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Determination of coupling loss factors for two beams using FEM

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Coupling loss factors (CLF) are most important for computing sound and vibration using SEA. CLF and internal loss factors (ILF) constitute a matrix of energy balance equations. CLF can be defined analytically, experimentally or using computational modeling. The latter way is most universal but less developed at present. Determination of CLF for two beams at right angle using FEM is presented in this paper. It is shown that CLF which are derived from numerical simulating are differ from analytical values. But numerical CLF in energy balance equations for structure of four beams provide better results than analytical CLF.

INTRODUCTION

The most common methods for computing of vibration and sound radiation of complex structures are Finite Element Method (FEM) and Energy Method (EM). EM is often designated as Statistical Energy Analysis (SEA), taking into account some assumptions when applying the method. Commercial packages that implement these two methods are designed: ANSYS, NASTRAN, ABACUS (FEM), AutoSEA, SEAM, SEADS (SEA) and others.

FEM is more exact and multi-purpose but quite expensive method since it requires very detailed structure modeling and high-performance computers. Therefore FEM is usually applied at low frequencies. EM is approximate and less expensive method. It is caused by simplified modeling, less amount of input data, moderate requirements for computers.

Using EM one should know coupling loss factors (CLF) for structure elements (subsystems). CLF can be calculated analytically from coefficients of energy propagation via junctions of subsystems. However these coefficients are known for several simple types of junctions: L-, T- X-shaped rigid junctions of semi-infinite beams and plates and for some other junctions [1]. It is insufficiently for practical computing of vibration in complex structures like ships for example. CLF can be defined from experiment. This way is the same in theoretical basis as numerical modeling described below, but it requires realization of the complicated and expensive physical experiments.

Universal numerical way for CLF determination is based on numerical modeling of subsystems in junction. Using FEM, vibratory energies of subsystems are calculated. Then CLF are defined from a set of liner equations in which coefficients are energies.

An example of CLF determination using FEM for plates (building structures) is presented in [2]. Plates' dimensions were variable values. It meets actual situation, when structure characteristics are not known exactly. This approach is absolutely correct for practical computing but it does not reveal clearly how analytical and numerical CLF correlate and how correlate final results of EM calculation (subsystems energies) using analytical and numerical CLF. In this paper the comparison is carried out by the example of beams in L-shaped junction where beams characteristics are deterministic values.

1. CLF DETERMINATION FOR TWO SUBSYSTEMS

For CLF determination, sets of energy balance equations are formed for cases of energy injection via each subsystem separately. For example, for two subsystems we have a set of four equations for determination of four unknowns: two CLF (η_{12}, η_{21}) and two internal loss factors (ILF: η_1, η_2):

$$\begin{aligned}
&\eta_1 E_{11} + \eta_{12} E_{11} - \eta_{21} E_{21} = W_1 / \omega;\\
&\eta_2 E_{21} + \eta_{21} E_{21} - \eta_{12} E_{11} = 0;\\
&\eta_1 E_{12} + \eta_{12} E_{12} - \eta_{21} E_{22} = 0;\\
&\eta_2 E_{22} + \eta_{21} E_{22} - \eta_{12} E_{12} = W_2 / \omega,
\end{aligned} \tag{1}$$

where E_{11}, E_{12} E_{21}, E_{22} are the vibratory energies of subsystems 1 и 2 (first index) when energy is injected in subsystems 1 и 2 (second index), respectively; W_1 and W_2 are the input powers, ω is the circular frequency. Energies and input powers are defined by FEM.

Analytical solutions of (1) are:

$$\eta_{12} = \frac{(W_2/\omega)E_{21}}{E_{22}E_{11} - E_{21}E_{12}}; \tag{2}$$

$$\eta_{21} = \frac{(W_1/\omega)E_{12}}{E_{22}E_{11} - E_{21}E_{12}}; \tag{3}$$

$$\eta_1 = \frac{(W_1/\omega)E_{22} - (W_2/\omega)E_{21}}{E_{22}E_{11} - E_{21}E_{12}}; \tag{4}$$

$$\eta_2 = \frac{(W_2/\omega)E_{11} - (W_1/\omega)E_{12}}{E_{22}E_{11} - E_{21}E_{12}}. \tag{5}$$

When ILF are known one can use only one equation from each pair to find two unknowns η_{12} and η_{21}, for example:

$$\begin{aligned}
&\eta_2 E_{21} + \eta_{21} E_{21} - \eta_{12} E_{11} = 0,\\
&\eta_1 E_{12} + \eta_{12} E_{12} - \eta_{21} E_{22} = 0.
\end{aligned} \tag{6}$$

From these equations we can find:

$$\eta_{12} = \frac{E_{21}(\eta_2 E_{22} + \eta_1 E_{12})}{E_{11}E_{22} - E_{12}E_{21}}; \qquad \eta_{21} = \frac{E_{12}(\eta_1 E_{11} + \eta_2 E_{21})}{E_{11}E_{22} - E_{12}E_{21}}. \tag{7}$$

When ILF in subsystems are the same we have:

$$\eta_{12} = \frac{\eta E_{21}(E_{22} + E_{12})}{E_{11}E_{22} - E_{12}E_{21}}; \qquad \eta_{21} = \frac{\eta E_{12}(E_{11} + E_{21})}{E_{11}E_{22} - E_{12}E_{21}}. \tag{8}$$

2. EXAMPLE OF CLF DETERMINATION FOR TWO BEAMS

Let us define by an example CLF for two beams joined at right angle (fig. 1). Stiff junction is simply supported. In such a structure only transverse (bending) vibrations arise under the action of transverse force. This circumstance simplifies calculations and analysis but does not confine conclusions. Beams length is 1 m, cross-section is 5×1 cm. The beams are joined along long side of the cross-section. Material is steel, internal loss factor is 0.01.

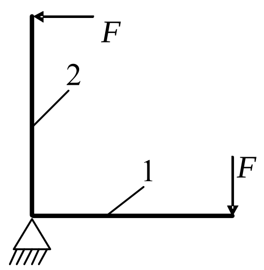


Fig. 1.

Structure for FEM determination of CLF

At FEM modeling, the beams were divided into standard beam elements. Length of each element is about 1 cm. Beam's vibration energy (E_b) was calculated from complex displacements in nodes of a mesh by formula

$$E_b = \frac{1}{2}\omega^2 \sum_{n=1}^{N} M_n \xi_n^2 , \tag{9}$$

where ξ_n is the displacement amplitude in node, M_n is the beam's mass per a node, N is the number of nodes in mesh.

Calculations were carried out at natural frequencies of the structure and the results were summed in octave bands. CLF were calculated by formulas (8).

Analytically CLF were defined by formula:

$$\eta_{ij} = \frac{\tau_{ij}\, c_{gi}}{\omega L_i} , \tag{10}$$

where τ_{ij} is the energy transmission coefficient of bending waves from beam i into beam j; c_{gi} is the group speed of bending waves in beam i, L_i is the length of beam i. For semi-infinite beams with the same cross-section, joined at right angle, the analytical value of energy transmission coefficient is 0.5 [3].

The results of FEM and analytical CLF calculations are presented in fig. 2. The difference of the results is obvious. Let us calculate vibratory energy in structure consisting of several beams using CLF derived by FEM and analytically for the purpose of CLF verification.

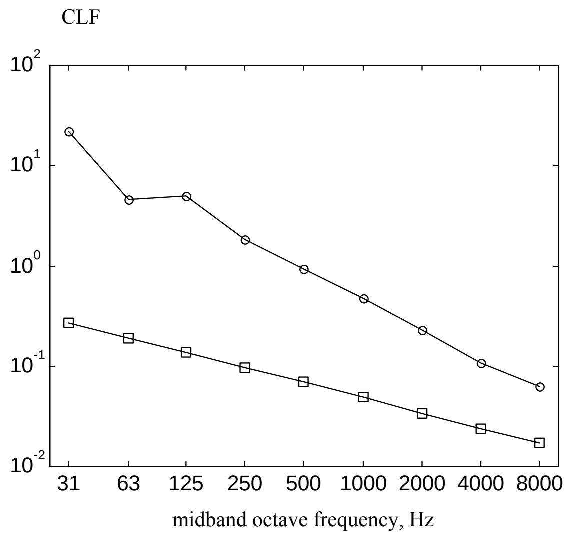


Fig. 2.

CLF calculated

by FEM

analytically

3. COMPUTING OF STRUCTURE VIBRATORY ENERGY

The structure consisting of four identical beams with length 1 m, coupled in sequence at right angles (fig. 3) was used for calculations. Properties of the beams are the same as before when we defined CLF in section 2.

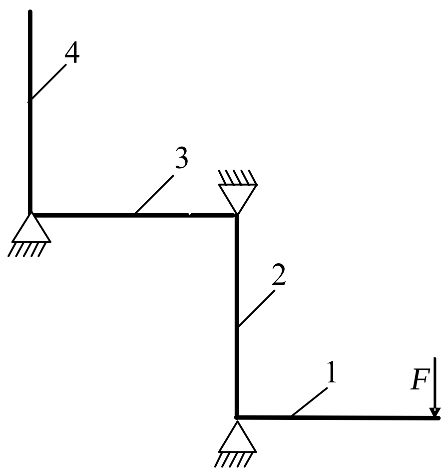


Fig. 3.

Structure consisting of four beams used for calculations for the purpose of CLF verification

Calculations were carried out using EM (approximate method) and FEM (more accurate method). Set of energy balance equations for the structure in fig. 3 is the following:

$$\omega\begin{pmatrix} \eta_1+\eta_{12} & -\eta_{21} & 0 & 0 \\ -\eta_{12} & \eta_2+\eta_{21}+\eta_{23} & -\eta_{32} & 0 \\ 0 & -\eta_{23} & \eta_3+\eta_{32}+\eta_{34} & -\eta_{43} \\ 0 & 0 & -\eta_{34} & \eta_4+\eta_{43} \end{pmatrix}\begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{pmatrix}=\begin{pmatrix} W_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{11}$$

To compare EM and FEM results, input power derived by FEM for structure consisting of four beams when applied force is 1 N was used in right part of equations (11).

Vibratory energies of beams calculated by FEM and EM are presented in fig. 4. One can see that the results agree well if CLF derived from FEM are used in EM. Small difference (less then 1 dB) may be caused by the following. When CLF were defined using FEM we simulated fragment of a whole structure. Therefore we did not take into account energy outflow from this fragment into joined structures and a reverse energy inflow; boundary conditions are not the same as in a whole structure. When using analytical CLF, EM results for most distant beam are underestimated (fig. 4(4)).

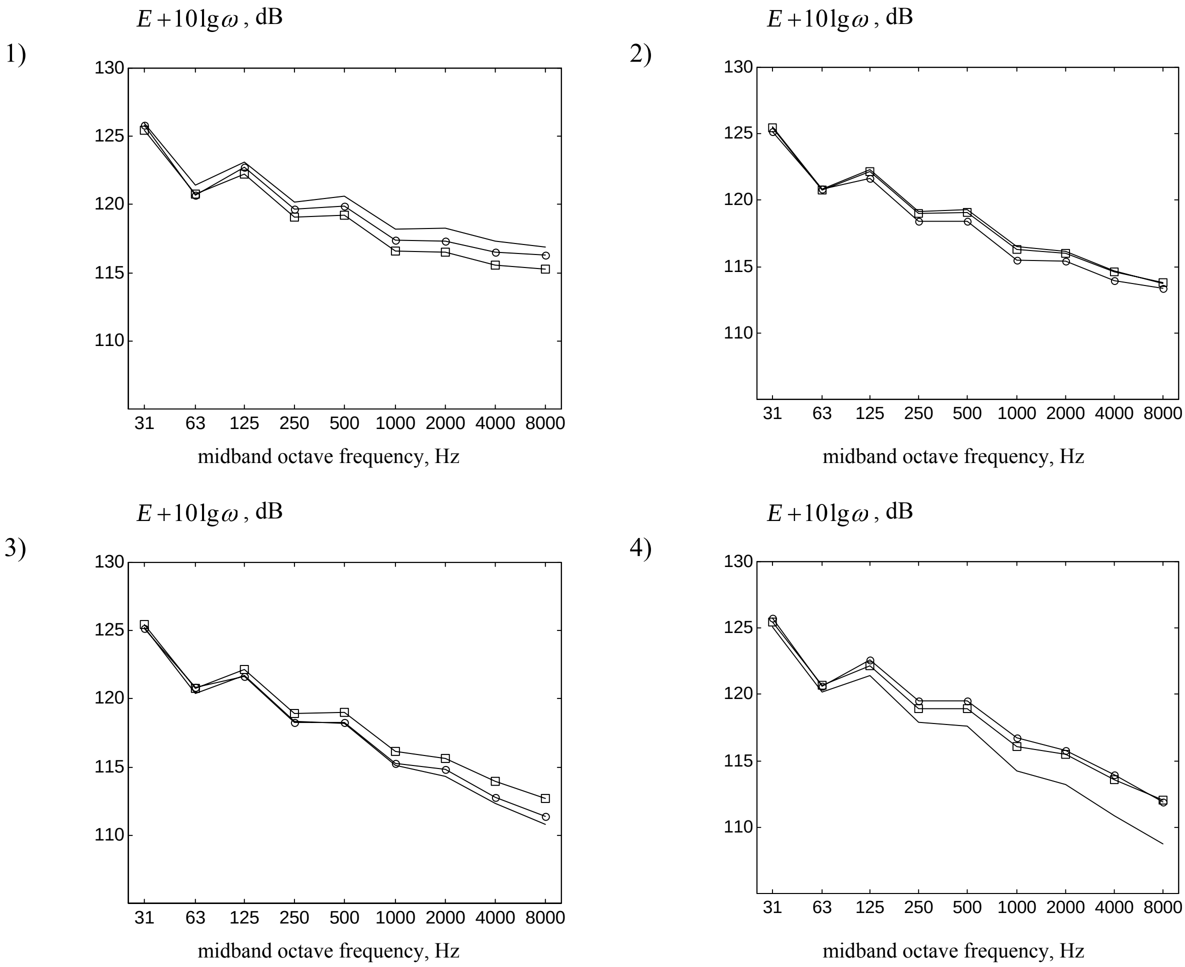
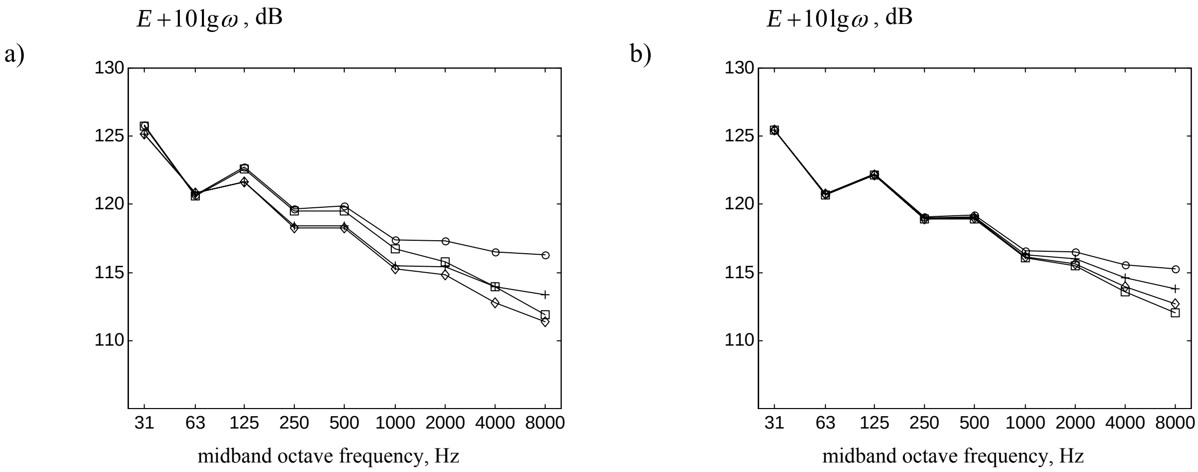


Fig. 4. Vibratory energy of beams 1…4 in structure consisting of four identical beams

EM calculation, CLF from FEM
EM calculation, CLF analytical
FEM calculation

The same calculated vibratory energies, but grouped in a different way, are presented in fig. 5. One can see that energies of beams 2, 3 and 4 calculated by FEM at frequencies 2–8 kHz are nearly the same and less than energy of beam 1 (fig. 5). This result agrees with theoretical conception in what vibration decrease in structure with periodical obstacles (in this case, obstacles are junctions) can be seen at initial (one or two) obstacles. Farther there is no essential vibration reduction [4]. When CLF derived from FEM modeling are used in EM, the calculation results show the same tendency. If analytical CLF are used then vibration reduction far away from excitation point is appreciably more (fig. 5c).



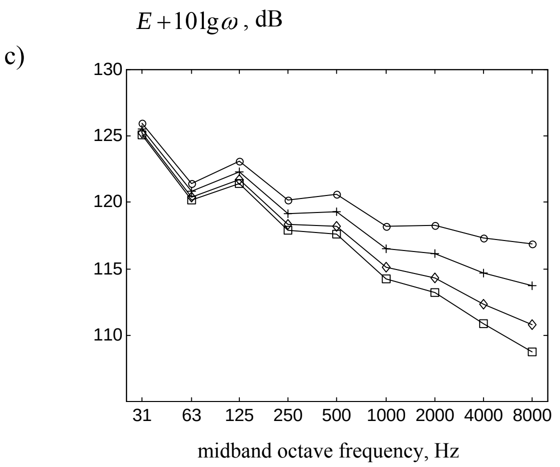
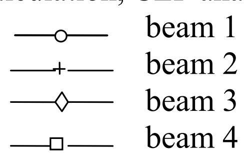


Рис. 5. Vibratory energy of beams 1…4 in structure consisting of four identical beams

a) FEM calculation
b) EM calculation, CLF from FEM
c) EM calculation, CLF analytical



Thus EM provides good results of vibration calculation in complex structure if CLF are derived from FEM modeling of fragment of the structure. In presented example the results of FEM and EM calculations practically coincide. If analytical CLF are used worse results are obtained for subsystem which is placed far away from excitation point.

We should note that at low frequencies of studied range the results of two EM calculations are nearly the same in spite of a great difference between CLF. It is important here that CLF is more than ILF and $\eta_{ij} = \eta_{ji}$.

To provide good EM results, it is also important to define correctly input power. In the presented example, input power for EM calculation was derived from FEM modeling.

CONCLUSION

In the presented example, usage of CLF derived from FEM simulation provides better results of vibration computing in complex structure than usage of known analytical CLF. In addition CLF from FEM were obtained without any assumptions.

EM results agree well with FEM simulation in spite of the fact that the CLF were determined from FEM modeling of fragment of a whole structure. In this case, energy outflow from fragment into joined structures and a reverse energy inflow are not taken into account and boundary conditions are not the same as in a whole structure.

REFERENCES

1. S. N. Ovsyannikov. Vibration propagation in buildings (in Russian). Tomsk Architectural University, 2000.
2. C. Hopkins. Statistical energy analysis of coupled plate systems with low modal density and low modal overlap. JSV. 2002, V. 251, No 2, pp. 193–214.
3. L. Cremer, M. Heckl, E. E. Ungar. Structure-Borne Sound, 2nd edition, Springer, Berlin, 1988.
4. Lyapunov V. T., Nikiforov A. S. Vibroinsulation in ship structures (in Russian). Leningrad, «Sudostroenie», 1975.