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Application of the wavelet transform analysis to Lamb modes signals in plates

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The paper proposes a wavelet spectral analysis method of Lamb waves in thin isotropic plates. A numerical code is developed on the Comsol Multiphysics software to solve, by the finite element method, the equation of motion and determine the displacement field. The post processing of the obtained displacement field is done, on the Matlab software, by a method based on the wavelet transform (WT). The considered application aims to determine the dispersion curves of symmetric S0 and antisymmetric A0 Lamb modes of a plane steel plate. These curves are obtained by the WT of displacement field and are compared to the analytic curves. Several mother wavelets are tested showing that the complex mother wavelet Shan 1-1.5 gives the better agreement.

Keywords: lamb wave, ultrasound, finite element method, wavelets.

INTRODUCTION

In industry, ultrasonic waves are commonly used for detecting and characterizing defects in materials. Lamb waves are particularly suitable for large structures like plates and sheets since they can propagate over long distances without significant attenuation. In order to set and optimize nondestructive testing (NDT) techniques, based on the propagation and diffraction of Lamb waves, it is necessary to provide propagation models and analysis methods of these waves. The modelling is generally performed by the finite element method (FEM) leading to the displacement signals in the studied structure. The processing of these predicted Lamb signals was made extensively by the Fourier transform [1, 2] while the wavelet transform (WT) analysis is under development since a few years. It was Daubechies [3] and Newland [4] who firstly introduced WT into the study of vibrational signals in the early 1990s. Then the WT has been introduced to the time-frequency representation of transient waves propagating in a dispersive medium. Hayashi et al. [5] give an estimation of thickness and elastic properties of metallic foils by the WT of laser-generated Lamb waves. Jeong et al. [6] use the Gabor wavelet to draw A0 mode dispersion curve in composite laminates. The effectiveness of WT analysis for studying wave dispersion was evaluated by Y. Y. Kim and E. H. Kim [7]. Many authors use the WT for the detection of a delamination, a crack, a defect or fatigue in plates, in composite structure or in pipes [8–13].

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In this work, we present a finite element (FE) modelling and propose a wavelet transform (WT) analysis of Lamb waves in isotropic plates. The FE modelling is developed on the Comsol Multiphysics software to compute the displacements field in plates. The obtained displacements are then processed by WT in the Matlab software to determine the dispersion curves of the propagating Lamb modes. Several real and complex mother wavelets are tested to determine the most suitable one for the Lamb signal analysis. The proposed FE modelling and WT processing are tested on a steel plate without defects.

1. LAMB WAVES THEORY

1.1. Lamb equation

We consider a Lamb wave propagating in thin isotropic plate of thickness e = 2d along the x direction of a Cartesian coordinate axis, Figure 1.

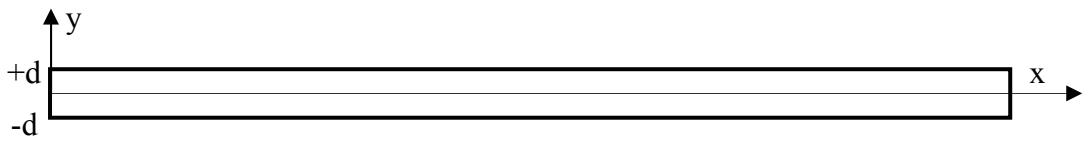

Figure 1. Schematic of the considered isotropic plate

The boundary conditions applied to the stress-free faces of the plate lead to the characteristic equations (Rayleigh-Lamb equations) [14–15]:

$$\begin{aligned} &\left(k^2+s^2\right)^2\cosh(qd)\sinh(sd)+4k^2qs\sinh(qd)\cosh(sd)=0, && (a)\\ &\left(k^2+s^2\right)^2\sinh(qd)\cosh(sd)-4k^2qs\cosh(qd)\sinh(sd)=0, && (b)\\ &s^2=k^2-k_T^2,\ q^2=k^2-k_L^2, \end{aligned} \tag{1}$$

where k is the wave number, k_L (k_T) is the longitudinal (shear) wave number.

The roots (k) of equation (1) can be real, pure imaginary or complex, corresponding respectively to propagating, no propagating and evanescent modes. Only the propagating modes (Lamb modes) are considered here.

1.2. Dispersion curves

The numerical resolution of equation (1) permits to obtain the dispersion curves for symmetric and antisymmetric Lamb modes. In the case of an isotropic steel plate, these curves are presented in figure 2.

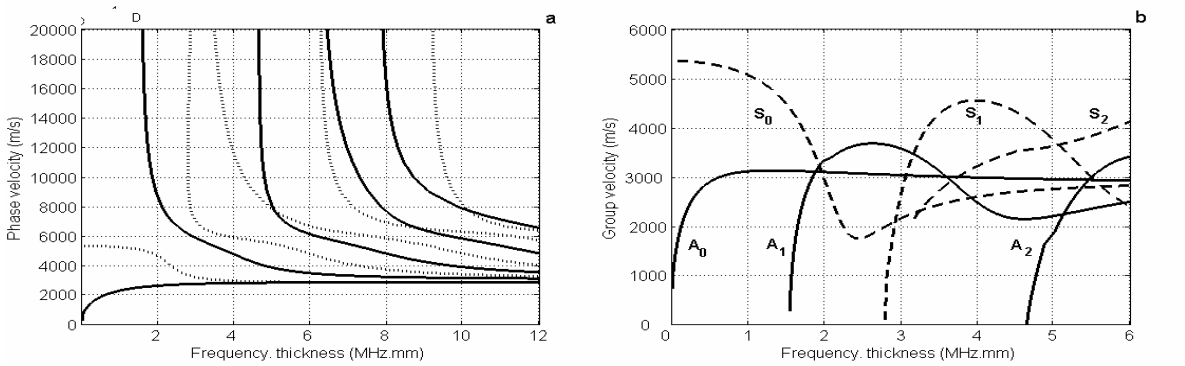

Figure 2. Exact dispersion curves: phase (a) and group (b) velocities versus the product frequency-thickness f.e
for a steel plate (v_L = 6144 m/s, v_T = 3095 m/s, e = 6 mm, ρ = 7850 kg/m^3)

1.3. The displacement field

The expressions of displacements u_{sx} and u_{sy} of symmetric modes are given by:

$$u_{sx} = Ak\left[\frac{\cosh(qy)}{\sinh(qd)} - \frac{2qs}{k^2+s^2}\frac{\cosh(sy)}{\sinh(sd)}\right]ie^{i[kx-\omega t]},$$
$$u_{sy} = Aq\left[\frac{\sinh(qy)}{\sinh(qd)} - \frac{2k^2}{k^2+s^2}\frac{\sinh(sy)}{\sinh(sd)}\right]e^{i[kx-\omega t]}, \quad (2)$$

where ω is the angular frequency

The expressions of displacements u_{ax} and u_{ay} of antisymmetric modes are simply obtained by changing in the expressions (2), the subscripts (*s*) by (*a*) and (sinh) by (cosh) and vice versa.

1.4. Modelling of Lamb waves propagation: the finite element method

The spatial discretization of a plate and the application of the virtual works theorem allow writing the motion equation in the following matrix form (damping is not considered in this study) [16]:

$$[M]\{\ddot{U}\} + [K]\{U\} = \{F\}, \quad (3)$$

where $[M]$ is the global mass matrix, $[K]$ is the global stiffness matrix, $\{U\}$ is the displacement vector, $\{\ddot{U}\}$ is the acceleration vector and $\{F\}$ is the vector of applied forces.

To solve the equation (2) and find the displacement field $\{U\}$, we used the Newmark method. The construction of the solution at time $t+\Delta t$ is done from the vectors at time *t*: $\{U_t\}$, $\{\dot{U}_t\}$ and $\{\ddot{U}_t\}$ according to the following algorithm [17]:

$$\{\dot{U}_{t+\Delta t}\} = \{\dot{U}_t\} + \Delta t\left((1-\alpha)\{\ddot{U}_t\} + \alpha\{\ddot{U}_{t+\Delta t}\}\right),$$
$$\{U_{t+\Delta t}\} = \{U_t\} + \Delta t\{\dot{U}_t\} + \Delta t^2\left(\left(\frac{1}{2}-\beta\right)\{\ddot{U}_t\} + \beta\{\ddot{U}_{t+\Delta t}\}\right), \quad (4)$$

where α and β are Newmark integration coefficients, Δt is the time step.

The coefficients α and β govern the stability, accuracy and numerical dissipation of the integration method. The four well-known variants of Newmark method are: the Newmark explicit method (α=1/2, β=0), the Fox-Goodwin method (α=1/2, β=1/12), the linear acceleration method (α=1/2, β=1/6) and the constant average acceleration method (α=1/2, β=1/4).

2. POST PROCESSING OF DISPLACEMENTS FIELD

2.1. Bi-dimensional Fourier transform: 2DFT

The bi-dimensional Fourier transform of the space-time evolution of displacement $u(x,t)$ is defined by the formula:

$$F_u(\omega,k) = \int_{-\infty}^{+\infty} u(t,x)e^{-(j\omega t - kx)}dtdx\,, \quad (5)$$

where ω is the angular frequency and j is the complex number such as $j^2 = -1$.

2.2. Wavelet transform (WT)

The wavelet transform (WT) was introduced by Morlet in the early 1980 to study seismic signals [18]. Later Grossmann, Meyer, Mallat and Daubechies established a proper mathematical foundation for wavelets [19]. Since then, wavelets have been extensively employed in signal processing applications. In WT, a varying window function is used, which called the mother wavelet. A wavelet is defined using two parameters: a scaling parameter a, which is the inverse of the frequency, corresponds to a dilatation or compression in time of the window function and a translation parameter b, which translates the window function along the time axis.

The continuous WT of a signal $u(t)$ is defined by [20]:

$$Wf\,(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} u(t)\psi^*\left(\frac{t-b}{a}\right)dt\,, \quad (6)$$

where $\psi(t)$ is the wavelet function, $\psi^*(t)$ is the complex conjugate of $\psi(t)$.

In the case of Lamb waves, the localization of the peak on the scalogram indicates the arrival time of the group velocity corresponding to the parameter b at the frequency corresponding to the scale parameter a [6, 21]. In other words, for each frequency f, the localization of the maximum value of the modulus of the wavelet coefficients $Wf\,(a,b)$ gives us the arrival times t_{d1} and t_{d2} corresponding to two points of the upper face of the plate M_1 and M_2 for a specific distances of propagation d_1 and d_2 (Figure 3). The frequency-dependant group velocity $v_g(f)$ is deduced from the following equation:

$$v_g(f) = \frac{d_2 - d_1}{t_{d2} - t_{d1}}, \tag{7}$$

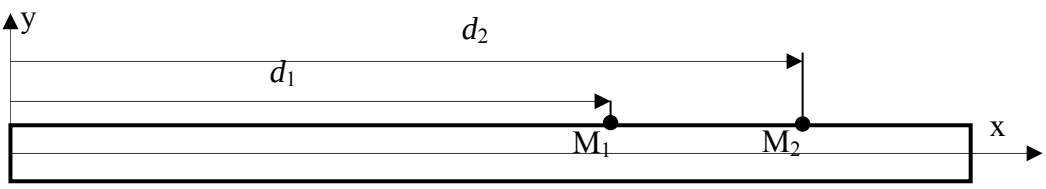

Figure 3. Schematic of the considered isotropic plate

3. NUMERICAL SIMULATION

3.1. The studied application

The studied application aims to determine the dispersion curves of S0 and A0 Lamb modes of a safe plane steel plate (Figure 1) with thickness e = 2d = 6 mm, Young's modulus E = 2e11 Pa, Poisson's ratio ν = 0.33, density ρ = 7850 kg/m^3, longitudinal velocity v_L = 6144 m/s and shear velocity v_T = 3095 m/s.

The simulation uses a finite element model, implanted in the Comsol Muliphysics code. The mesh must be able to represent the physical characteristics of the wave propagation. We choose a quadrilateral mesh and the smallest wavelength λ_{min} must contain at least 10 spatial steps. Then the spatial and time steps $\Delta x, \Delta y$ and Δt must verify the conditions:

$$\max(\Delta x, \Delta y) \prec \frac{\lambda_{min}}{10}, \ \Delta t \prec 0,7 \frac{\min(\Delta x, \Delta y)}{v_L}, \tag{8}$$

where λ_{min} is the smallest wavelength and v_L is the longitudinal wave velocity.

3.2. Generation of Lamb modes

To generate the S0 or A0 Lamb modes, we apply on the left edge of the plate (x=0, y) the analytical displacements (equation 2), normalized by the power flow through the plate thickness (figure 4a, 4b). The spatial distribution of the displacements is applied during 10 cycles tone burst (for t=0 to t=10/f) weighted by a Hanning window centered on the excitation frequency. The product frequency-thickness f.e varies from 0.1 to 2.2 MHzmm by a step of 0.1 MHzmm.

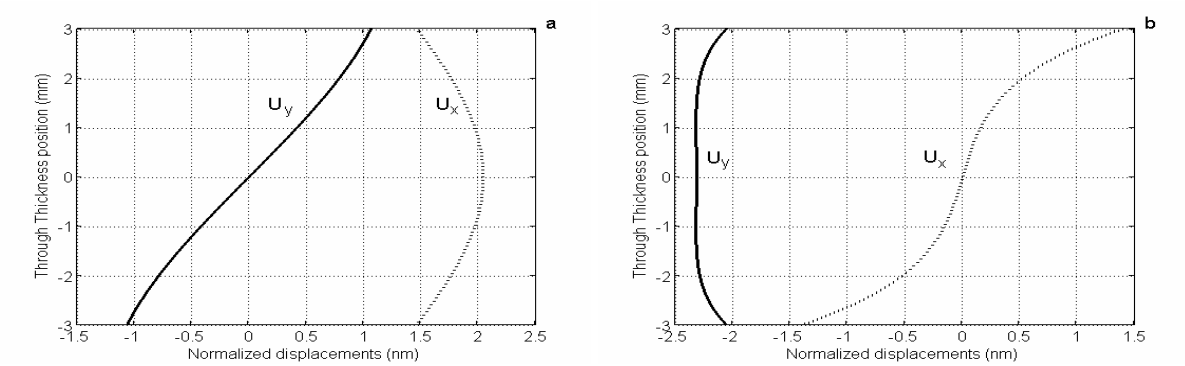

Figure 4. Normalized displacements at 1.5 MHzmm applied to the left edge of the plate to generate S0 mode (a) and A0 mode (b)

3.3. Dispersion curves

For the S0 and A0 Lamb modes, and for the values of the frequency-thickness product varying from 0.1 to 2.2 MHzmm with a sampling step of 0.1 MHzmm, we pick up, on the upper face of the plate, the time evolution of displacements $u_{y1}(t)$ and $u_{y2}(t)$ at two points M_1 and M_2 located respectively at $d_1 = 50$ mm and $d_2 = 100$ mm. The Figure 5 presents these displacements at 1.5 MHzmm collected on the points M_1 and M_2 for the A0 Lamb mode.

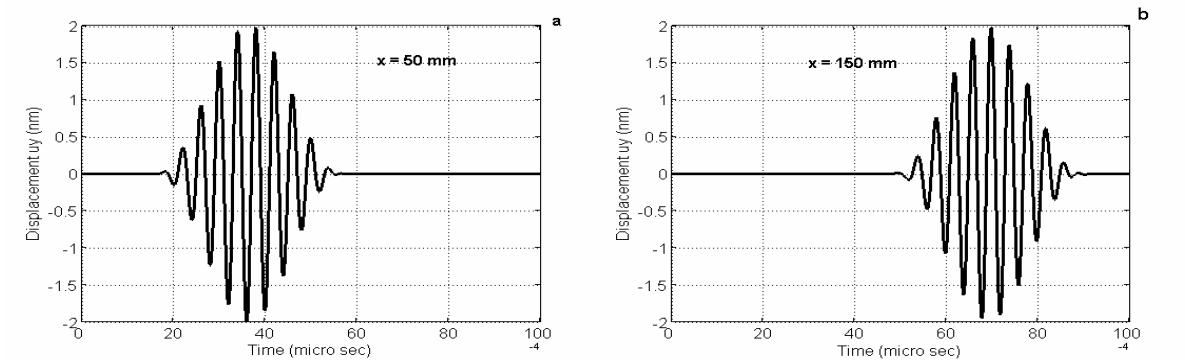

Figure 5. The out of plane displacements u_y pick up on the upper face of the plate at 1.5 MHzmm for the A0 mode at M_1 (x=50 mm) (a) and M_2 (x=150 mm) (b)

To determine the dispersion curves of the excited modes, we apply the WT to these obtained displacements. Several mother wavelets (real and complex) are tested: Morlet, Gaussian, Meyer, DMeyer, Mexican_hat and Shannon.

The figure 6 presents, in the case of the Gauss1 mother wavelet, the plots of wavelet coefficients of displacements at the points M_1 and M_2 at 1.5MHz.mm for the A0 mode.

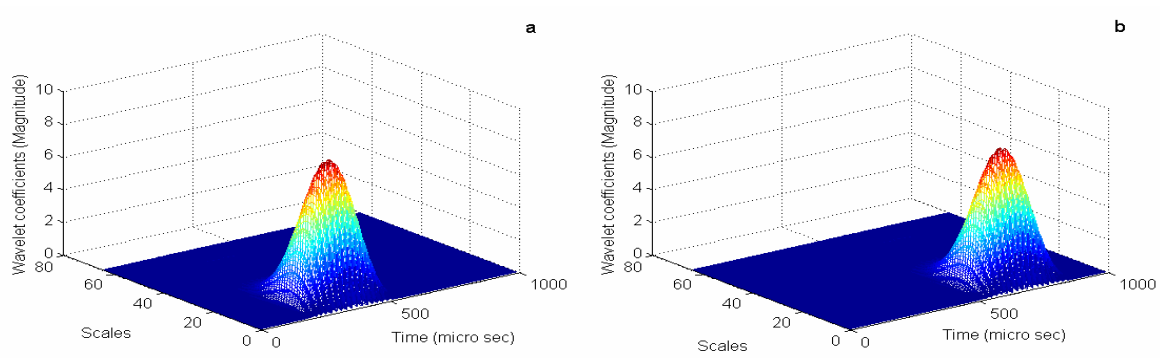

Figure 6. Gauss1 wavelet coefficients of displacements: for the point M_1 (x = 50 mm) (a) and for the point M_2 (x = 150 mm) at 1.5 MHzmm forA0 Lamb mode in a steel plate

Figures 7a and 7b present the "coefficient lines" in the case of Gauss1 mother wavelet for the scale a = 11 corresponding to the peak value of wavelet coefficients. Figure 7c and 7d show these "coefficient lines" in the case of Shan1-1.5 mother wavelet for the scale a = 59.

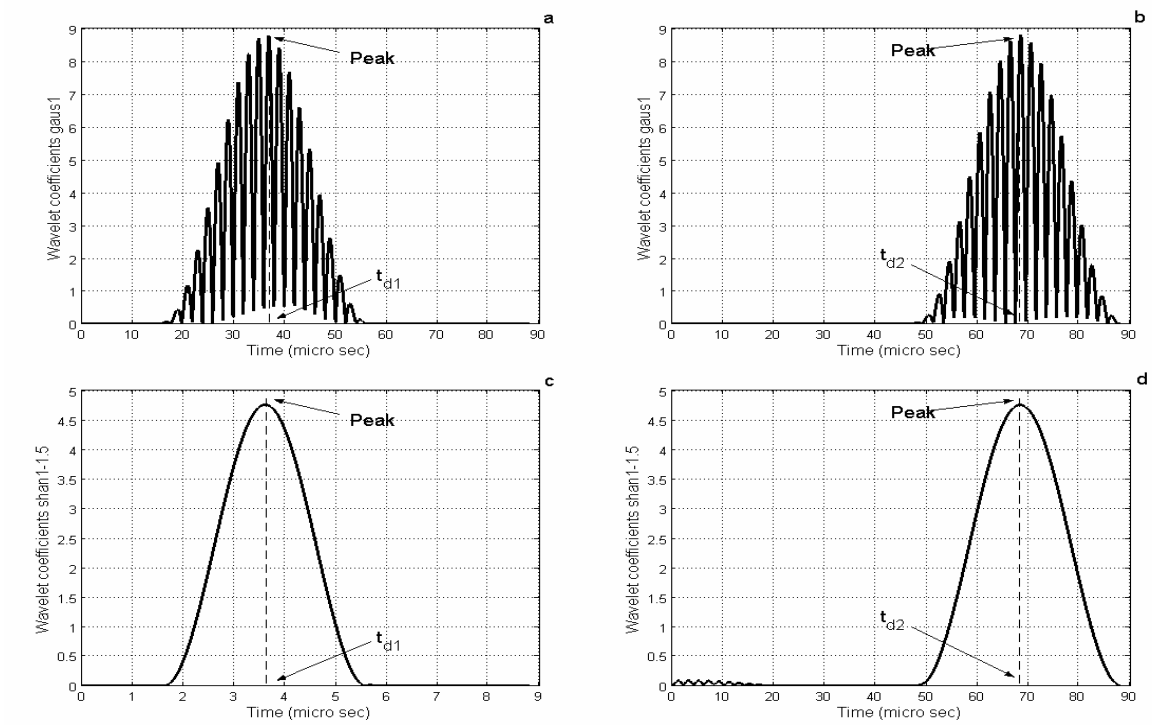

Figure 7. "Coefficient lines" for the A0 mode in a steel plate at 1.5 MHzmm. Gauss1 wavelet: at M_1 (x=50 mm) (a) and M_2 (x=150 mm) (b) Shan1.1.5 wavelet: at M_1 (x=50 mm) (c) and M_2 (x=150 mm) (d)

From peak values of the wavelet coefficients (figures 7a and 7b (7c and 7d)), we compute the arrival times t_{d1} and t_{d2} corresponding to the points M_1 and M_2 for the Gauss1 mother wavelet (Shan1.1.5 mother wavelet). The corresponding group velocities $v_g(f)$ are deduced according to equation (7).

The table 1 presents the obtained results for the various used mother wavelets. The relative errors on velocities are calculated in comparison to the exact velocities deduced from dispersion curves. The analysis of these results shows clearly that the Shannon (Shan1-1.5) complex mother wavelet gives the best values of the group velocity. This is certainly due to the fact that the mother wavelet Shan 1-1.5 is complex and it presents a maximum of resemblance to the Lamb signals.

Figure 8 presents, for the Lamb modes S0 and A0, a superimposition of dispersion curves deduced from the Shannon wavelet analysis to analytic dispersion curves. The agreement between these curves is very good.

Table 1. Group velocity values for the S0 and A0 Lamb modes, computed for various mother wavelets at 1 MHz.mm

	Mode S0				**Mode A0**			
Wavelet	t_{d1} **(µs)**	t_{d2} **(µs)**	**Group velocity** v_g (m/s)	**Relative error**	t_{d1} **(µs)**	t_{d2} **(µs)**	**Group velocity** v_g (m/s)	**Relative error**
Morlet	41,25	60,3	5249,34	3,64%	45,8	77,1	3194,89	2,67%
Gauss1	**39,75**	**58,95**	**5208,33**	2,83%	**47,3**	**78,6**	3194,89	2,67%
Gauss2	41,25	60,45	5208,33	2,83%	45,8	77,1	3194,89	2,67%
Gauss3	39,75	58,95	5208,33	2,83%	47,3	78,6	3194,89	2,67%
Gauss4	41,25	60,45	5208,33	2,83%	45,8	77,1	3194,89	2,67%
Meyr	43,5	62,7	5208,33	2,83%	48,2	79,5	3194,89	2,67%
dmey	41,25	60,45	5208,33	2,83%	45,9	77,2	3194,89	2,67%
mexh	41,25	60,45	5208,33	2,83%	45,8	77,1	3194,89	2,67%
shan1-1.5	**39,75**	**59,7**	**5012,53**	-0,29%	**46,4**	**78,5**	3115,26	0,11%
shan1-1	41,1	60,3	5208,33	2,83%	45,8	78,6	3048,78	-2,03%
shan1-0.1	39,3	58,95	5089,06	0,47%	47,2	78,6	3184,71	2,34%
shan2-3	39,75	60,3	4866,18	-3,93%	47,2	78,6	3184,71	2,34%
shan1-0.5	39,45	59,4	5012,53	-1,04%	45,9	78,6	3058,10	-1,73%

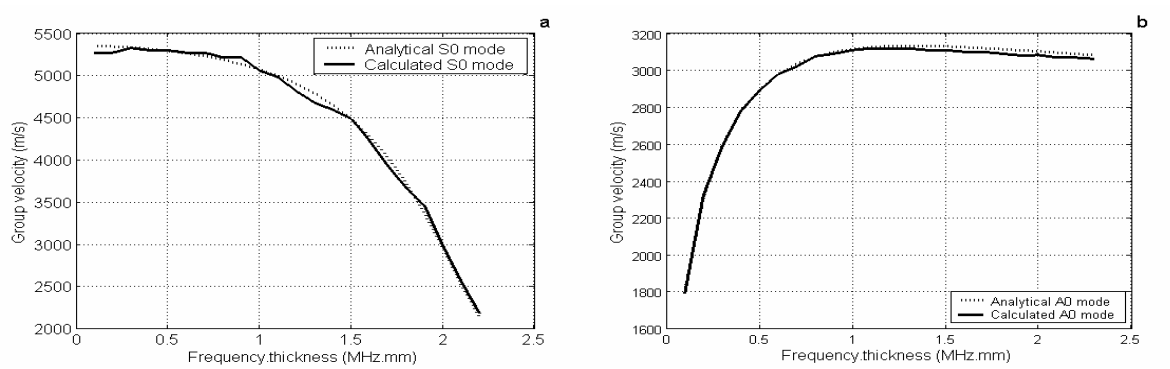

Figure 8. Dispersion curves: analytical (…) and calculated by shan1-1.5 WT (__) for S0 (a) and A0 (b) Lamb mode

CONCLUSION

We proposed in this paper a WT spectral analysis of Lamb modes in isotropic plates. The displacements predicted by FE modelling were processed by the WT. We determined by the WT the dispersion curves of S0 and A0 Lamb modes in steel plate without defects. Several mother wavelets were tested and we showed that the complex Shannon Shan1-1.5 mother wavelet gave the best values of group velocities. The presented application demonstrates the robustness of the WT processing and shows that the complex Shannon Shan1-1.5 mother wavelet is the most suitable for the WT analysis of Lamb modes signals. As a second application of the proposed WT processing method, we are currently working on the propagation of Lamb modes A0 and S0 in a plate with an internal defect. The complex mother wavelet Shan 1-1.5 is used for the post processing of the FEM predicted displacement field in order to compute the power coefficients of the reflected and the transmitted Lamb modes by the defect.

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