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Computing of coupling loss factors using FEM, probabilistic approach

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Technique for coupling loss factors (CLF) determination with taking into account uncertainty of subsystems is presented. Two beams in junction are considered by an example. Random variables are beams length, which obey the normal distribution law. Finite Element Method (FEM) is used for CLF computing. To verify the technique validity computation example is presented for structure that consists of four sequentially joined beams with uncertain length. Calculations are carried out using two techniques: FEM and Energy Method (EM). It is shown that computing results by two techniques agree well both for mean values and for dispersion.

INTRODUCTION

The most common methods for computing of vibration and noise in complex structures are Finite Element Method (FEM) and Energy Method (EM). EM is often called as Statistical Energy Analysis (SEA) taking into account some assumptions when applying the method. Commercial packages that implement these two methods are designed: ANSYS, NASTRAN, ABACUS (FEM), AutoSEA, SEAM, SEADS (SEA) and others.

FEM is more accurate and multipurpose but more expensive method since it requires detail structure modeling and high-performance computers. Computation costs increase with frequency. Therefore FEM is usually applied at low frequencies. At middle and high frequencies EM is more appropriate method. EM is approximate and comparatively inexpensive. Cost effectiveness is determined by simplified structure modeling, fewer amounts of input data, less computer expenses.

When applying EM, one should know coupling loss factors (CLF) of subsystems in structure. CLF can be determined analytically, from experiment and by using numerical calculations. Analytical CLF are defined from transmission coefficients of energy via subsystems junction. But these coefficients are known for several simple junctions: L-, T- X-shaped rigid junctions of semi-infinite beams and plates and for some other junctions [1, 2, 3]. It is usually insufficiently for practical computing of vibration in complex structures like ships for example. CLF determination from experiment requires complicated and expensive physical experiments.

Most universal numerical way for CLF determination is based on numerical modeling (FEM for example) of subsystems in junction. Using FEM, vibratory energies of subsystems are calculated. Then CLF are defined from a set of liner equations in which coefficients are energies. No additional assumptions are used in this procedure. Example of CLF computing for two beams in junction using FEM was presented in [4]. It was shown that using in EM CLF calculated by FEM provides more accurate results for complex structure than applying analytical CLF.

This conclusion was obtained for deterministic input data. However properties of real structures: geometrical dimensions, material properties, external forces, load application points, etc. — are not known exactly. Besides, technological factors of random origin like welding stresses influence on mechanical properties of structures. In fact, uncertainty is essential property of real structures. Because of all the random factors there are no identical structures, which are formally identical [5, 6].

An example of FEM computing for structure with uncertain properties is presented in [7]. Periodical structure which consists of four beams having equal mean lengths and some lengths dispersion was considered. Ten structures, which are samples from random assembly, were calculated. Results are characterized by a large variation. It demonstrates, that probabilistic approach is adequate for computing vibration in real structures.

When using EM, input data are CLF, internal loss factors (ILF) and input power. These input data substantively are random variables, which properties can be defined experimentally or evaluated numerically. An example of CLF determination using FEM for junctions of building slabs, which dimensions are random variables, is presented in [8]. In [9] uncertainty of plates properties was simulated by point masses randomly distributed over plates' surface. In this paper probabilistic approach is applied for CLF determination of two beams in junction applying FEM. Computing example for complex structure, where random CLF are used, is presented

1. CLF DETERMINATION USING FEM

For CLF determination, sets of energy balance equations are formed for cases of energy injection via each subsystem separately. For example, for two subsystems we have a set of four equations for determination of four unknowns: two CLF (η_{12}, η_{21}) and two internal loss factors (ILF: η_1, η_2). When ILF are known and they are equal, analytical solution of energy balance equations is the following [4].

$$\eta_{12} = \frac{\eta E_{21}\left(E_{22} + E_{12}\right)}{E_{11}E_{22} - E_{12}E_{21}}\,;$$

$$\eta_{21} = \frac{\eta E_{12}\left(E_{11} + E_{21}\right)}{E_{11}E_{22} - E_{12}E_{21}}\,, \tag{1}$$

where E_{11}, E_{12} E_{21}, E_{22} are the vibratory energies of subsystems 1 и 2 (first index) when energy is injected in subsystems 1 и 2 (second index), respectively, which are defined by FEM.

Using this procedure, CLF for two beams in junction were obtained. Substitution of these CLF into energy balance equations for complex structure is provided results, which agree well with more accurate FEM solution [4].

2. CLF DETERMINATION FOR TWO BEAMS HAVING RANDOM PROPERTIES AND STATISTICAL PROPERTIES OF CLF

Let us consider CLF for beams in junction at right angle discussed in [4], fig. 1, taking into account uncertain beams length. Stiff junction is simply supported. In such a structure only transverse (bending) vibrations arise under the action of transverse force. This circumstance simplifies calculations and analysis but does not confine conclusions. Beams length is 1 m, cross-section is 5×1 cm. The beams are joined along long side of the cross-section. Material is steel, internal loss factor is 0.01.

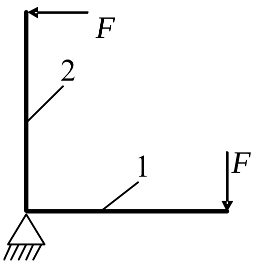

Fig. 1.

Structure for FEM determination of CLF

To obtain CLF statistical properties, several calculations of vibratory energy were carried out by FEM. At that random sets of input data (random sets of beams length) were used. Next, CLF were calculated by formulas (1). In this way a set of CLF samples was obtained. Statistical properties of entire CLF assembly can be evaluated from these samples.

At FEM simulation, the beams were divided into standard beam elements. Length of each element is about 1 cm. Beam's vibration energy was calculated from complex displacements in nodes. Calculations were carried out at natural frequencies of the structure and the results were summed in octave bands.

Nine calculations for nine random sets of beams length (table 1) were executed. Beams lengths were random samples from normal distribution with mean value 1 m and dispersion 0.05 m.

Table 1. Beams length (m), composing structure in fig. 1
(samples from normal distribution with mean value 1 m and dispersion 0.05 m)

	sample								
	1	2	3	4	5	6	7	8	9
beam 1	1.067	0.995	1.018	0.919	1.076	1.021	1.010	0.959	1.008
beam 2	0.986	1.094	0.944	1.043	0.949	0.989	1.037	1.074	0.992

Computing results are presented in fig. 2. It can be seen, that range of random CLF is quite wide, especially in frequency bands which contain several first resonant frequencies of beams (125–500 Hz). A little derivation from equality of beams leads to drastic CLF decreasing. This result agrees with theoretical concept, confirmed experimentally, concerning vibration transmission via obstacle [2]. CLF range becomes narrower with frequency increasing. Mean values approach to analytical CLF at higher frequencies. CLF difference in pairs η_{12} and η_{21} is not significant.

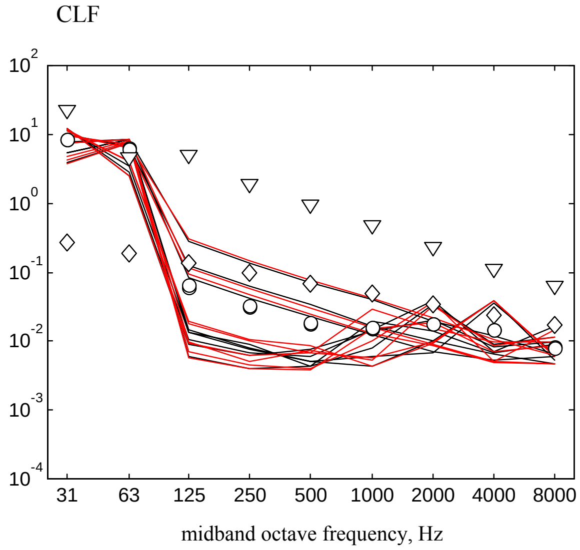

Fig. 2.

CLF for two beams with mean value 1 m and dispersion 0,05 m, FEM computing:

—— nine randon values η_{12},

—— nine randon values η_{21},

○ mean values η_{12}, η_{21}, as well as:

▽ CLF for equal beams,

◇ analytical CLF

Using obtained results, CLF statistical properties (distribution law, mean value, dispersion) were determined. If beams length samples were obtained from normally distributed assembly, then the most suitable distribution law for CLF is normal distribution of decimal logarithm of CLF. The same distribution law is pointed out in [7]. I.e. CLF statistical properties — mean value and standard deviation — are calculated for values $\lg \eta_{12}$ and/or $\lg \eta_{21}$, which are approximately the same in considered example. Statistical properties of $\lg(\eta_{12}/\eta_{21})$ are obey the normal distribution law as well.

3. COMPUTING OF VIBRATORY ENERGY OF COMPLEX STRUCTURE

3.1. Structure for calculation

To verify obtained CLF, calculations were carried out for the structure presented in fig. 3.

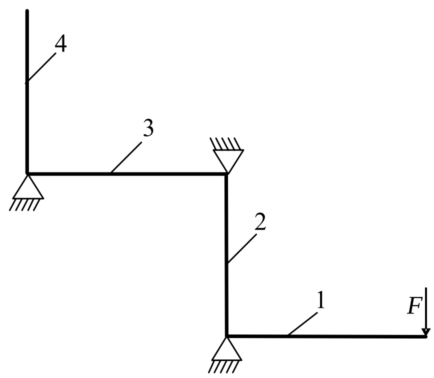

Fig. 3.
Structure consisting of four beams used for calculations for the purpose of CLF verification

3.2. FEM computing

FEM computing were carried out for 10 samples of the structure, in which beams length are random samples from normal distribution with mean value 1 m and dispersion 0.05 m (table 2), i.e. beams statistical properties are the same as under CLF determination.

Table 2. Beams length (m), composing structure in fig. 3
(samples from normal distribution with mean value 1 m and dispersion 0.05 m)

sample	beam 1	beam 2	beam 3	beam 4
1	0.9892	1.0071	1.0235	1.0501
2	1.0493	1.0350	0.9771	1.0060
3	1.0505	1.0574	0.9558	1.0926
4	0.9786	1.0182	0.9459	1.0120
5	1.0587	0.9244	1.0271	1.0177
6	1.0285	0.9319	0.9700	0.9944
7	0.9656	1.0308	0.9913	1.0109
8	0.9420	1.0415	0.9268	1.0102
9	0.9697	0.8558	1.0357	1.0653
10	1.0476	0.9948	0.9756	0.9647

Computing results for beam 4 (most distant from the point of energy injection) are presented in fig. 4 (these results along with description of calculating procedure were presented in previous authors' paper [7]).

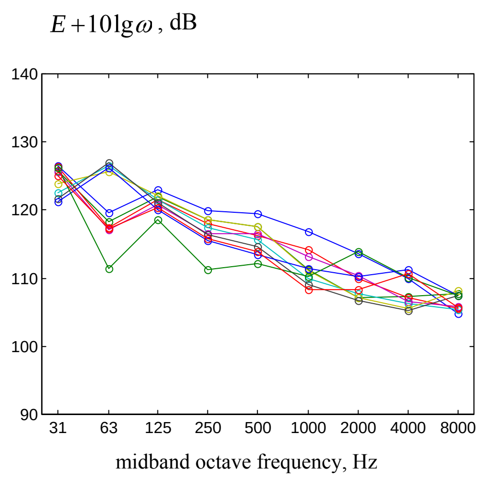

Fig. 4.
Vibratory energy of beam 4 (fig. 3),
FEM computing with 10 random sets of input data (table 2)

3.3. EM computing

3.3.1. Energy balance equations

Energy balance equations for the structure in fig. 3 is the following

$$\omega\begin{pmatrix} \eta_1+\eta_{12} & -\eta_{21} & 0 & 0 \\ -\eta_{12} & \eta_2+\eta_{21}+\eta_{23} & -\eta_{32} & 0 \\ 0 & -\eta_{23} & \eta_3+\eta_{32}+\eta_{34} & -\eta_{43} \\ 0 & 0 & -\eta_{34} & \eta_4+\eta_{43} \end{pmatrix}\begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{pmatrix}=\begin{pmatrix} W_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{2}$$

where $\eta_1 \dots \eta_4$ are the ILF in beams; $\eta_{12} \dots \eta_{43}$ are the CLF; W_1 is the power injected into beam 1 from force F; $E_1 \dots E_4$ are the unknown vibratory energy; ω is the circular frequency.

Random values in equations (2) are CLF and input (injected) power. ILF we shall consider as deterministic value.

To obtain final result — beams energies in the form of mean values and confidence intervals — one should obtain some assembly of energy balance equation solutions. In these equations input data (CLF, input powers) are samples from assemblies of random values. To generate the samples, information about statistical properties (distribution law, mean value, dispersion) of input data is used. Statistical properties of CLF and input power can be obtained from FEM.

3.3.2. Coupling loss factors

Using CLF statistical properties, which were derived from FEM calculations (section 2), random CLF samples were generated for substitution in energy balance equations. One value from pair of decimal logarithm CLF ($\lg\eta_{ij}$) was generated using statistical properties of $\lg\eta_{12}$ or $\lg\eta_{21}$. Second CLF η_{ji} was obtained using statistical properties of $\lg(\eta_{12}/\eta_{21})$. In that way real relationship — $\eta_{ij} \approx \eta_{ji}$ at middle and high frequencies — was attained. CLF calculated by formulas

$$\eta_{ij}=10^{\left(\lg\eta_{ij}\right)_g};$$
$$\eta_{ji}=\eta_{ij}\cdot 10^{\left(\lg\left(\eta_{ij}/\eta_{ji}\right)\right)_g}, \tag{3}$$

where index "g" means that value in brackets is obtained as random sample.

Some examples of CLF and relationship of CLF pair (generated values) are presented in fig. 5 and 6. In the same figures, for visual comparison, the maximum and minimum values, which were obtained in section 2 from nine samples only, are presented.

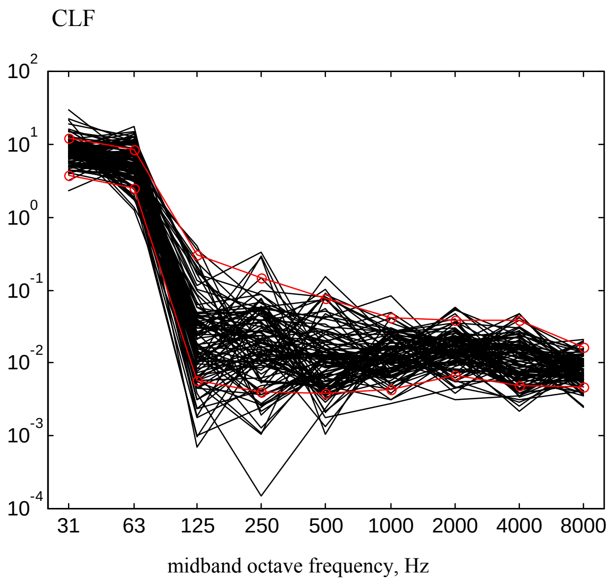

Fig. 5. CLF for two beams in junction:

— η_{ij} random samples generated (100 random values at each frequency)

–o– maximum and minimum η_{12}, calculated by FEM (from fig. 2)

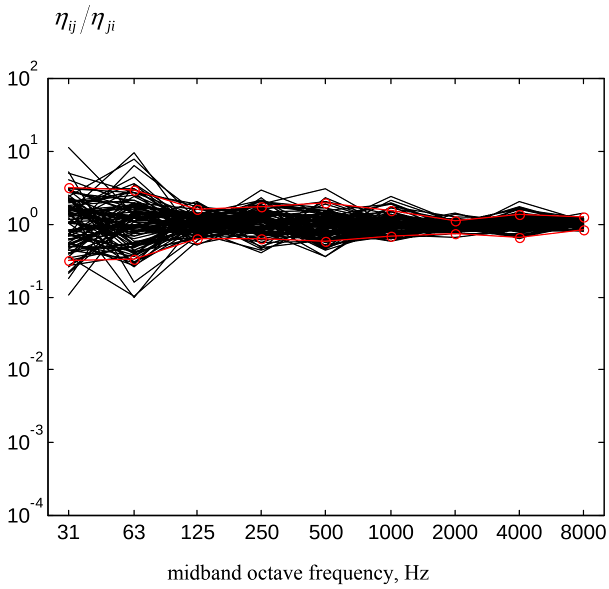

Fig. 6.
Relationship of CLF in pair η_{ij}/η_{ji}

— η_{ij}/η_{ji} random samples generated (100 random values at each frequency)

–o– maximum and minimum η_{12}/η_{21}, calculated by FEM

3.3.3. Input power

To obtain statistical properties of input power, FEM computing was carried out for beam simply supported at one end (fig. 7). Force acts at the other end. Substitution of entire structure by fragment is made taking into account that entire complex structure (like ship or plane) simulation is practically impossible in a wide frequency range including middle and high frequencies. One has to restrict the simulation by involving a part of a structure into the model.

Beam length is random variable with mean value 1 m and dispersion 0.05 m (table 3). Using FEM simulation results, input power was calculated by formula $W = \mathrm{Re}\left(F \cdot v^*\right)/2$, where F is the acting force, which is specified in FEM simulation (1 N in our case), v^* is the complex-conjugate velocity in node where force acts.

It was found in this example that input power calculated from fragment simulating practically coincide with input power, obtained from entire structure (fig. 8 and 9) excluding low frequencies where one first resonance frequency is in the band (63 Hz).

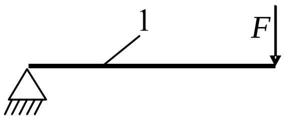

Fig. 7. Structure for FEM determination of input power

Table 3. Beams length (m), fig. 7
(samples from normal distribution with mean value 1 m and dispersion 0.05 m)

sample	1	2	3	4	5	6	7	8	9	10
length	0.966	0.949	0.938	1.014	0.979	1.003	0.982	0.977	1.019	1.036

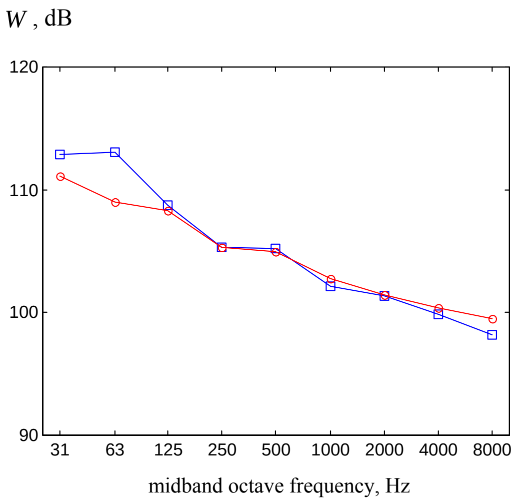

Fig. 8.

Input power (FEM computing):
- one beam simulation, mean values over the 10 random beam lengths (table 3);
- four beams simulation, mean values over the 10 random structures (table 2)

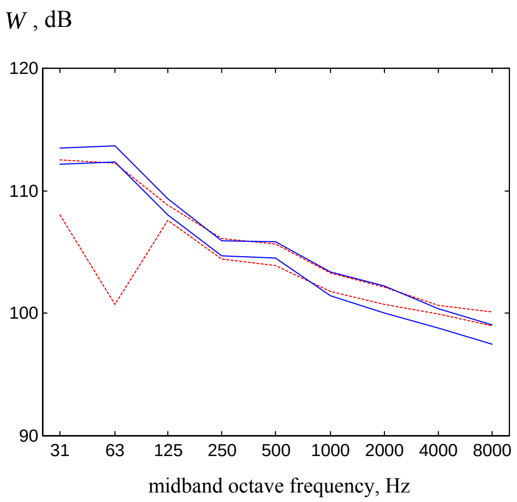

Fig. 9.

Maximum and minimum input power (FEM computing):
- one beam simulation, 10 random beam lengths (table 3);
- four beams simulation, 10 random structures (table 2)

Using FEM results, statistical properties of input power were determined in the same way as for CLF. Random samples of input power were generated for substitution in energy balance equations.

It should be noted that input power dispersion is much less than CLF dispersion in frequency range 125–8000 Hz (compare fig. 2 and 9). Therefore input power dispersion does not affect final results significantly. One could neglect this dispersion and to use mean values of input power in further calculations.

3.3.4. Comparison of EM and FEM calculating results

EM calculations were carried out for the following input data.

- CLF: 100 samples for each junction from CLF distribution with properties defined in section 3.3.2. In this way one of two CLF was generated. The second one was defined from relationship of CLF in pair (see section 3.3.2).
- Input power: 100 samples from distribution with properties defined in section 3.3.3 for one beam.
- ILF: deterministic value, $\eta_i = 0{,}01$.

Thus, 100 EM calculations were carried out. In each calculation CLF and input power were random values simultaneously. 100 EM results are compared with 10 FEM results, presented in section 3.2.

Vibratory energies of beams 1…4 in some octave band are presented in fig. 10. In fig. 11 the same data are presented in the other form: mean values and confidence intervals of vibratory energies for each beam.

It can be seen that EM and FEM results agree well both in mean values and in deviations. Relatively large discrepancy is observed at low frequencies (63 Hz), that is caused first of all by inaccuracy of input power determination using fragment (one beam) of the entire structure. In frequency range 125–500 Hz EM yields underestimated energy for distant beams (3, 4). It may be caused by modest accuracy of CLF statistical properties when we defined them using FEM: 9 samples is not large amount of sampling for so large CLF dispersion, fig. 2.

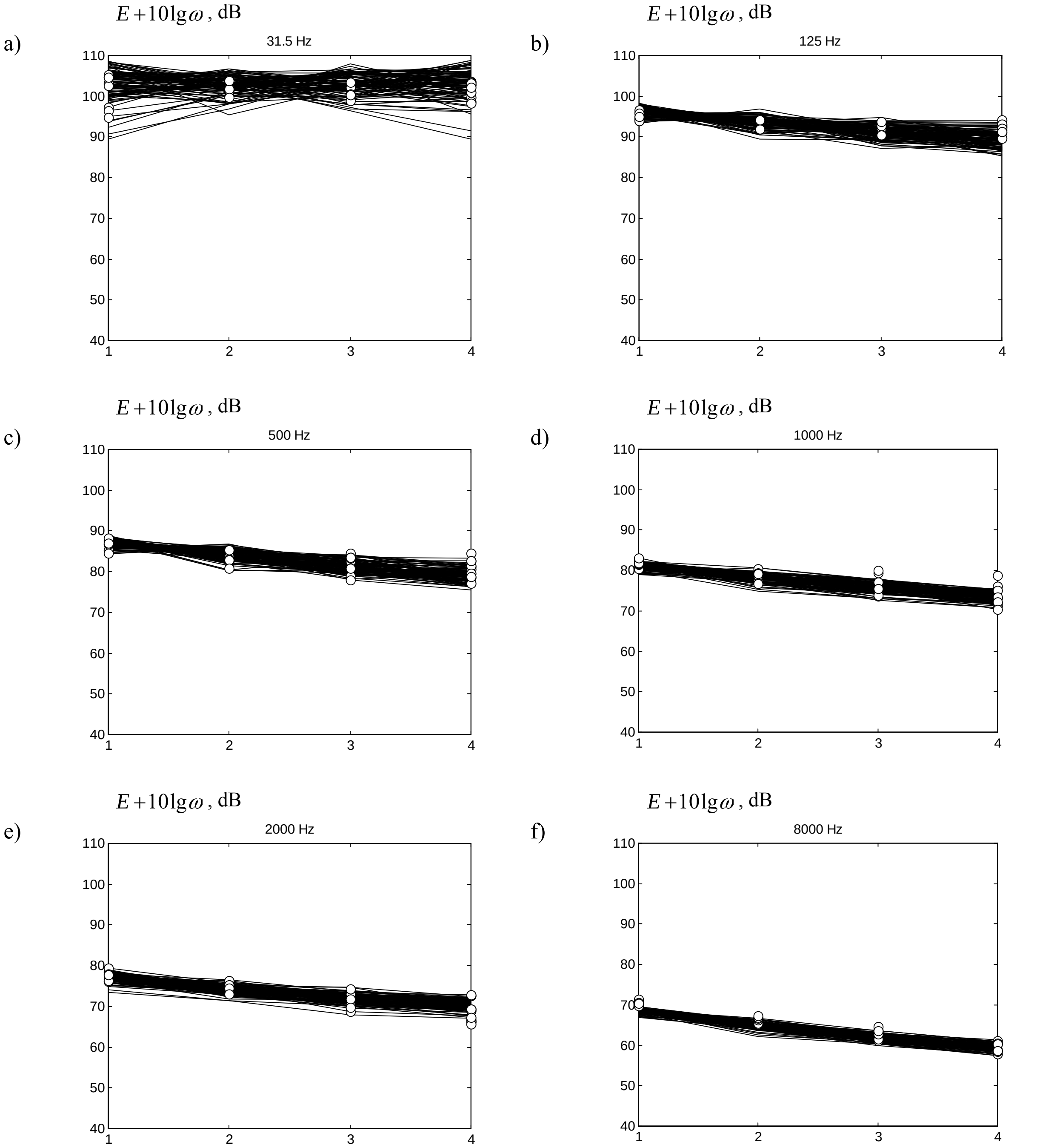

Fig. 10. Vibratory energy of beams 1…4 in frequency ranges:
solid lines — EM calculations using CLF and input power generated on the base of CLF and input power statistical properties derived from FEM computing (100 calculations);
○ — FEM computing (10 расчетов);
a) 31,5 Hz, b) 125 Hz, c) 500 Hz, d) 1000 Hz, e) 2000 Hz, f) 8000 Hz

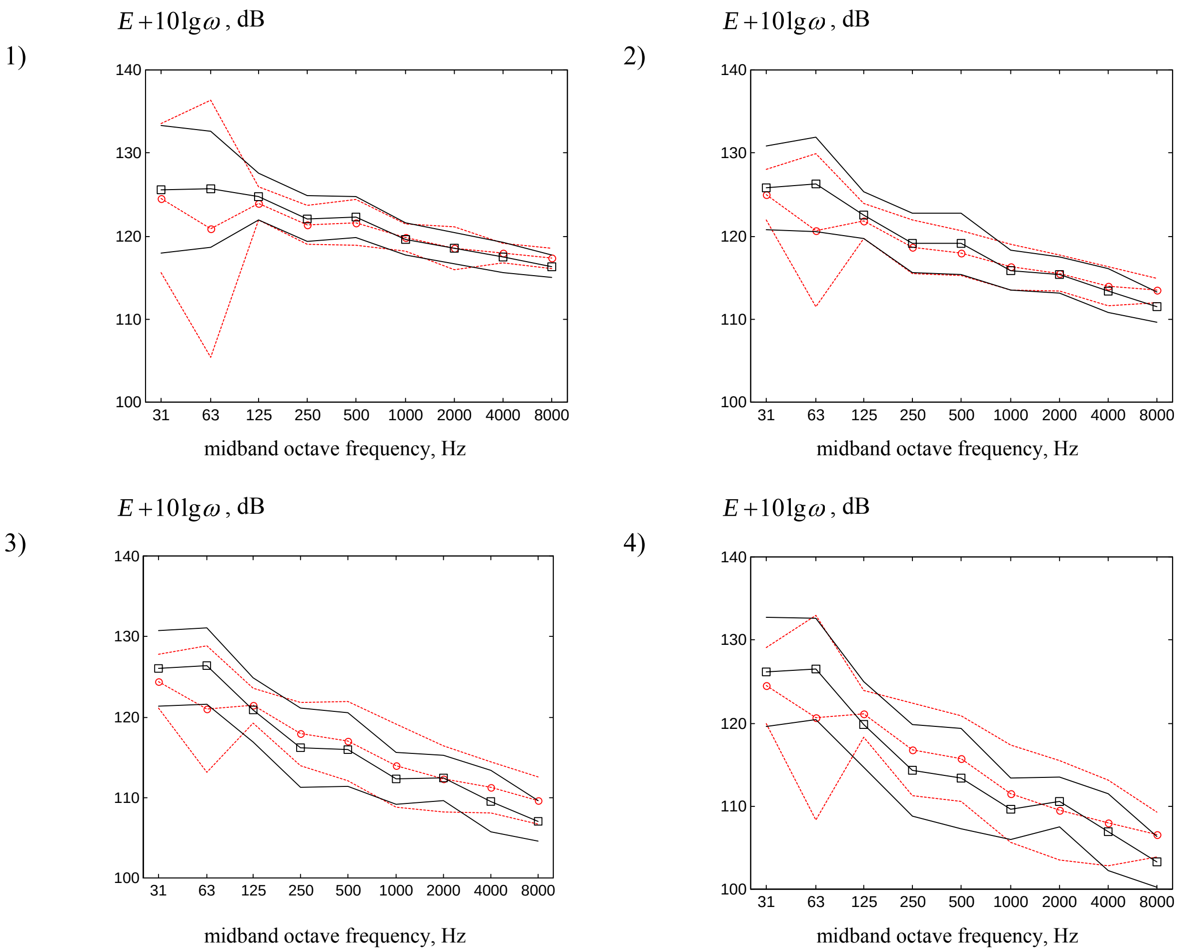

Fig. 11. Mean values (—□—) and confidence intervals for confidence probability 0,95 (------) for energies of beams in whole structure from EM calculation using random CLF and input power; —○—, ------ the same values from FEM calculation for 10 random modification of the structure; input power is obtained from one beam simulation;
1) beam 1, 2) beam 2, 3) beam 3, 4) beam 4

CONCLUSIONS

An example of CLF computing, by using FEM, for subsystems with uncertain properties is presented in the paper. CLF statistical properties: distribution law, mean value, dispersion — for two beams in junction having random length are defined. These properties are used for probabilistic EM calculations of structure which consists of four beams. EM results are compared with FEM results for the same structure in which beams lengths are random values as well. It was found that EM and FEM results agree well both in mean values and in deviations. At that, EM is comparatively inexpensive technique.

Thus, FEM application for CLF determination allows not only to calculate CLF for junctions when there are no analytical solutions, but also to evaluate statistical properties of CLF as random values. Next, these properties are used in EM computing of complex structure as input data. As a result one obtains not only some mean values but possible range of results as well.

Representation of final results in the form of mean value and standard deviation adequately describes predictable state of real structures having uncertain properties. Applied methodology can be used for probabilistic computing of more complex structures.

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