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Viscoelastic damping and piezo-electric control of structures subjected to aerodynamic noise

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Analytical and numerical simulations are carried out in order to identify physical parameters affecting acoustic and motion control by viscoelastic piezo-electric and material damping. Numerical simulations in terms of critical parameters, such as relaxation functions of the structure and piezo-electric devices, aerodynamic coefficients, Mach number, are carried out to evaluate their sensitivity to system responses, sensing and structural control. Computational simulations indicate that light weight piezo-electric viscoelastic devices can be effectively used to actively or passively control flight vehicle structural response to aerodynamic noise.

INTRODUCTION

Aerodynamic noise, i. e. flow generated acoustical disturbances in skin panels, ducts, combustors, helicopter and turbine blades, etc., is ever present in flight vehicles and may lead to breaches in structural integrity as well as crew and passenger discomfort. While efficient aerodynamic design may lead to some noise reduction, its ultimate disposal can only be achieved through light weight energy dissipation devices, such as material damping or piezo-electric generated potentials used for either active or passive motion and sound control.

Although these noise problems are inherently stochastic, the present pilot simulations are deterministic in order to reduce the number of contributing parameters and to gain fundamental insight into the physical phenomena. The present analysis, then, deals with the union and interaction of several areas: aeroacoustics, aeroelasticity, viscoelastic materials, piezoelectric effects and damping to produce motions of small amplitudes and decaying sound transmissions. Aerodynamic noise generated by a variety of flows has been studied extensively since first systematically analyzed in [1] and [2] and subsequently expanded among others in [3, 4, 5, 6]. Additional extensive aerodynamic noise treatises may be found in [7, 8, 9, 10, 11]. The theory of aeroelasticity is well established and may be found described in detail in such classical texts as [12, 13, 14, 15].

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Analyses of viscoelastic damping effects in [16, 17, 18, 19, 20, 21, 22] have shown that energy dissipation due to material and/or structural damping may produce either stabilizing or destabilizing contributions to the system's self-excited dynamic motion depending on phase relationships of the state variables. For instance, this phenomenon leads to viscoelastic flutter velocities which are either smaller or larger than corresponding elastic ones for aerodynamically, dynamically and geometrically identical lifting surfaces.

Piezoelectric control of elastic and viscoelastic structures has been demonstrated in numerous publications, which have been discussed in [23]. Recent formulations and analyses of piezo-aero-thermo-viscoelastic effects in [24, 25, 26, 27, 28] have demonstrated that sufficient power can be generated by viscoelastic piezo-electric light weight material strips to effectively influence and control static and dynamic motion.

Viscoelastic material properties may be found for polymers in [29] and [30] and for metals in [31]. Piezo-viscoelastic properties are displayed in [32, 33, 34, 35].

1. ANALYSIS

In this first of a series of studies on the interaction of viscoelastic, structural and piezo-electric damping and its interaction with aerodynamic noise, only simple idealized conditions are considered in order to gain a fundamental understanding of the governing physics and mathematics of these phenomena. To this end, only deterministic conditions will be considered here, although stochastic influences from material properties, temperatures, geometries, aerodynamic pressure fluctuations are undeniably present in real physical problems. (See [36] for a literature review, and analysis and evaluation of viscoelastic stochastic material property effects.)

It has been shown in [4] that the acoustic pressure, $\Delta p(x,t)$, problem formulated in [1] and [2] can be solved for a number of conditions. For illustrative purposes of the present paper, a simple harmonic motion version of the pressure differential over a thin plate given by Sears [37] is selected, where

$$\Delta p(x_1,t) \;=\; \frac{-2\,\imath\,\rho_o\,U_r\,a_3\,\exp[-\imath\,(\sigma_1+k_1\,U_r\,t)]}{\sqrt{[\imath\,\pi\,\sigma_1\,(1+M_r)(\zeta+1)]}}\;\exp\left[\frac{\imath\,M_r\,\sigma_1}{1+M_r}(\zeta+1)\right] \tag{1}$$

where $-b \le x_1 \le b$, $\zeta = x_1/b$, $\imath = \sqrt{-1}$, $\sigma_1 = k_1 b$, $k_1 = \omega/U_r$, a_3 is the x_3-component amplitude of the disturbance velocity and the free steam Mach number is $M_r = U_r/c_0$. The total force per unit length of x_2 over the plate is obtained by integration w. r. t. x_1 is

$$F_3(t) \;=\; \pi\,a_3\,\rho_o\,U_r\,b\exp\left[-\imath\,k_1\,U_r\,t\right]\;S_c(\sigma_1,M_r) \tag{2}$$

with

$$S_c(\sigma_1,M_r) \;=\; \frac{\exp\left[-\imath\,\sigma_1\right]}{\sigma_1\,\pi}\sqrt{\frac{2\imath}{M_r}}\,\mathcal{F}\left(\sqrt{\frac{4\,\sigma_1\,M_r}{\pi(1\,+\,M_r)}}\right) \tag{3}$$

for high frequencies and where $\mathcal{F}$ is the complex Fresnel integral [38] defined as

$$\mathcal{F}(A) \;=\; \int_0^A \exp(\imath\,\pi\,\widehat{\zeta}^2/2)\;d\widehat{\zeta} \tag{4}$$

For low frequencies, the Sears function becomes

$$S_c(\sigma_1, M_r) \;=\; \frac{S(\sigma_1/\beta_r^2)}{\beta_r}\left[J_o(\widetilde{\zeta}) \;+\; \imath\, J_1(\widetilde{\zeta})\right] \exp\left[-\imath\,\sigma_1\, f(M_r)/\beta_r^2\right] \tag{5}$$

with

$$\widetilde{\zeta} \;=\; M_r^2\,\sigma_1/\beta_r^2 \qquad \beta_r \;=\; \sqrt{1\,-\,M_r^2} \tag{6}$$

$$f(M_r) \;=\; (1\,-\,\beta_r)\,\ln(M_r) \;+\; \beta_r\,\ln(1\,+\,\beta_r) \;-\; \ln 2 \tag{7}$$

and the J_n are Bessel functions.

The large deformation constitutive relations at constant temperatures in curvilinear coordinates θ^i for viscoelastic piezo-electric materials have been formulated in [23] as

$$\tau_j^i(\theta,t) \;=\; \int_{-\infty}^{t} \phi_{jl}^{ik}(\theta,t,t')\,\frac{\partial\gamma_k^l(\theta,t')}{\partial t'}\,dt' \tag{8}$$

$$-\int_{-\infty}^{t} \phi_{jl}^{i\mathbf{E}}(\theta,t,t')\,\frac{\partial\widehat{E}^l(\theta,t')}{\partial t'}\,dt' - \int_{-\infty}^{t} \phi_{jl}^{i\mathbf{T}}(\theta,t,t')\,\frac{\partial\mathcal{A}T(\theta,t')}{\partial t'}\,dt'$$

$$D^i(\theta,t') \;=\; \int_{-\infty}^{t} \phi_l^{ik\mathbf{E}}(\theta,t,t')\,\frac{\partial\gamma_k^l(\theta,t')}{\partial t'}\,dt' \;+\; \int_{-\infty}^{t} \phi_l^{i\mathbf{E}}(\theta,t,t')\,\frac{\partial\widehat{E}^l(\theta,t')}{\partial t'}\,dt' \tag{9}$$

where τ_j^i are the Cauchy stress tensors, $\gamma_k^{l\mathbf{e}}$ the Green-Zerna large strain tensors, ϕ_{jl}^{ik} the viscoelastic relaxation functions, $\phi_{jl}^{i\mathbf{E}}$ the electrical viscoelastic relaxation functions or the piezoelectric stress/charge matrix, $\phi_l^{i\mathbf{E}}$ the dielectric permittivity matrix, $\widehat{E}^l$ the electrical field intensity vectors, and D^i the electric displacement vectors. Field equations and associated general boundary conditions for displacements, stress and electric potentials have been discussed in detail in [23]. Eqs. (8) and (9) can, if desired, be reduced to linear expressions.

All relaxation functions depend on environmental conditions and in nonlinear cases on strain rates as well, such that excluding material aging and manufacturing effects any $\phi(\theta,t,t') = \phi[\theta,t,t',$ $M(\theta,t'),T(\theta,t'),\dot{I}(\theta,t')]$, with $M(\theta,t',T)$ the moisture content, $T(\theta,t',M)$ the temperature, $\dot{I} = \{\dot{I}_i(\theta,t)\}$ the three fundamental strain rate invariants. In particular, relaxation functions denoted by the superscript $\mathbf{E}$, but also all other ϕs, can additionally and independently be made functions of an invariant of $\widehat{E}^l$ (the viscoelastic electrical field intensity vectors), such as $I_e^{\mathbf{E}}(\theta,t) = tr\{\widehat{E}^l\}$, resulting in nonlinear material property electrical intensity effects or conversely exhibiting a dependency on electric displacements through $I_d^{\mathbf{E}}(\theta,t) = tr\{D^l\}$. The generic $I^{\mathbf{E}}$ represents either of these electric invariants. (See Appendix A for further details.)

The functional dependence of any and all of the above nonlinear anisotropic relaxation functions can in general be defined as

$$\phi_{jl}^{ik}(\theta,t,t',M,T,\dot{I},\dot{I}^{\mathbf{E}}) \;=\; \tag{10}$$

$$\sum_{\mathbf{m}=0}^{\mathbf{M}_{\underline{jl}}^{ik}} \sum_{\mathbf{p}=0}^{\mathbf{P}_{\underline{jl}}^{ik}} \sum_{\mathbf{q}=0}^{\mathbf{Q}_{\underline{jl}}^{ik}} \sum_{\mathbf{r}=0}^{\mathbf{R}_{\underline{jl}}^{ik}} B_{\underline{jl}\mathbf{mpqr}}^{\underline{ik}}(\theta, t, t', M, T)\ (\dot{I}_1 - 3)^{\mathbf{m}}\ (\dot{I}_2 - 3)^{\mathbf{p}}\ (\dot{I}_3 - 1)^{\mathbf{q}}\ (\dot{I}^{\mathbf{E}} - 3)^{\mathbf{r}}$$

where underscores indicate no summations over the affected indices, and where the parameters $\mathbf{M}_{jl}^{ik}$, $\mathbf{P}_{jl}^{ik}$, $\mathbf{Q}_{jl}^{ik}$ and $\mathbf{R}_{jl}^{ik}$ and the functions $B_{jl\mathbf{mpqr}}^{ik}$ are material dependent and represent distinct functional sets for each of the directional ϕ_{jl}^{ik}, $\phi_{jl}^{i\mathbf{E}}$, $\phi_{l}^{ik\mathbf{E}}$, $\phi_{j}^{i\mathbf{E}}$ and $\phi_{jl}^{ik\mathbf{T}}$, which must be experimentally generated for each material and for each set of environmental conditions. For linear materials, none of the parameters are dependent on any of the invariants of strains and/or of the electric field intensity and displacement. Additionally, for small deformations the stresses τ_j^i and strains γ_j^i reduce to their Cartesian engineering counterparts σ_{ij} and ϵ_{ij} and $x_i = \theta^i$.

Thus, linear and nonlinear isotropic and anisotropic piezoelectric viscoelastic material (PVM) behaviors are completely defined in terms of the modeled constitutive relations, but experimental determination of the Eqs. (10) material property functions B remains to be carried out and represents a formidable task.

The simplest 1-D isothermal linear viscoelastic representations of moduli E and relaxation functions ϕ then become

$$E^{\mathbf{m}}(t) \;=\; \frac{d\phi^{\mathbf{m}}(t)}{dt} \;=\; \sum_{n=0}^{N^{\mathbf{m}}} \phi_n^{\mathbf{m}}\ \exp(-t/\tau_n^{\mathbf{m}}) \tag{11a}$$

$$E^{\mathbf{p}}(t) \;=\; \frac{d\phi^{\mathbf{p}}(t)}{dt} \;=\; \sum_{n=0}^{N^{\mathbf{p}}} \phi_n^{\mathbf{p}}\ \exp(-c^{\mathbf{p}}\, t/\tau_n^{\mathbf{p}}) \tag{11b}$$

$$E^{\mathbf{E}}(t) \;=\; \frac{d\phi^{\mathbf{E}}(t)}{dt} \;=\; \sum_{n=0}^{N^{\mathbf{E}}} \phi_n^{\mathbf{E}}\ \exp(-c^{\mathbf{E}}\, t/\tau_n^{\mathbf{E}}) \tag{11c}$$

with $\tau_0^{\mathbf{m}} = \tau_0^{\mathbf{p}} = \tau_0^{\mathbf{E}} = \infty$ and where the superscripts $\mathbf{m}$, $\mathbf{p}$ and $\mathbf{E}$ respectively refer to structural, piezo-electric device and to piezoelectric voltage-deformation viscoelastic material properties. The additional factor $c^{\mathbf{E}} > 0$ allows for a time shift of the PVM properties relative to the structural ones, and a value of unity indicates that the relaxation times $\tau_n^{\mathbf{E}}$ are sufficient to describe any time shifts between material and piezoelectric viscoelastic responses.

If structural damping is included, then the modulus and relaxation function representing structural viscoelastic properties must be modified to include the 90° out of phase component due to Coulomb friction, or the leading terms must read

$$\left[{}^0\mathcal{C}_{jl}^{ik}(\theta, t')\ + \imath\, g_{jl}^{ik}\right] \qquad \text{or} \qquad [\phi_0^{\mathbf{m}}\ +\ \imath\, g] \tag{12}$$

where g_{jl}^{ik} and $g \geq 0$ are the time and frequency independent structural damping coefficients [19], [20].

In order to reduce the number of parameters to a minimum, a deterministic problem consisting of a rigid lifting surface on a flexible support (Fig. 1) is used as a typical vehicle for the present diagnostic fundamental sensitivity study. The joints between rods L_1 and L_2 exhibit structural damping and the rods L_2 are viscoelastic with attached viscoelastic piezoelectric devices. The L_1 rods are rigid and two degrees of freedom are possible in the form vertical and rotational motions are possible by distinct elongations of the L_2 rods. The 2 DOF configuration can be achieved by

either placing the center of pressure off center by modifying the Δp pressure distribution of Eq. (1) to an unsymmetric one or more simply preserving the symmetric Δp and introducing unequal viscoelastic relaxation functions or distinct cross sectional areas or both for the L_2 rods.

If one designates the vertical and rotational motions respectively by $h(t)$ and $\alpha(t)$, then the lift force $\mathcal{L}$ on the plate with surface area $2\,c\,b$ due to aerodynamic noise, aero-acoustics and motion is

$$\mathcal{L}(t) \;=\; \underbrace{F_3(t)\,b}_{\substack{\text{aerodynamic}\\ \text{noise}}} \;+\; \underbrace{C_0\,\sin\left\{\alpha_r + \alpha(t) + \tan^{-1}\left[\frac{\dot{h}(t)}{U_r}\right]\right\}}_{\text{lift}}$$

$$\simeq\; F_3(t)\,b \;+\; C_0\,\left[\alpha_r + \alpha(t) + \frac{\dot{h}(t)}{U_r}\right] \tag{13}$$

where

$$C_0 \;=\; \frac{2\,\pi\,\rho_o\,M_r^2\,c_0^2\,b^2}{\sqrt{1-M_r^2}} \qquad \text{for} \qquad M_r < 1 \tag{14}$$

and

$$C_0 \;=\; \frac{4\,\rho_o\,M_r^2\,c_0^2\,b^2}{\sqrt{M_r^2-1}} \qquad \text{for} \qquad M_r > 1 \tag{15}$$

Similarly the aerodynamic moment is defined as

$$\mathcal{M}(t) \;=\; e(t)\,\mathcal{L}(t) \tag{16}$$

where $e(t)$ is the moment arm between the center of pressure and the rotation point on L_1.

The uniform strain in the homogeneous L_2 bars can be determined geometrically as

$$\epsilon_{33}(t) = \frac{\pm\,L_1\,\sin\left[\alpha(t)\right] \;+\; h(t)}{L_2} \;\simeq\; \frac{\pm\,\alpha(t)\,L_1 \;+\; h(t)}{L_2} \tag{17}$$

The governing relations of this linear system now become

$$\underbrace{I_h\,\ddot{h}}_{\text{inertia}} \;+\; \underbrace{\mathcal{L}[F_3,\alpha_r,\alpha,\dot{h}]}_{\text{pressure}+\text{lift}} \;=$$

$$\underbrace{K_h\int_0^t \widetilde{\phi}(t-t')\,\frac{d}{dt'}\left[\pm\alpha(t') \;+\; \frac{h(t')}{L_2}\right]\,dt'}_{\text{force due to viscoelastic L}_2\text{ rods}} \;-\; \underbrace{K_{h1}\int_0^t \phi_1^{\mathbf{E}}(t-t')\,\frac{d\widehat{E}(t')}{dt'}\,dt'}_{\text{piezoelectric force}} \tag{18}$$

$$I_\alpha\,\ddot{\alpha} \;+\; \mathcal{M}[F_3,\alpha_r,\alpha,\dot{h}] \;=$$

$$K_\alpha\int_0^t \widetilde{\phi}(t-t')\,\frac{d}{dt'}\left[\pm\alpha(t') \;+\; \frac{h(t')}{L_2}\right]\,dt' \;-\; K_{\alpha 2}\int_0^t \phi_2^{\mathbf{E}}(t-t')\,\frac{d\widehat{E}(t')}{dt'}\,dt' \tag{19}$$

$$D(t) \;=\; K_D \int_0^t \phi_3^{\mathbf{E}}(t-t')\,\frac{d}{dt'}\left[\pm\alpha(t') \;+\; \frac{h(t')}{L_2}\right] dt' \;+\; K_{D4}\int_0^t \phi_4^{\mathbf{E}}(t-t')\,\frac{d\widehat{E}(t')}{dt'}\,dt' \tag{20}$$

where either $D(t)$ or $\widehat{E}(t)$ or the total displacement of the L_2 rods is prescribed for either control or damping. The effective relaxation function $\widetilde{\phi}$ due to L_2 and piezoelectric device responses with areas A_{L_2} and A_P is

$$\widetilde{\phi}(t) \;=\; \frac{A_{L_2}\,\phi(t) + A_P\,\phi_P(t)}{A} \quad \text{with} \quad A \;=\; A_{L_2} + A_P \tag{21}$$

The generic aerodynamic pressure (noise) F_3, while in the previous section was associated with a simple harmonic pressure fluctuation of Eq. (1), can in the above relations be generated by any other deterministic or stochastic noise source. In a more general sense, for a flexible lifting surface on flexible or rigid supports, the unknowns become position dependent, i. e. $h(x_1, x_2, t)$, $\alpha(x_2, t)$ and $\widehat{E}(x_1, x_2, t)$ or $D(x_1, x_2, t)$. In either form, the three linear coupled relations (18) - (20) must be solved simultaneously for α, h and D or $\widehat{E}$. This can be accomplished in either the time domain by a repetitive Runge-Kutta scheme or by finite differences, or not in real time space, but rather in the integral transform space, by applying Laplace or Fourier transforms.

The first scheme results in the simultaneous solution of Eqs. (18) – (20) together with the following

$$\begin{array}{lllllllll} y_1 & = & h & \quad y_4 & = & \alpha & \quad y_7 & = & \widehat{E} \\ y_2 & = & \dot{y}_1 & \quad y_5 & = & \dot{y}_4 & & & \\ y_3 & = & \dot{y}_2 & \quad y_6 & = & \dot{y}_5 & & & \end{array} \tag{22}$$

Alternately, in order to avoid integral-differential equations, one can represent the anisotropic integral constitutive relations (8) and (9) as differential equations in the form

$$P_\tau\{\tau_j^i\} \;=\; \widetilde{Q}_{jl}^{ik}\{\gamma_k^l\} \;-\; Q_{jl}^{i\mathbf{E}}\{\widehat{E}^l\} \tag{23}$$

$$P_D\{D^i\} \;=\; Q_l^{ik\mathbf{E}}\{\gamma_k^l\} \;+\; Q_l^{i\mathbf{E}}\{\widehat{E}^l\} \tag{24}$$

where

$$P_\tau \;=\; \sum_{\mathbf{n}=0}^{N^\tau} a_{\mathbf{n}}^\tau\,\frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \qquad Q_{jl}^{ik} \;=\; \sum_{\mathbf{n}=0}^{N_{\underline{jl}}^{\underline{ik}}} b_{\underline{jl}\mathbf{n}}^{\underline{ik}}\,\frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \tag{25}$$

with similar definitions for P_D and the other Q operators. Note that the leading term of any $\widetilde{Q}_{jl}^{ik}$ contains the structural damping contributions of Eqs. (12), if any, resulting in $b_{ilN}^{ik} = b_{ilNR}^{ik} + \imath\, b_{ilNI}^{ik}$. This approach results in a large number of first order simultaneous differential equations, since in order to accurately represent real viscoelastic materials over their entire time range, the Ns need to take on values from 25 to 30. Shorter time ranges, or course, require considerably smaller summation limits and hence much lower order DEs with $1 \le N \le 4$ or 5 for one to three time decades, but the N values increase nonlinearly for longer times. For materials exhibiting

instantaneous responses to sudden loads, the N values in any one of the Eqs. (23) – (25) are equal to each other, but not necessarily equal in value from one of these relations to another [39], [40].

For instance, the 1-D equivalent DE representation of the integral relations (18) – (20) then becomes

$$\sum_{\mathbf{n}=0}^{N^\tau} a_{\mathbf{n}}^{\tau} \frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \left\{ I_h \, \ddot{h} \, + \, \mathcal{L}[F_3, \alpha, \dot{h}] \right\} \, =$$

$$K_h \sum_{\mathbf{n}=0}^{N^\tau} b_{\mathbf{n}}^{\tau} \frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \left\{ \pm\alpha \, + \, \frac{h}{L_2} \right\} \, - \, K_{h1} \sum_{\mathbf{n}=0}^{N^\tau} b_{\mathbf{n}}^{\tau\mathbf{E}} \frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \left\{ \widehat{E} \right\} \tag{26}$$

$$\sum_{\mathbf{n}=0}^{N^\sigma} a_{\mathbf{n}}^{\sigma} \frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \left\{ I_\alpha \, \ddot{\alpha} \, + \, \mathcal{M}[F_3, \alpha, \dot{h}] \right\} \, =$$

$$K_\alpha \sum_{\mathbf{n}=0}^{N^\sigma} b_{\mathbf{n}}^{\sigma} \frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \left\{ \pm\alpha \, + \, \frac{h}{L_2} \right\} \, - \, K_{\alpha 2} \sum_{\mathbf{n}=0}^{N^\sigma} b_{\mathbf{n}}^{\sigma\mathbf{E}} \frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \left\{ \widehat{E} \right\} \tag{27}$$

$$\sum_{\mathbf{n}=0}^{N^D} a_{\mathbf{n}}^{D} \frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \left\{ D \right\} \, = \, K_D \sum_{\mathbf{n}=0}^{N^\sigma} b_{\mathbf{n}}^{D} \frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \left\{ \pm\alpha \, + \, \frac{h}{L_2} \right\} \, + \, K_{D4} \sum_{\mathbf{n}=0}^{N^D} b_{\mathbf{n}}^{D\mathbf{E}} \frac{d^{\mathbf{n}}}{dt^{\mathbf{n}}} \left\{ \widehat{E} \right\} \tag{28}$$

This now results in a set of three simultaneous DEs of order $N^\tau + 2$, $N^\sigma + 2$ or $N^D + 1$, which ever is the largest. The equivalent set of Eqs. (22) correspondingly changes to

$$\begin{array}{ccccccccc} y_1 & = & h & Y_1 & = & \alpha & X_1 & = & \widehat{E} \\ y_2 & = & \dot{y}_1 & Y_2 & = & \dot{Y}_1 & X_2 & = & \dot{X}_1 \\ & \vdots & & & \vdots & & & \vdots & \\ y_{N^\tau} & = & \dot{y}_{N^\tau - 1} & Y_{N^\sigma} & = & \dot{Y}_{N^\sigma - 1} & X_{N^D} & = & \dot{X}_{N^D - 1} \end{array} \tag{29}$$

Thus the original three high order DEs (26) to (28) have been reduced to a set of $N^\tau + N^\sigma + N^D + 3$ simultaneous first order DEs.

As an alternate approach in the real time domain, it is possible to retain the integrals in Eqs. (18) – (20) and evaluate the system in terms of finite differences in time using the analysis developed in [41].

The second solution protocol involving integral Fourier transforms (FT), provided U_r and e are

time independent, leads to the following for the rigid lifting surface

$$\begin{bmatrix} \overbrace{-\omega^2 I_h + \imath\,\omega\left(\frac{-K_h\,\overline{\widetilde{\phi}}}{L_2}+\frac{C_0}{U_r}\right)}^{\overline{\overline{a}}_{11}} & \underbrace{C_0 \mp \imath\,\omega\,K_h\,\overline{\widetilde{\phi}}}_{\overline{\overline{a}}_{12}} & \underbrace{\imath\,\omega\,K_{h1}\,\overline{\overline{\phi}}_1^{\mathbf{E}}}_{\overline{\overline{a}}_{13}} \\ \underbrace{\imath\,\omega\left(\frac{-K_\alpha\,\overline{\widetilde{\phi}}}{L_2}+\frac{C_0\,e}{U_r}\right)}_{\overline{\overline{a}}_{21}} & \underbrace{-\omega^2 I_\alpha \mp \imath\,\omega\,K_\alpha\,\overline{\widetilde{\phi}} + C_0\,e}_{\overline{\overline{a}}_{22}} & \underbrace{\imath\,\omega\,K_{\alpha 2}\,\overline{\overline{\phi}}_2^{\mathbf{E}}}_{\overline{\overline{a}}_{23}} \\ \underbrace{\frac{\imath\,\omega\,K_D\,\overline{\overline{\phi}}_3^{\mathbf{E}}}{L_2}}_{\overline{\overline{a}}_{31}} & \underbrace{\pm\imath\,\omega\,K_D\,\overline{\overline{\phi}}_3^{\mathbf{E}}}_{\overline{\overline{a}}_{32}} & \underbrace{\imath\,\omega\,K_{D4}\,\overline{\overline{\phi}}_4^{\mathbf{E}}}_{\overline{\overline{a}}_{33}} \end{bmatrix} \begin{Bmatrix} \overline{\overline{h}} \\ \overline{\overline{\alpha}} \\ \overline{\overline{\widehat{E}}} \end{Bmatrix} =$$

$$\begin{Bmatrix} \underbrace{-b\,\overline{\overline{F}}_3 - C_0\,\overline{\overline{\alpha_r}}}_{\overline{\overline{c}}_1} \\ \underbrace{-e\left(b\,\overline{\overline{F}}_3 + C_0\,\overline{\overline{\alpha_r}}\right)}_{\overline{\overline{c}}_2} \\ \underbrace{\overline{\overline{D}}}_{\overline{\overline{c}}_3} \end{Bmatrix} \tag{30}$$

with the caveat of Eqs. (12) that the leading term of the Prony series defining $\widetilde{\phi}$ may contain frequency independent imaginary structural damping contributions. Eqs. (30) can be reduced to 1-DOF ones by setting either $h(t)$ or $\alpha(t)$ equal to zero and removing the corresponding relation containing the $\ddot{h}$ or $\ddot{\alpha}$ term. For instance, if one considers only the vertical motion $h(t)$, then the equations reduce to

$$\overline{\overline{a}}_{11}\,\overline{\overline{h}} + \overline{\overline{a}}_{13}\,\overline{\overline{\widehat{E}}} = \overline{\overline{c}}_1 \tag{31}$$

and

$$\overline{\overline{a}}_{31}\,\overline{\overline{h}} + \overline{\overline{a}}_{33}\,\overline{\overline{\widehat{E}}} = \overline{\overline{D}} \tag{32}$$

which gives the solutions for prescribed $D(t)$ as

$$\overline{\overline{h}} = \left(\frac{\overline{\overline{c}}_1}{\overline{\overline{a}}_{13}} - \frac{\overline{\overline{D}}}{\overline{\overline{a}}_{33}}\right) \div \left(\frac{\overline{\overline{a}}_{11}}{\overline{\overline{a}}_{13}} - \frac{\overline{\overline{a}}_{31}}{\overline{\overline{a}}_{33}}\right) \tag{33}$$

$$\overline{\overline{\widehat{E}}} = \frac{\overline{\overline{c}}_1}{\overline{\overline{a}}_{13}} - \frac{\overline{\overline{a}}_{11}}{\overline{\overline{a}}_{13}}\,\overline{\overline{h}} \tag{34}$$

or for predetermined $\widehat{E}(t)$, the "simpler" set becomes

$$\overline{\overline{h}} = \frac{\overline{\overline{c}}_1 - \overline{\overline{a}}_{13}\,\overline{\overline{\widehat{E}}}}{\overline{\overline{a}}_{11}} \tag{35}$$

where $\overline{\overline{D}}$ is defined by Eq. (32) and with similar expressions for the other possible imposed control conditions. Fourier transform inversions of Eqs. (30) or the reduced sets of Eqs. (33) and (34) or (35) can, unfortunately, only be achieved numerically with the use of fast Fourier transforms (FFT). For long time scales and a large number of frequency decades such computations can be extremely computationally time consuming. Another approach is to use the inversion scheme based on the approximation $F(t) \approx \left[\imath\,\omega\,\overline{\overline{F}}(\omega)\right]_{t=.5/\imath\,\omega}$ developed in [42]. However, while highly simple and convenient, the method's accuracy needs to be tested on a case by case basis in dynamic problems.

While the above relations only show electrical field intensity vectors, strains are directly translatable into voltages $\mathbf{V}$, or vice versa, through the voltage-deformation (strain) constitutive relations

$$\mathbf{V}(t) \;=\; \frac{1}{A}\left(\int_0^t \psi_P^{\mathbf{E}}(t-t') \times \frac{\partial}{\partial t'}\left\{\int_0^{t'} \left[A_{L_2}\,\phi(t-s) + A_P\,\phi_P^{\mathbf{E}}(t-s)\right] \frac{\partial \epsilon_{33}(s)}{\partial s}\,ds\right\}\,dt'\right) \tag{36}$$

where $\psi_P^{\mathbf{E}}$ is the piezoelectric creep function. The isotropic constitutive relations, Eqs. (13), (14) and (36), show that piezo-electro-viscoelasticity requires four distinct relaxation functions for material characterization (structure, piezo device, voltage-deformation).

2. DISCUSSION OF RESULTS

For a simple harmonic input (SHI), $F(t) = \Re\{F_A(\imath\,\Omega)\,\exp(\imath\,2\,\pi\,\Omega\,t)\}$ where F_A is the complex amplitude, all responses will be simple harmonic in these linear systems, with the forcing frequencies Ω (Hz). The FT of any variable then can be used to evaluate complex amplitudes from appropriate relationships, since the FT of the exponential functions factor out. For instance, from Eq. (36) one obtains for SHI

$$V_A(\imath\,\omega) \;=\; -\frac{\omega^2}{A}\,\overline{\overline{\psi}}_P\,\left[A_{L_2}\,\overline{\overline{\phi}} + A_P\,\overline{\overline{\phi}}_P^{\mathbf{E}}\right]\,\epsilon_{A33}(\imath\,2\,\pi\,\Omega) \tag{37}$$

where $V(t) = \Re\{V_A(\imath\,\omega)\exp(\imath\,2\,\pi\,\Omega\,t)\}$ and with similar expressions for all other variables.

As an illustrative example consider a simulation study rigid model on 2–D flexible supports (Fig. 1) with vertical motion $h(t)$ and angle of attack (rotation) $\alpha(t)$ in order to minimize the number of contributing parameters. Fig. 2 depicts the generic viscoelastic relaxation functions of Eqs. (11) and for sensitivity studies of Eqs. (34), (35) and (36), the following parametric values were selected:

$c = 10^3 \quad c_2^E = c_3^E = c_4^E = 2\times 10^3 \quad \rho \;=\; 1.3\,\mathrm{kg/m^3}$

$E_\infty = 0.1 \quad E_0 = 1 \quad (E_\infty^E)_2 = (E_\infty^E)_3 = (E_\infty^E)_4 = 0.5$

$(E_1^E)_2 = (E_1^E)_3 = (E_1^E)_4 = 0.9 \quad \tau = 100\,\mathrm{sec}$

$\tau_2^E = \tau_3^E = \tau_4^E = 50\,\mathrm{sec} \quad D_A \;=\; 0.01\,\mathrm{m} \quad b \;=\; 0.3\,\mathrm{m}$

Some significant parametric variation results for an SHI $D(t)$ are displayed in Figs. 3 – 7, 12 and 13. The parameters, such as the disturbance amplitude a_3 of Eq. (1) & Fig. 3, stiffness K_h of Eq. (30) & Figs. 4 and 12, the free stream velocity U_r of Eq. (7) & Figs. 5 and 13, and the

piezo-electric stiffness K_D of Eq. (30) & Fig. 6, all contribute to the piezo-electric voltage in the low ranges of the SHI frequecies Ω and cease to be of any influence for $\Omega > 30$ Hz. A similar statement can be made about the vertical displacements h of Eq. (30) & Figs. 12. Fig. 7 is a composite plot of $\widehat{E}$ vs. time for SHM, Eq. (37) and $D_A = 1$, and it is to be noted that the response voltage amplitudes and periods are very sensitive to SHI frequencies Ω.

Figs. 8 – 11 and 14 show FT voltage and displacement sensitivities for the same parameters, but for a Heaviside step function input for $D(t) = 0$ for $t \leq 0$ and $D(t) = D_A$ for $t \geq 0$. The same pattern of decreasing parametric influence for FT frequencies above 30 Hz is observed.

It must be remembered that in self-excited problems of this type more damping or stiffness or inertia or aerodynamic noise can lead to either more or less stable configurations as the responses are predominately influenced by the phase relations among these forces.

The pilot results and protocols developed here have applicability to naval (submarines and surface ships), air and land vehicles structures and their components, as well as to space antennas and solar sails, regarding their survivability, failure probabilities and structural control. The results presented here are specifically for 2–D viscoelastic flutter. However, by proper definition of the aerodynamic lift and moment functions in Eqs. (18) and (19) the analyses can be extended to acoustic noise inputs. Furthermore, the analyses are also applicable to fluid and solar wind fluctuations or meteorite impacts on large very flexible space structures or simply to the use of piezo-electric devices to maintain shapes of space antenna dishes.

No experimental results are available for comparison purposes of these theoretical analyses and simulations.

CONCLUSIONS

Light weight piezo-electric viscoelastic devices can be effectively used to actively or passively control flight vehicle structural response to aerodynamic noise. This can be accomplished by providing suitable piezo-electric input voltages, by prescribed electric displacements or by controlling the output emf through appropriate resistors. Extensions of these pilot simulations to more complex structural components will be the subject of future research.

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APPENDIX A – GENRALIZED NONLINEAR KELVIN MODELS

Relaxation functions can also be derived from generalized nonlinear Kelvin models (GKM) as

$$\frac{\partial \phi_{jl}^{ik}(\theta,t,t')}{\partial t'} = {}^{0}C_{jl}^{ik}(\theta,t') + \sum_{n=1}^{N_{\underline{jl}}^{\underline{ik}}} \left\{ \frac{1}{{}^{n}\eta_{\underline{jl}}^{\underline{ik}}(\theta,t')} \exp\left[-[{}^{n}F_{\underline{jl}}^{\underline{ik}}(\theta,t) - {}^{n}F_{\underline{jl}}^{\underline{ik}}(\theta,t')]\right] \right\} + \frac{1}{{}^{N_{\underline{jk}}^{\underline{il}}+1}\eta_{\underline{jk}}^{\underline{il}}(\theta,t')} \tag{38}$$

where

$$ {}^{n}F_{jk}^{il}(\theta,t) = \int^{t} \frac{ds}{{}^{n}\lambda_{jk}^{il}(\theta,s)} \qquad \text{and} \qquad {}^{n}\lambda(\theta,t') = \frac{{}^{n}\eta(\theta,t,t',M,T,\dot{I},\dot{I}^{\mathbf{E}})}{{}^{n}E(\theta,t,t',M,T,\dot{I},\dot{I}^{\mathbf{E}})} \tag{39}$$

For isothermal conditions and linear nonhomogeneous materials, each summation term in Eqs. (38) reduces to

$$ {}^{n}\phi_{jl}^{ik}(\theta,t,t') = \frac{1}{{}^{n}\eta_{\underline{jl}}^{\underline{ik}}(\theta)} \exp\left[-\frac{(t-t')}{{}^{n}\lambda_{\underline{jl}}^{\underline{ik}}(\theta)}\right] \tag{40}$$

and results in convolution integral constitutive relations.

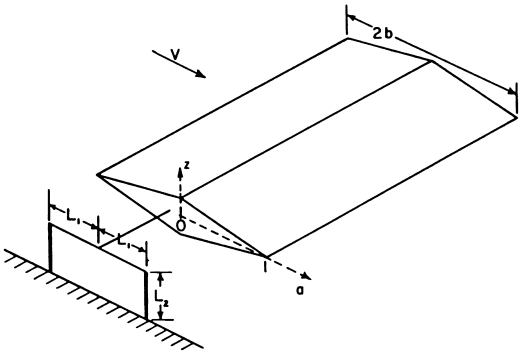
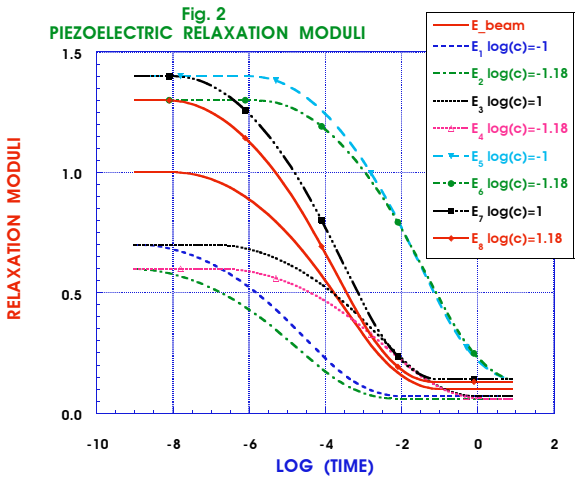
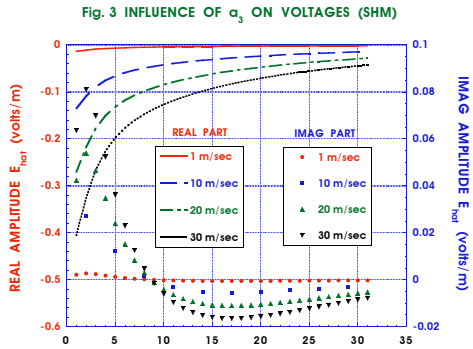
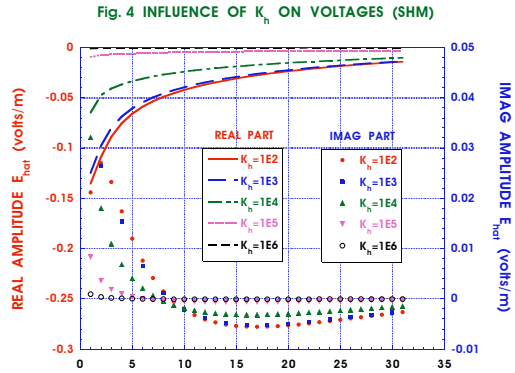
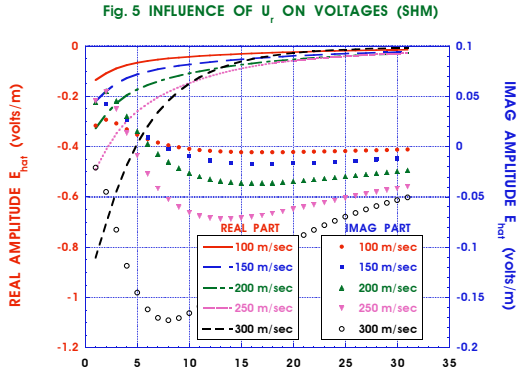


Fig. 1 Rigid Airfoil on Viscoelastic Supports









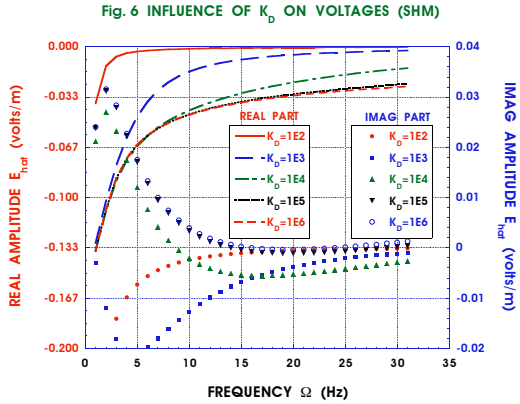


Fig. 7 HARMONIC INPUT / OUTPUT VOLTAGES

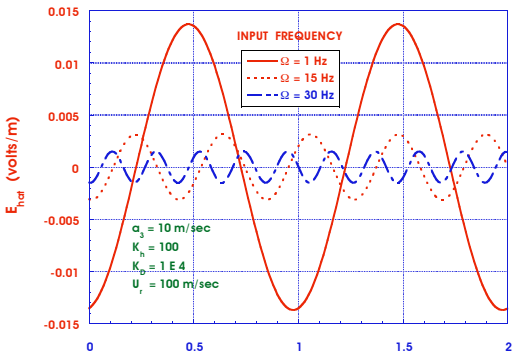


Fig. 8 INFLUENCE OF a_3 ON FT VOLTAGES (STEP)

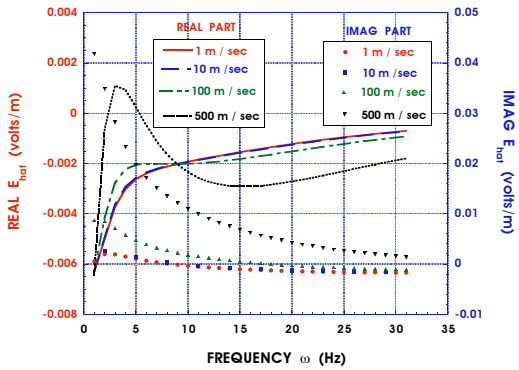


Fig. 9 INFLUENCE OF $k_1 = \omega / U_r$ ON FT VOLTAGES (STEP)

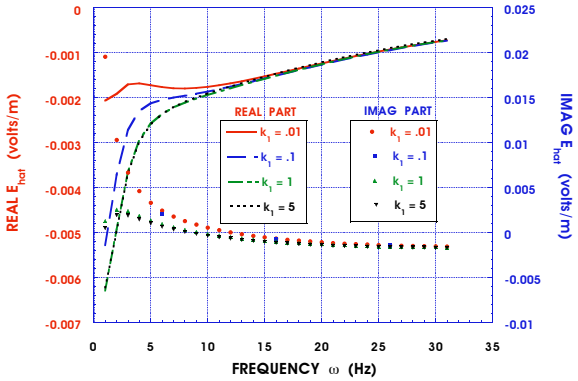


Fig. 10 INFLUENCE OF K_h ON FT VOLTAGES (STEP)

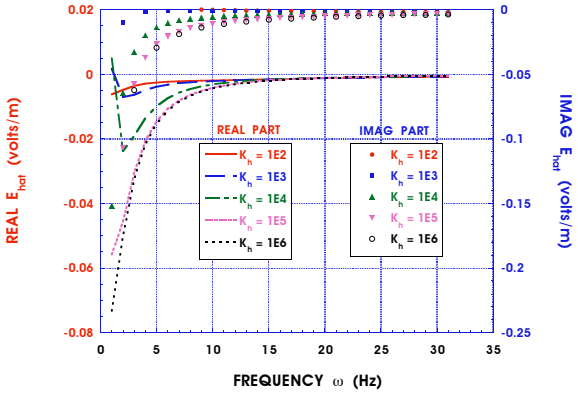


Fig. 11 INFLUENCE OF U_r ON FT VOLTAGES (STEP)

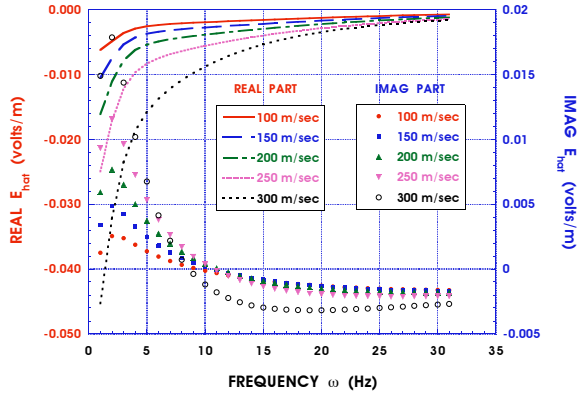


Fig. 12 INFLUENCE OF K_h ON DISPLACEMENTS (SHM)

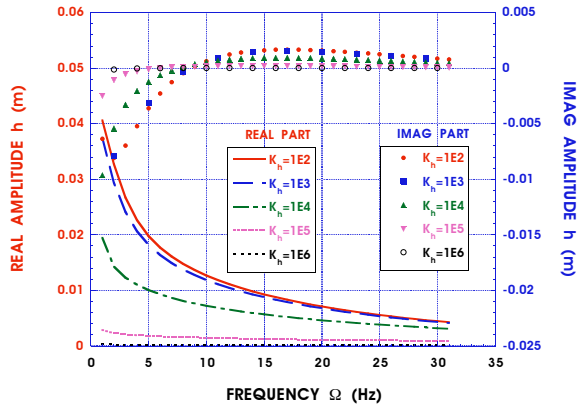


Fig. 13 INFLUENCE OF U_r ON DISPLACEMENTS (SHM)

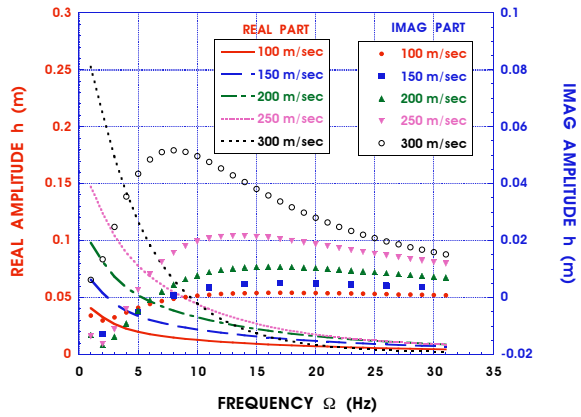
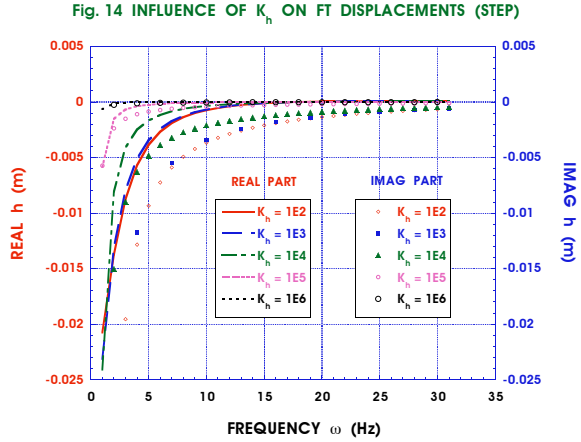



Fig. 14 INFLUENCE OF K_h ON FT DISPLACEMENTS (STEP)