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## Power systems harmonics and inter-harmonics identification: a power quality issue

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The discrete Hartley transform (DHT) is a real-valued transform and is closely related to the familiar Fourier transform (FT). This paper presents the application of DHT for power system harmonics and inter-harmonics identification and measurements. Unlike the DFT, DHT can identify these harmonics without any restriction on the frequency of the harmonic to be identified. Also DHT can easily identify the harmonics and inter-harmonics directly without any mathematical models for any one of them. The proposed algorithm uses directly the samples of the voltage or current waveform at the location where the power quality criteria are to meet. In this paper, the convolution property of the DHT is used in the identification and measurement process. Because the Hartley transform is a real transform, it is more computationally efficient than the Fourier and Laplace transforms. Different examples are presented. Effects of critical parameters on the performance of the proposed algorithm are discussed.

### INTRODUCTION

The widespread use of electronic equipment of all types in generation and transmission of power in electric power systems injects different types of harmonics into electric power system networks. The presence of harmonics in the network causes many problems to the customers fed from that network, starting from home appliances to interfacing with the communication equipment and temperature rise in all the electrical elements connected to the network. To make the network more reliable and secure, the term power quality is used and applied now in many distribution networks, and limits are designed for total harmonic distortion (THD), which should be respected by utility companies. To eliminate harmonic propagation into the network filters are used. To design such filters an accurate method is needed to measure the harmonic levels in the network. Over the past two decades, many techniques have been used to identify and measure harmonic contamination. Reference [1] comprises of three new signal-processing techniques for power quality analysis including harmonic identification and measurements: the continuous wavelet transform (CWT), the multi-resolution analysis and the quadratic transform. The conclusion reached is that the continuous wavelet transform produces errors in the harmonic measurements while the multi-resolution analysis produces frequency uncertainty. Reference [2]

presents an approach based on non-linear least errors square (LES) parameter estimation for frequency and harmonic evaluation using artificial neural networks. The system used this technique is a real-time system with a very fast response, but the complexity of the neural network increases as the number of harmonic components increases.

Kalman filtering algorithm is widely applied for the identification and measurements of power system harmonic [3–5]. This technique is more suitable, when the voltage or current signals are time varying during the data window size, and tracks well the harmonic components, but needs appropriate initialization for the filter and accurate model for the waveform under study. Reference [5] presents the applications of the linear Kalman filtering algorithm for identifying the amplitude and damping factors of the inter-harmonic signals that contaminate the voltage or current signal waveforms. Reference [6] presents a new approach to detect, localize, and investigate the feasibility of classifying various types of power quality disturbances. The approach is based on wavelet transform analysis, particularly the dyadic- orthonormal wavelet transform. The key idea underlying the approach is to decompose a given disturbance signal into other signals which represent a smoothed version and a detailed version of the original signal. The decomposition is performed using multi-resolution signal decomposition techniques. The proposed technique is tested for detecting and localizing disturbances with actual power line disturbances. In order to enhance the detection outcomes, the squared wavelet transform coefficients of the analyzed power line signal are utilized. Reference [7] presents an enhanced measurement scheme on the harmonics in power system voltages and currents which are not limited to stationary waveforms, but can also estimate harmonics in waveforms with time-varying amplitudes. The proposed scheme is based on Parseval's relation and the energy concept which defines a “group harmonic” identification algorithm for the estimation of the energy distribution in the harmonics of time-varying waveforms. The scheme is tested on a portable computer-based harmonic recorder providing on-site harmonic data collection and precise harmonic identification.

The discrete Fourier transform (DFT) and fast Fourier transform (FFT) are static parameter estimation techniques used widely in harmonic identification and measurement, but they need some approximations [8, 9]: The waveform is stationary and periodic, the sampling frequency is greater than twice the highest frequency to be evaluated, the number of periods sampled needs to be exactly an integer, and only integer multiple of the fundamental frequency are taken into account in the harmonic survey. Therefore, the waveform must not contain a frequency that are not an integer multiples of the fundamental frequency. There are three major pitfalls in the application of FFT; namely, aliasing, leakage, and picket fence effect [8]. The “picket fence effect” occurs if the analyzed waveform includes a frequency, which is not one of the discrete harmonic of the fundamental, such as interharmonic frequencies, voltage flicker frequencies and any other transient frequencies. Reference [8] proposes an algorithm based on Parseval's relation and the energy concept, which defines a “group of harmonics” for the estimation of the energy

distribution in the harmonics of time-varying waveforms. Reference [9] presents the application of multi-rate digital signal processing techniques to measurement of power system harmonic level. Reference [10] presents an adaptive infinite impulse response (IIR) line enhances (LE) comb filter configuration and its parametisation. This technique is used to resolve the problem of power system harmonic signal retrieval. Test results, in this reference, showed that the proposed ALE comb filter is capable of tracking the fundamental frequency of inverter output voltage and harmonic line current in a power system environment.

This paper presents the application of DHT for power system harmonic and inter-harmonic identification and measurements. Unlike the DFT, DHT can identify the inter-harmonic and harmonic without any restriction on the frequency of the harmonic to be identified. Also DHT can easily identify the harmonics and inter-harmonics directly without mathematical models for them. The proposed algorithm uses directly the samples of the voltage or current waveform at the location where the power quality criterion is to meet. In this paper, the convolution property of the DHT is used in the identification and measurement process. Because the Hartley transform is a real transform, it is more computationally efficient than the Fourier and Laplace transforms. Different examples are presented in the paper. Effects of critical parameters on performance of the proposed algorithm are discussed.

## 1. THE DISCRETE HARTLEY TRANSFORM [10–12]

The discrete Hartley transform (DHT) of a real function  $f(\tau)$  is given by

$$H(v) = M^{-1} \sum_{\tau=0}^{M-1} f(\tau) \text{cas}(2\pi v / M). \quad (1)$$

The inverse is

$$f(v) = M^{-1} \sum_{\tau=0}^{M-1} H(\tau) \text{cas}(2\pi v / M). \quad (2)$$

Here

$$\text{cas}(x) = \cos(x) + \sin(x).$$

The even and odd parts of a function are given by

$$f_e(v) = 0.5 [H(v) + H(M - v)], \quad (3)$$

$$f_o(v) = 0.5 [H(v) - H(M - v)]. \quad (4)$$

The magnitude of the function is

$$A(v) = \sqrt{[f_e(v)]^2 + [f_o(v)]^2}. \quad (5)$$

The phase angle is

$$\tan \phi(v) = \frac{f_o(v)}{f_e(v)}. \quad (6)$$

In the preceding equations,  $M$  is the total number of available samples,  $\tau$  is the sample order,  $\tau = 0, 1$ , and  $\nu$  is the order of the component to be identified. Using DHT we do not need to model the signal considered. What we need is the output of the algorithm at the frequency being evaluated. If there is an output at a specific frequency, then the wave contains that frequency. If the output is zero, then the signal does not contain that frequency.

Equations (5) and (6) give the amplitude and phase angle of the harmonic component of order  $\nu$  regardless the frequency of the component and its relation to the fundamental component as in DFT or FFT. Note that since DHT is purely real, then the computational burden is correspondingly less. In fact if  $M$  is the total number of samples, then the number of operations required to take the DFT or inverse DFT is  $M \log_2 M$  complex multiplication (using FFT), this corresponding to  $M \log_2 M$  real multiplication using the fast Hartley transform [12]. Also, using DHT algorithm, we do not need to model the signal under consideration. What we need is the output of the algorithm at the frequency being evaluated. If there is an output this means that the wave contains that frequency, but if the output is zero, then the signal does not contain that frequency, unlike the well known algorithms used earlier, an accurate model for the signal to a certain harmonic frequency is required. This harmonic frequency is designed by the experience of the wave analyzer expert. Despite of the DFT and FFT popularity, the DHT has the ability to identify the inter-harmonic. In the following section we offer simulated examples that contain harmonic and inter-harmonic components.

## 2. SIMULATED EXAMPLES

### Example 1. Harmonic contaminated signal

The voltage signal at a certain bus in a power system has the following form:

$$v(t) = \cos(\omega_0 t + 30^\circ) + 0.5 \cos(3\omega_0 t) + 0.15 \cos(9\omega_0 t + 10^\circ) + 0.1 \cos(11\omega_0 t - 15^\circ),$$

$$\omega_0 = 2\pi f_0, \quad f_0 = 50 \text{ Hz}.$$

The above signal is sampled at 1000 Hz,  $\Delta T = 1$  ms, and number of samples – 20 (one cycle on 50 Hz nominal frequency) is used to estimate the harmonic contents of the signal. Note that, the window size, in DHT, must be an integer number of cycles. In the following sections, we discuss the effects of number of samples on the estimated parameters. The proposed DHT successfully estimated exactly the magnitude and phase angle of each harmonic component. In the following section we discuss the effects of sampling frequency on the estimated parameters.

### *Effects of sampling frequency*

In real life samples of the voltage or current signal in the form of digits are available, which are obtained at a specified sampling frequency. The order of harmonics in that signal is not known. According to the sampling theorem, the sampling frequency must be higher than double the expected higher frequency in the waveform. If the sampling frequency of the samples is known in advance, then we can easily calculate the expected higher frequency in the signal, just by simply dividing the sampling frequency by two. Here, a question can be raised, what is the sampling frequency that can be used at the beginning of the sampling process? The experience of the analyst suggests that the highest frequency expected in the signal, or the order of harmonic can be obtained from analysis of this signal. As such, one can decide the sampling frequency he can use. In the above simulated example the highest frequency in the voltage waveform is  $11 \times 50 = 550$  Hz, and hence we expect the sampling frequency to be not less than 1100 Hz. Table 1 gives the effects of the sampling frequency on the estimates, where we start the estimates with a small sampling frequency 500 Hz and we end up with 3000 Hz. Examining this table reveals the following remarks:

- Bad estimates for harmonic components are obtained at sampling frequency less than 1500 Hz. This is true, since the sampling theory is not met.
- Good estimates are obtained at sampling frequency greater than 1500 Hz, and the estimates are accurate enough.
- During the estimation process at a sampling frequency less than 1500 Hz, the proposed algorithm estimates other harmonic components. This is true, due to aliasing effect, another signal could be obtained, with more harmonics than that exist in the actual signal.

Table 1. Effects of sampling frequency, data window size = 1 cycle (20 ms)  
( $V$  and  $\phi$  are the harmonic voltage amplitude and its phase angle)

| Harm.<br>№ | Fs=500 Hz |        | Fs=1000 Hz |        | Fs=1500 Hz |        | Fs=2000 Hz |        | Fs=2500 Hz |        | Fs=3000 Hz |        |
|------------|-----------|--------|------------|--------|------------|--------|------------|--------|------------|--------|------------|--------|
|            | $V$       | $\phi$ | $V$        | $\phi$ | $V$        | $\phi$ | $V$        | $\phi$ | $V$        | $\phi$ | $V$        | $\phi$ |
| 1          | 1.197     | 21.8   | 1.0        | 30.0   | 1.0        | 30.0   | 1.0        | 30.0   | 1.0        | 30.0   | 1.0        | 30.0   |
| 3          | 0.5       | 0.0    | 0.5        | 0.0    | 0.5        | 0.0    | 0.5        | 0.0    | 0.5        | 0.0    | 0.5        | 0.0    |
| 9          | 1.197     | -22.0  | 0.25       | 12.0   | 0.15       | 10.0   | 0.15       | 10.0   | 0.15       | 10.0   | 0.15       | 10.0   |
| 11         | 1.197     | 22.0   | 0.25       | -12.0  | 0.1        | -0.15  | 0.1        | -0.15  | 0.1        | -0.15  | 0.1        | -0.15  |

### Effects of Number of Samples

Another test is conducted in this section, where we change the number of samples from  $m = 25$  samples to 150 samples with sampling frequency 2500 Hz, data window size changes from half cycle to 3 cycles. Table 2 gives the results obtained for each harmonic component. Examining this table reveals the following remarks:

- A bad estimate is obtained for harmonics phase angle, when the data window size is not an integer number of cycles, meanwhile good estimates are obtained for the harmonics magnitude at all number of samples.
- The only restriction on this algorithm is that the data window size should be an integer number of cycles.
- If a data window size, which is a fraction of cycles, the proposed algorithm estimates different harmonics order during the estimation process.

Table 2. Effects of number of samples on the estimated harmonics, sampling frequency = 2500 Hz

| Harm.№ | m = 25 |        | m = 50 |        | m = 75 |        | m = 100 |        | m = 125 |        | m = 150 |        |
|--------|--------|--------|--------|--------|--------|--------|---------|--------|---------|--------|---------|--------|
|        | V      | $\phi$ | V      | $\phi$ | V      | $\phi$ | V       | $\phi$ | V       | $\phi$ | V       | $\phi$ |
| 1      | 1.0    | -120   | 1.0    | 30.0   | 1.0    | -120   | 1.0     | 30.0   | 1.0     | -120   | 1.0     | 30.0   |
| 3      | 0.5    | -90.0  | 0.5    | 0.00   | 0.5    | -90    | 0.5     | 0.00   | 0.5     | -90    | 0.5     | 0.00   |
| 9      | 0.15   | -100   | 0.15   | 10.0   | 0.15   | -100   | 0.15    | 10.0   | 0.15    | -100   | 0.15    | 10.0   |
| 11     | 0.10   | -75.0  | 0.1    | -15.0  | 0.1    | -75    | 0.1     | -15.0  | 0.1     | -75    | 0.1     | -15.0  |

### Example 2 Signal contaminated with inter-harmonics

We assume that the voltage signal on example 1 is contaminated with inter-harmonics as

$$v(t) = 0.2 \cos(0.2\omega_0 t + 25^\circ) + 0.15 \cos(0.6\omega_0 t) + 1 \cos(\omega_0 t + 30^\circ) + 0.5 \cos(3\omega_0 t) + 0.15 \cos(9\omega_0 t + 10^\circ) + 0.1 \cos(11\omega_0 t - 15^\circ).$$

This signal is sampled at 100 Hz only to identify the inter-harmonics contents, since we are interested only in the identification and measurement of inter-harmonics, and the largest inter-harmonic frequency expected to be in the waveform is 30 Hz. As such the sampling frequency must be greater than twice this frequency, and a window size of 100 samples is used. The frequency in DHT is adjusted, at the beginning of the estimation, to be equal to the inter-harmonics frequencies expected to be in the signal waveform. In this example, the DHT is adjusted to be equal to 10 Hz and 30 Hz. The proposed algorithm estimates exactly the inter-harmonic components in this waveform.

If one assumes that there are no inter-harmonics in the waveform, but in real time there are inter-harmonics, and tries to estimate the harmonic components, bad estimates may be obtained. A test has been carried out where, in the above example, we estimate the harmonic components and inter-harmonics are not accounted for. The proposed algorithm produces inaccurate estimates for the harmonics voltage magnitude and phase angle. However, when the sampling frequency is increased to 4000 Hz and 400 samples (5 cycle's data window size is used), accurate estimates are obtained. Meanwhile, if at the beginning of the estimation process the inter-harmonics are first identified and then filtered out from the signal waveform, accurate estimates will be obtained.

### Example 3. Actual recorded data

The proposed approach is tested using actual recorded data for three phases voltage of 500 kV, the voltages samples are obtained during the switching on of the 500 kV supply to a transmission line. The Electromagnetic Transient Program (EMTP) is used to compute the voltage at a certain bus on the system. The sampling frequency used in EMTP was 10000 Hz ( $\Delta T=0.1$  ms), and a number of samples equals to 400 samples is used (data window size = 2 cycles).

The proposed algorithm is able to identify up to 50 harmonics. Table 3 gives the magnitude and phase angle of the harmonics content of each phase for a harmonic magnitude greater than 0.5 per cent. Examining this table reveals the following:

- The fundamental components are unbalanced in magnitude, but they almost have 120° displacements.
- Although the three phases have the same order of harmonic, their magnitudes are not equal.
- The second harmonic is the dominant harmonics components in the three phases.

Table 3. Harmonic contents of the waveform,  
sampling frequency = 10000 Hz, m = 400 samples

| Harmon.<br>order | Phase A |        | Phase B |         | Phase C |          |
|------------------|---------|--------|---------|---------|---------|----------|
|                  | V       | $\phi$ | V       | $\phi$  | V       | $\phi$   |
| 1                | 100.8   | – 4.9  | 104.34  | – 124.9 | 99.7    | 113.5    |
| 2                | 13.4    | 48.7   | 14.56   | – 29.7  | 19.93   | – 169.00 |
| 3                | 2.77    | 110.6  | 0.614   | – 6.96  | 3.56    | – 119.8  |
| 4                | 1.36    | 71.6   | 1.202   | – 37.1  | 2.63    | – 79.6   |
| 5                | 1.30    | 81.8   | 0.365   | 26.85   | 1.65    | – 108.98 |
| 6                | 0.897   | 89.33  | 0.3605  | 9.4     | 1.26    | – 114.99 |
| 7                | 0.955   | 83.50  | 0.202   | – 27.42 | 0.949   | – 108.5  |
| 8                | 0.829   | 94.14  | 0.0953  | 80.75   | 0.744   | – 108.6  |

### 3. CONCLUSIONS

The main contribution of this paper is the application of a new digital signal processing algorithm, Hartley transform, for identification and measurements of power system harmonics and inter-harmonics for power quality analysis. Unlike other techniques, this technique does not require a mathematical model for the voltage or current waveform under investigation. This technique uses the available digital samples of the waveform directly, provided that the sampling frequency satisfies the sampling theorem. The DHT is a real valued transformation, which makes this transform computationally efficient and always offers a computational advantage of two as compared to DFT. Meanwhile it has the ability to identify the harmonics and inter-harmonics contents of the waveform regardless the value of the frequency of these components with respect to the fundamental components. The only restriction of the algorithm is that the data window size must be an integer number of cycles, like the DFT. The proposed algorithm is tested using different examples; two of them are simulated examples that contain harmonics and inter-harmonics, while the third example is actual recorded data from the real life of a 500 kV transmission system. The performance of the algorithm appears to be satisfactory.

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