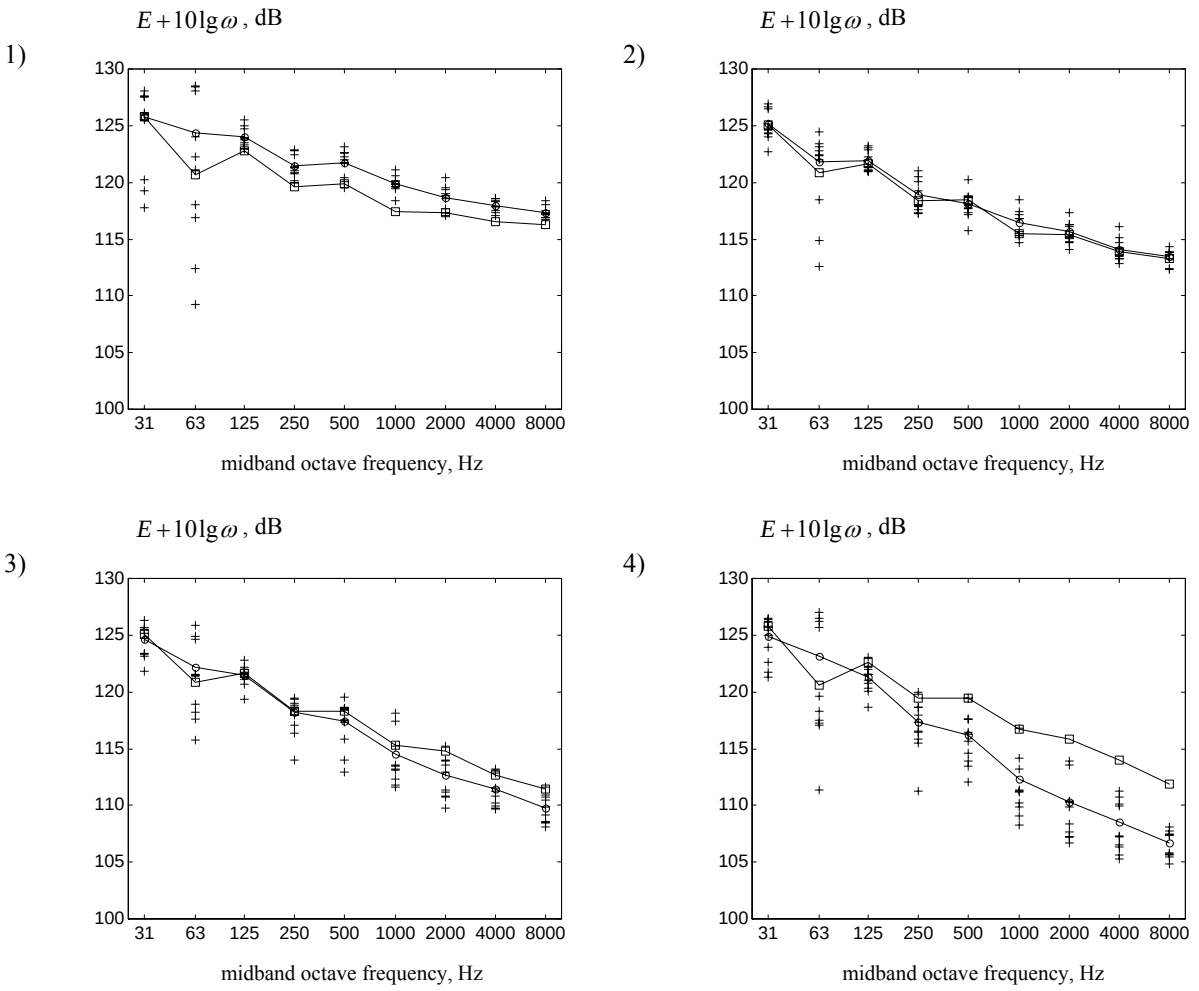

Fig. 2. Vibration energy of beams 1…4, FEM computing

++++ individual results for ten modifications of the structure (table 1),

—o— average energies over 10 random modifications,

—□— equal beams (1 m)

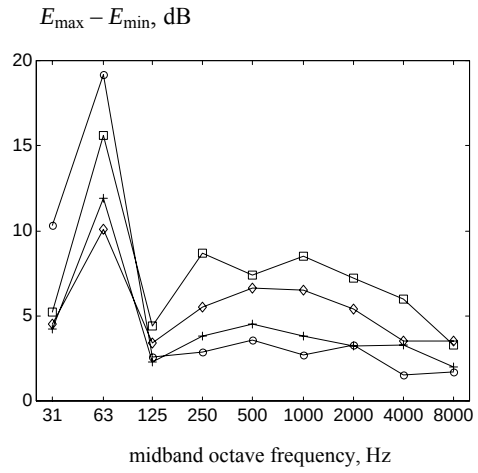

Fig. 3. Difference between maximum and minimum energies of beam calculated FEM for 10 random modifications of the structure

—○— beam 1
—+— beam 2
—◇— beam 3
—□— beam 4

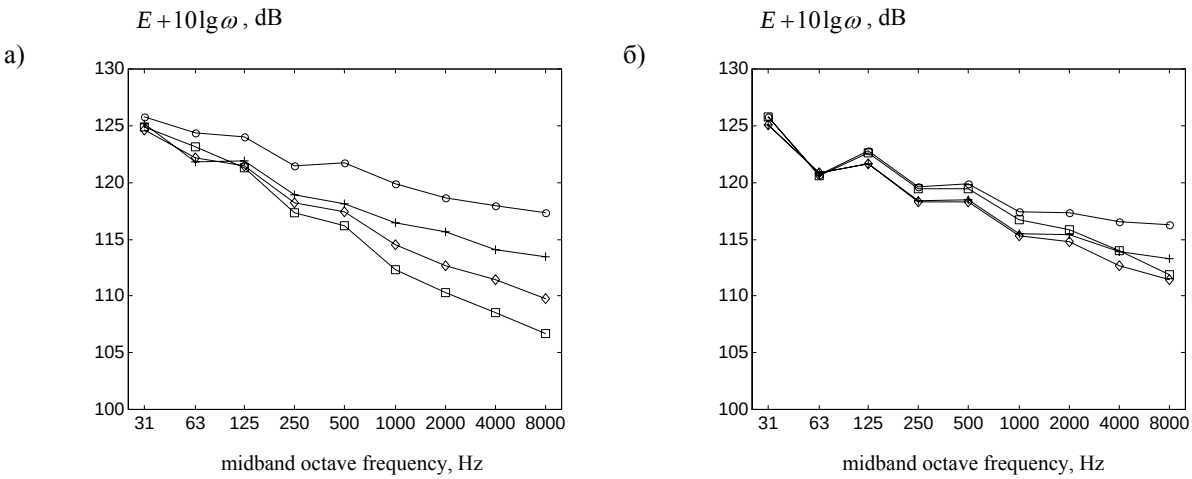

Fig. 4. Vibration energy of beams 1…4, FEM computing
a) average values over 10 random modifications, б) equal beams (1 m),
—○— beam 1, —+— beam 2, —◇— beam 3, —□— beam 4

3. SEA CALCULATION

Set of energy balance equations for the structure presented in fig. 1 is

$$\omega \begin{pmatrix} \eta_1 + \eta_{12} & -\eta_{21} & 0 & 0 \\ -\eta_{12} & \eta_2 + \eta_{21} + \eta_{23} & -\eta_{32} & 0 \\ 0 & -\eta_{23} & \eta_3 + \eta_{32} + \eta_{34} & -\eta_{43} \\ 0 & 0 & -\eta_{34} & \eta_4 + \eta_{43} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{pmatrix} = \begin{pmatrix} W_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{2}$$

where $\eta_1 \ldots \eta_4$ are the internal loss factors in beams; $\eta_{12} \ldots \eta_{43}$ are the coupling loss factors for beams; W_1 is the input power injected into beam 1 by force F; $E_1 \ldots E_4$ are the unknown vibration energies of beams; ω — circular frequency.

Coupling loss factors are calculated by formula

$$\eta_{ij} = \frac{\tau_{ij}\, c_{gi}}{\omega L_i}, \tag{3}$$

where τ_{ij} is the transmission coefficient of bending waves energy from beam *i* into beam *j*, c_{gi} is the group velocity of bending waves in beam *i*, L_i is the length of beam *i*.

Transmission coefficient for beams joined at right angle was derived analytically [2]: $\tau = 0.5$ for beams of identical cross-section, like in the case under consideration.

Calculations show that influence of beams length varying in moderate bounds (as in table 1), does not practically affect the results of SEA calculations. Therefore SEA results, presented below, concern the structure which consists of equal beams only.

Input power, injected into the structure under the action of force, was defined by FEM and approximately. Using FEM results, input power was calculated as follows:

$$W = \frac{1}{2}\mathrm{Re}\left(F \cdot v^*\right), \tag{4}$$

where *F* is the force affecting the structure which is specified under FEM modeling (1 N), v^* is the complex conjugate velocity at driving point. Calculations were carried out, as before, at resonant frequencies, results were summarized in octave bands.

Input power was defined using FEM for the following structures:

– structure composing of four beams (fig. 1),
– structure composing of two beams (fig. 5a),
– one beam (fig. 5б).

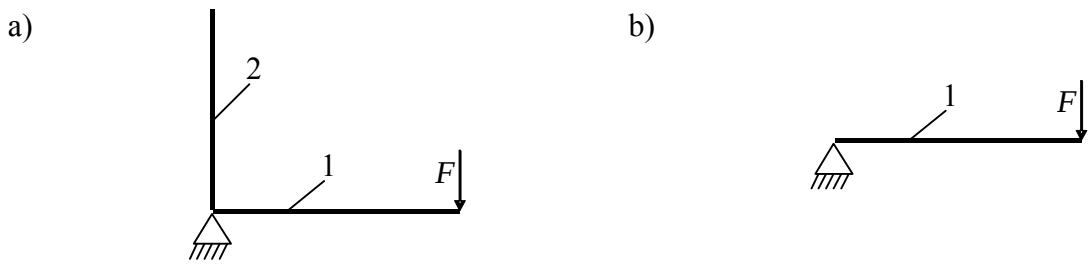

Fig. 5. Structures for FEM calculations of input power

In all cases, input power was calculated for ten modifications of the structure with different beams lengths. Beams lengths, as before, were random samples from normal distribution with mean value 1 m and dispersion 0.05 m. Beams lengths used in calculations are presented in tables 1, 2 and 3.

Table 2. Beams lengths (m), composing structure of two beams, fig. 5a
(random samples from normal distribution with mean value 1 m and dispersion 0.05 m)

	modification									
	1	2	3	4	5	6	7	8	9	10
beam 1	1.067	0.995	1.018	0.919	1.076	1.021	1.010	1.100	0.959	1.008
beam 2	0.986	1.094	0.944	1.043	0.949	0.989	1.037	1.027	1.074	0.992

Таблица 3. Beam length (m), fig. 5b
(random samples from normal distribution with mean value 1 m and dispersion 0.05 m)

modification	1	2	3	4	5	6	7	8	9	10
length	0.966	0.949	0.938	1.014	0.979	1.003	0.982	0.977	1.019	1.036

Calculations show that dispersion of input power is relatively small. Difference between maximum and minimum input power calculated over the ten random samples is about 2 dB at middle and high frequencies for all three structures (fig. 6). Vibration energy changes approximately in the same bounds. As an example, vibration energy of beam 4 calculated using ten input powers, derived for model in fig. 5b, is presented in fig. 7. Following calculations use average input power for each structure.

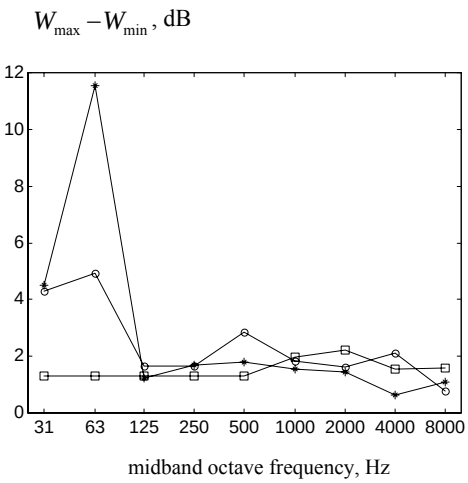

Fig. 6. Difference between maximum and minimum input power calculated over the ten random samples

four beams
two beams
one beam

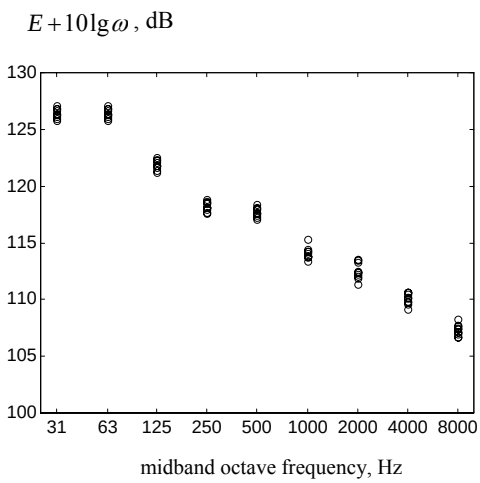

Fig. 7. Vibration energy of beam 4 using input power calculated when force acts on one beam, fig. 5b (10 random modifications of beam length, table. 3)

Input power in octave frequency bands was calculated approximately by the formula

$$W = \frac{F^2 \operatorname{Re} Z_F}{2|Z_F|^2} \cdot N(\Delta f), \tag{5}$$

where Z_F is the impedance of a beam with respect to a force, $N(\Delta f)$ is the number of resonant frequencies in octave band (Δf is the bandwidth). Impedance was defined at resonant frequencies i.e. minimum impedance of a beam, excited at free end, was used [3]:

$$Z_{F\min} = \frac{\omega m L \eta}{4}, \tag{6}$$

where m is the mass of a beam per length unit (1 m), η is the internal loss factor;

$$N(\Delta f) = L \Delta f \sqrt[4]{\frac{m}{B\omega^2}}, \tag{7}$$

where B is the bending stiffness of a beam.

Input powers calculated by FEM and approximately (formulas (5 – 7)) are presented in fig. 8. Input powers calculated in a different ways (exactly and approximately) and using models of different complexity (fig. 1, 5) are nearly the same.

SEA results for vibration energies of beams in comparison with FEM are presented in the next section.

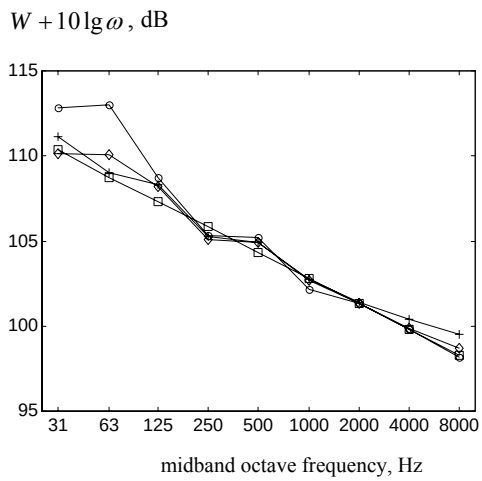

Fig. 8. Input power calculated FEM and approximately

- four beams, fig. 1 (FEM)
- two beams, fig. 5a (FEM)
- one beam, fig. 5b (FEM)
- one beam, fig. 5b (approximately, form. 5–7)

4. COMPARISON OF SEA AND FEM RESULTS

Vibration energy of beams 1…4 in the structure depicted in fig. 1, which is calculated using SEA and FEM is presented in fig. 9. One can see the following:

1. SEA and FEM (average over ten random modifications of the structure) results coincide well. The difference for the most distant beam (4) is not more than 2 dB at middle and high frequencies.
2. SEA results are in the range between maximum and minimum FEM results with rare exception.

3. SEA and FEM (average) results coincide in a wide frequency range including low frequencies where only one resonance frequency of the beam is in octave bands (31,5, 63 and 125 Hz). At the same time there are two resonance frequencies of the entire beam structure (fig. 1) in octave bands 31,5 и 63 Hz and three frequencies are in band 125 Hz. It is generally considered that SEA is applicable when resonant frequencies density is higher. Presented results allow us to hope to extend frequency range of SEA application, but additional studies are necessary.

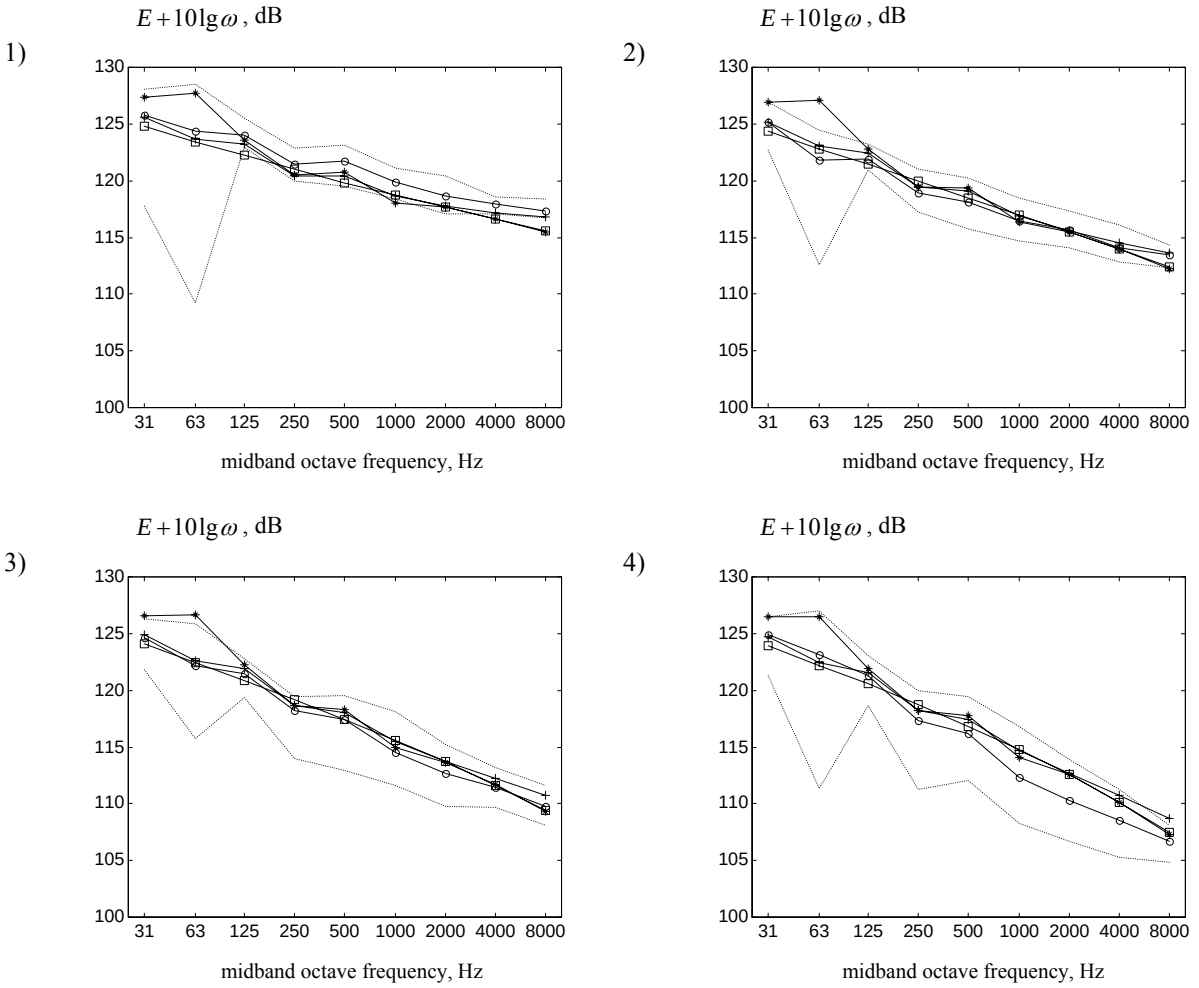

Fig. 9. Vibration energy of beams 1…4 composing structure in fig. 1

—+— SEA, input power is calculated by FEM for entire structure in fig. 1

—✳— SEA, input power is calculated by FEM for one beam (fig. 5b)

—□— SEA, input power is calculated approximately by formulas 5–7

—○— FEM, average values over 10 random modifications of the structure

--------- FEM, minimum and maximum values over 10 modifications of the structure

CONCLUSION

FEM computation of vibration in formally periodical beam structure was carried out taking into account uncertainty of lengths of beams composing structure. Input data were presented as mean value and some dispersion. Calculated vibration energies are characterized by a large variation. Value of variation depends on dispersion of input data. Average energies appreciably differ from the energies calculated for equal beams.

In engineering practice, mean values for geometrics, material properties etc. are usually used as input data. It can lead to erroneous results for periodic structures since perfect identity (periodicity) of structure elements is rather improbable for real structures.

To obtain more realistic results for real structures, FEM calculations should be carried out using input data which take into account possible deviation of characteristics from mean values. Calculations become more expensive. However calculation examples, presented in this paper, show that SEA provides results, which agree well with average FEM results. SEA is essentially less expensive technique because of simplified modeling of a structure, fewer amounts of input data, lower requirements for computers.

One weak point of SEA is input power injected into the structure. This quantity is hardly can be defined quite accurate for real structures and forces using simplified engineering formulas. One can use FEM to calculate input power. At that, it seems, no need to simulate complex structure entirely. In presented example we included in FEM model only the component on which force acts, and input power was calculated sufficiently accurately.

Vibration and sound propagation processes in structures consisting of beams, plates, shells, spaces filled with fluid are similar. Therefore presented example qualitatively describes energy propagation not only in beams structures but in other types of structures as well. Calculation methodology and conclusions can be possibly applied more widely.

The study confirms that SEA is efficient basis for computation of vibration and sound in complex structures which consist of a lot of elements. In the SEA framework, to increase accuracy of calculations, one can rationally use FEM to calculate, for example, input power or coupling loss factor for complex junctions if there are no analytical formulas for transmission coefficients. For these sorts of calculations one can simulate fragments of complex structure (not a structure entirely), for example, machine foundation, elements in junction etc. Therefore computing expenses are comparatively moderate.

REFERENCES

1. Lyapunov V. T., Nikiforov A. S. Vibration isolation in ship structures. Leningrad, “Sudustroenie” 1975. (*In Russian*).
2. L. Cremer, M. Heckl, E. E. Ungar. Structure-Borne Sound, 2nd edition, Springer, Berlin, 1988.
3. Nikiforov A. S. Noise control of ship structures. Leningrad, “Sudustroenie” 1990. (*In Russian*)