Electronic Journal «Technical Acoustics»
http://www.ejta.org
2008, 6

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Thermoacoustic effects of transverse non-uniformity of mean temperature in channels without mass streaming

Received 16.03.2008, published 16.04.2008

Small thermoacoustic devices can fulfill a need for high-performance compact energy conversion systems. One of the miniaturization consequences is the appearance of strong transverse temperature gradients in the stacks of thermoacoustic engines. The thermoacoustic theory is extended in this paper to include a transverse gradient of the mean temperature field. The influence of this gradient on the acoustic velocity and temperature fluctuation amplitudes and on the flow of thermoacoustic enthalpy and acoustic energy is demonstrated under the standing-wave conditions and in the absence of mass streaming.

Key words: thermoacoustics, small-scale heat engine, acoustics of non-uniform media.

INTRODUCTION

Thermoacoustic devices represent a new class of energy conversion systems, where thermal energy is transformed into acoustic energy, or where sound is used to pump heat against temperature gradient. A typical arrangement of a simple standing-wave thermoacoustic device is illustrated in Fig. 1a. In the presence of a strong temperature gradient along the porous material (stack), high-amplitude sound in the gas inside the resonator can be generated. The history, physics, and theories of thermoacoustics, as well as descriptions of actual thermoacoustic systems, are presented in great details by Swift [1, 2].

Cooling devices based on thermoacoustic principles have found practical applications, and a number of medium- and large-scale thermoacoustic prime mover prototypes have been built. The demand for small-scale energy conversion systems with high energy density have been growing in last years [3]. Miniature thermoacoustic devices are promising candidates for this application. However, studies on small-scale thermoacoustic systems have started only recently, e.g., [4–6]. Scaling down thermoacoustic devices is challenging due to higher viscous and thermal losses and an increased role of previously unimportant effects.

One of such phenomena is the appearance of the strong mean temperature gradients in the transverse direction inside longitudinal channels. The currently used calculation methods in thermoacoustics involve an assumption that the mean temperature is independent on the transverse coordinate [7]. This assumption is well justified in properly designed macro-scale systems with large aspect ratios (long and narrow resonators) and efficient convective-type heat exchangers applied at the stack ends. However, in future miniature thermoacoustic devices, aspect ratios of resonators will have to shrink to compensate for a significant increase of viscous losses at small scales. Besides, the heat exchangers will have to be made of the conduction type due to limited space. These two factors will result in the appearance of strong transverse temperature gradients in the engine stacks (or regenerators), where acoustic power is produced in prime movers (or heat is pumped in cooling devices).

This study addresses some effects of the transverse mean temperature gradient on the acoustic velocity, temperature fluctuations, and acoustic power and enthalpy flow under the standing-wave conditions in a two-dimensional channel.

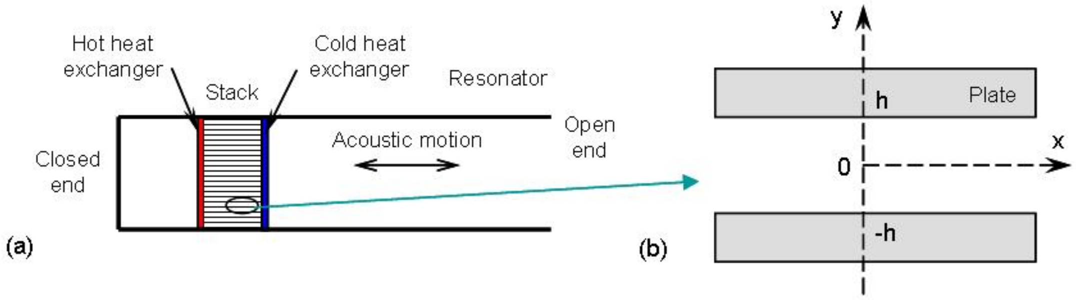

Figure 1. (a) Standing-wave engine arrangement, (b) coordinate system for analysis

1. THEORY

The geometry of the problem under analysis is shown in Fig. 1b. The primary direction for the first-order acoustic velocity oscillations is aligned with the channel parallel to the x-axis. The standing-wave phasing is assumed, i.e., there is 90° phase shift between the acoustic pressure and velocity fluctuations far from the walls in sufficiently wide channels. The velocity is zero at the walls due to no-slip conditions, and the gas temperature at the wall is anchored to the wall temperature. The pressure fluctuation p_1 depends on the horizontal coordinate only. Different from previously used assumptions in thermoacoustic theories, the mean temperature in this analysis depends not only on the longitudinal but also on the vertical coordinates. For simplicity, we assume linear variation of the mean temperature in the vertical direction:

$$T_m(x,y) = T_0\left(1+\frac{\partial T_m}{\partial x}\frac{x}{T_0}+\frac{\partial T_m}{\partial y}\frac{y}{T_0}\right), \qquad (1)$$

where the transverse gradient $\partial T_m / \partial y$ is constant.

The appropriate x-momentum and energy equations with dominant terms for the first-order complex amplitudes of the horizontal velocity u_1 and temperature T_1 fluctuations remain the same as in the classical thermoacoustic theory [2], with the exception that the mean density ρ_m and temperature T_m are now y-dependent,

$$i\omega\rho_m u_1 = -\frac{dp_1}{dx} + \mu\frac{\partial^2 u_1}{\partial y^2}, \qquad (2)$$

$$i\omega\rho_m\, c_p T_1 + \rho_m\, c_p \frac{\partial T_m}{\partial x} u_1 = i\omega\, p_1 + k\frac{\partial^2 T_1}{\partial y^2}, \tag{3}$$

where ω is the angular frequency of acoustic oscillations, i is the imaginary unity, and μ, k, c_p are the viscosity, heat conductivity, and specific heat capacity of the working fluid (ideal gas), respectively. The boundary conditions in the channel cross-section at $x = 0$ are the following:

$$u_1(-h) = u_1(h) = 0, \tag{4}$$

$$T_1(-h) = T_1(h) = 0, \tag{5}$$

where h is the channel half-width.

Equations (2) and (3) can be written in the dimensionless form, accounting for the transverse mean temperature gradient (Eq. 1):

$$(1 + ay')\frac{\partial^2 u'_1}{\partial^2 y'^2} - 2i\,u'_1 = -2i(1 + ay'), \tag{6}$$

$$(1 + ay')\frac{\partial^2 T'_1}{\partial^2 y'^2} - 2i\,\sigma T'_1 = -2i\,\sigma(1 + ay') + 2\sigma\frac{G}{Z'}u'_1, \tag{7}$$

where $a = \frac{\partial T_m}{\partial y}\frac{\delta_\upsilon}{T_0}$, $y' = \frac{y}{\delta_\upsilon}$, $u'_1 = \frac{u_1}{\hat{u}_1}$ $T'_1 = T_1\frac{\rho_0 c_p}{p_1(0)}$, $G = \frac{\partial T_m}{\partial x}\frac{c_0}{(\gamma - 1)T_0\omega}$, $\sigma = \left(\frac{\delta_\upsilon}{\delta_k}\right)^2$ is the Prandtl number, $\delta_\upsilon = \sqrt{\frac{2\mu}{\omega\rho_0}}$ is the viscous penetration depth, $\delta_k = \sqrt{\frac{2k}{\omega\rho_0\, c_p}}$ is the thermal penetration depth, $Z' = \frac{1}{\rho_0 c_0}\frac{p_1(0)}{\hat{u}_1(0)}$ is the dimensionless acoustic impedance, $\hat{u}_1 = \frac{i}{\rho_0\omega}\frac{dp_1}{dx}$ is the horizontal acoustic velocity amplitude in unrestricted geometry, γ is the ratio of specific heats, and c_0 is the speed of sound at T_0.

The analytical solution for the dimensionless acoustic velocity is obtained from Eq. (6), assuming constant fluid properties:

$$u' = i(1 + ay') + \sqrt{1 + ay'}\left\{A_1 J_1\left(d\sqrt{1 + ay'}\right) + A_2 Y_1\left(d\sqrt{1 + ay'}\right)\right\}, \tag{8}$$

where $J_1(z)$ and $Y_1(z)$ are the Bessel functions of the first and second kind and $d = \frac{2\sqrt{2}}{a}\exp\left(-i\frac{\pi}{4}\right)$. Constants A_1 and A_2 are determined from the boundary conditions $u'_1(-h') = u'_1(h') = 0$, where $h' = h/\delta_\upsilon$. In a particular case of the zero transverse mean temperature gradient, the solution for u' coincides with that derived previously [2]:

$$u'_1\big|_{a=0} = 1 - \frac{\cosh\{(1 + i)y'\}}{\cosh\{(1 + i)h'\}}. \tag{9}$$

The solution for the temperature fluctuation (Eq. 7) in case of non-zero a can be found in quadratures. Equations (6) and (7) can also be solved numerically.

The effect of the transverse mean temperature gradient is evaluated for the second-order thermoacoustic enthalpy flow $\dot{H}_2$ and acoustic energy flow $\dot{E}_2$ along the channel. Neglecting by the acoustic mass streaming, the expressions defining these energy flows can be written as follows [1]:

$$\dot{H}_2 = \frac{1}{2} c_p \int_S \rho_m \operatorname{Re}[T_1 \tilde{u}_1]\, dS \,, \tag{10}$$

$$\dot{E}_2 = \frac{1}{2} \int_S \operatorname{Re}[p_1 \tilde{u}_1]\, dS \,, \tag{11}$$

where the symbol tilde indicates the complex conjugate and the integration is carried out over the channel cross-sectional area S. These expressions can be presented in a dimensionless form dividing them by $\frac{1}{2}|p_1||\hat{u}_1|S$. Selecting $p_1(0)$ as a real number and assuming purely imaginary impedance Z' (relevant to standing waves), the dimensionless energy flows expressed via earlier defined u' and T' can be written as follows:

$$\dot{H}'_2 = \frac{1}{2h'} \int_{-h'}^{h'} h_2 dy' = \frac{1}{2h'} \int_{-h'}^{h'} \frac{-\operatorname{Im}[T'_1 \tilde{u}'_1]}{1+ay'} dy' \,, \tag{12}$$

$$\dot{E}'_2 = \frac{1}{2h'} \int_{-h'}^{h'} e_2\, dy' = \frac{1}{2h'} \int_{-h'}^{h'} \operatorname{Im}[u'_1] dy' \,, \tag{13}$$

where h_2 and e_2 are fluxes of thermoacoustic enthalpy and acoustic energy, respectively.

2. RESULTS

A two-dimensional channel (Fig. 1b) in a standing-wave engine with typical characteristics is considered. The Prandtl number and dimensionless impedance are taken as fixed parameters: $\sigma = 0.67$ (as for helium) and $Z' = 2i$ (standing wave). The imaginary impedance Z' does not imply absolute zero in the acoustic energy flow, since the actual y-dependent impedance has real components close to the walls of the channel.

The calculated dimensionless acoustic velocity and fluctuating temperature amplitudes are illustrated in Fig. 2 for the relative half-width of the channel $h' = 4$. Two values of parameter G are chosen, corresponding to the refrigerator ($G = -1$) and prime mover ($G = -5$) regimes. The dimensionless transverse mean temperature gradient is selected to be zero ($a = 0$) and non-zero ($a = 0.1$). In the absence of the transverse gradient, the profiles of acoustic velocity and fluctuating temperature amplitudes are symmetric relative to the channel centerline. At non-zero a, these profiles are distorted. The acoustic velocity fluctuations are not sensitive to the longitudinal temperature gradient.

The corresponding enthalpy and acoustic energy fluxes (h_2 and e_2 in Eqs. 12–13) are shown in Fig. 3. They have significant magnitudes at some distance from the walls. In wide channels and the selected standing-wave phasing, these fluxes will be close to zero in the most part of the channel, except for the near-wall regions.

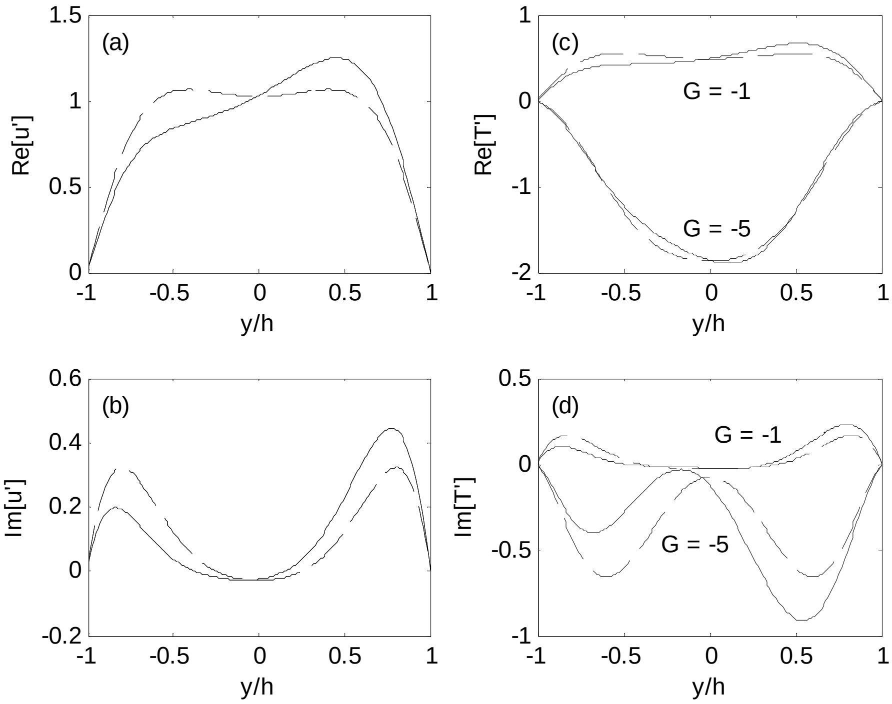

Figure 2. Dimensionless amplitudes of (a, b) acoustic velocity and (c, d) temperature fluctuations in the channel with $h' = 4$. Dashed curves – $a = 0$, solid curves – $a = 0.1$

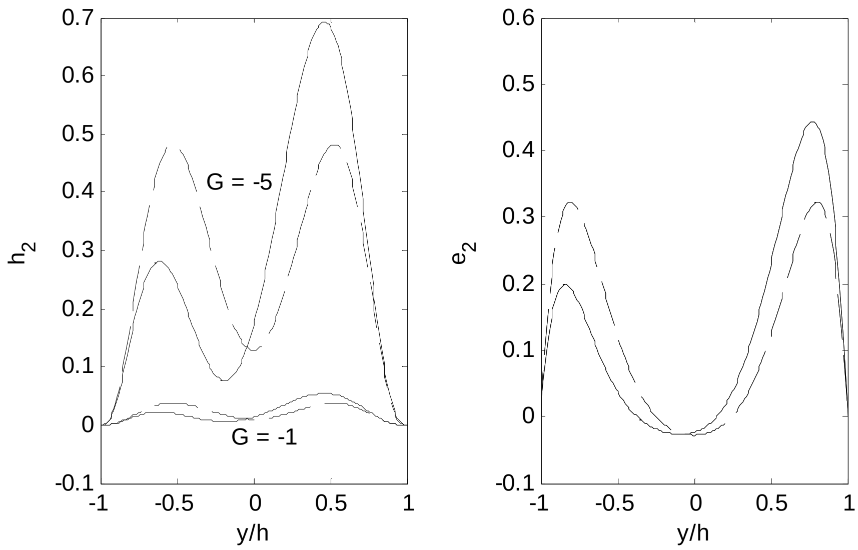

Figure 3. Fluxes of (a) thermoacoustic enthalpy and (b) acoustic energy in the channel with $h' = 4$. Dashed curves – $a = 0$, solid curves – $a = 0.1$

The difference between the enthalpy and acoustic energy flows at zero and non-zero transverse mean temperature gradients can be quantified using the following metrics:

$$x_H(a) = \frac{\dot{H}_2(a) - \dot{H}_2(0)}{\dot{H}_2(0)}, \tag{14}$$

$$x_E(a) = \frac{\dot{E}_2(a) - \dot{E}_2(0)}{\dot{E}_2(0)}. \tag{15}$$

These quantities are plotted in Fig. 4 for the analyzed above system parameters and variable h' and a. The effect of the mean temperature non-uniformity on the enthalpy and acoustic energy flows is considerable only at high values of the transverse temperature gradients (high a) and in sufficiently wide channels (high h'). However, if the reference temperature in a channel with transversely uniform temperature is selected as a temperature of one of the channel walls, then the effect considered here will be much more important even at low a and h'.

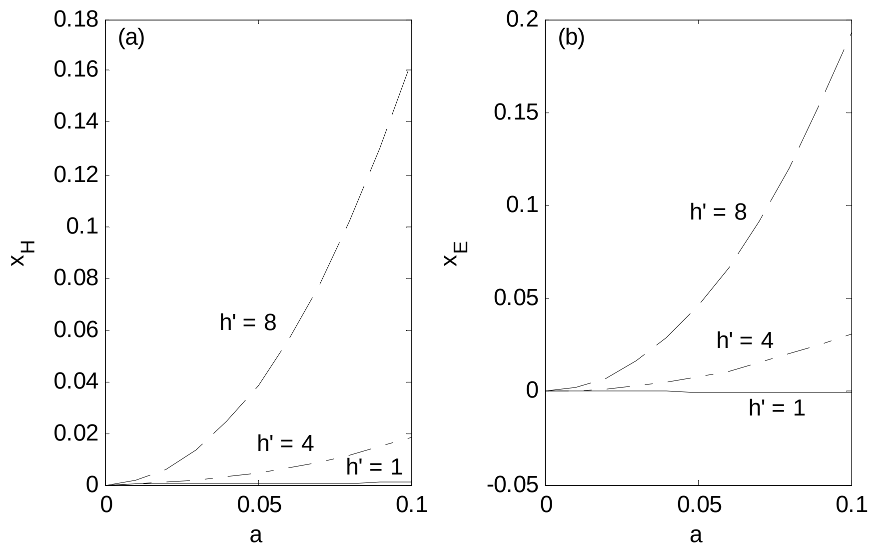

Figure 4. Corrections to (a) enthalpy flow and (b) acoustic energy flow

In this study the influence of the transverse gradient on the acoustic mass streaming in the channel is not accounted for. It can be is significant, since the driving mass fluxes are concentrated in the wall proximity [2]. In its turn, the acoustic streaming will result in the additional enthalpy flow component that will increase the deviation metric x_H. Moreover, a recently identified new type of streaming in regenerators with opposite mean flow streams in different channels of the stack [8] is likely to be coupled with transverse temperature gradients in individual channels.

CONCLUDING REMARKS

The presence of a transverse mean temperature gradient results in the distortion of amplitude profiles of the complex acoustic velocity and temperature fluctuations in a stack channel. The effect of this gradient on thermoacoustic enthalpy and acoustic energy flow is found to be modest for typical standing-wave engine parameters, if the appropriate characteristic temperature is selected in the equivalent channel without transverse temperature variation. However, the acoustic mass streaming, unaccounted in this paper, will be also affected by the mean temperature non-uniformity across the channel. This suggests future studies to include mass streaming in the presence of the transverse mean temperature gradient, which will lead to the additional enthalpy flow components.

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