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On arising of turbulence in viscous heat-conducting gas

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A mechanism of turbulence arising in viscous heat-conducting gas is considered. It is found, that linear Hooke's Law connecting the change of pressure and relative volume deformation is broken. The small acoustic disturbances of density, which take place in a viscous heat-conducting gas flow, produce nonproportional pulsation of pressure, thus turbulence appears. The results of computing and experiments are presented.

Keywords: turbulence, viscous heat-conducting gas.

INTRODUCTION

The problem of arising turbulence has occupied the minds of engineers and scientists since the end of the XIX-th century, but no result is obtained yet. V.V. Struminsky, a well-known researcher of turbulence, writes down as follows [1]: «Other important researches also started by Reynolds are connected with flow stability problems such as the problem of arising turbulence. The conclusions of linear theory of stability correspond only qualitatively to experiments conducted by Schubauer and Skramstad. Reynolds' critical numbers calculations by linear theory result in values which are two orders different from experimental ones. Initiated research on the application nonlinear theory are far from completion». It is stated in works of Lecont, Tindal, Raleigh, Meyer and other [2, 3] that sound influences the flame behavior in gas burners and jets. In works [4, 5, 6] and other influence of acoustic disturbance on arising turbulence in a boundary layer is stated, though influence mechanism is unclear.

How do small acoustic disturbances lead to the arising of hydrodynamic Tollmien-Schlichting waves with velocities of propagation much less the speed of sound? Let us consider these facts from the point of breaking linear Hooke's Law connecting change of pressure with relative volume deformation as it is established by the author [7].

THEORETICAL PART

One of the primary physical properties of liquids and gases is their compressibility. It is defined as the ability of a substance to change its volume under uniform pressure. It is traditionally considered that compressibility of gas is precisely described by linear approximation. According to it, change of pressure is connected with relative volume deformation as described by Hooke's Law

$$dp = -E\frac{dV}{V} = -E\frac{d\upsilon}{\upsilon} = E\frac{d\rho}{\rho}, \qquad (1)$$

where p — is pressure, E — is module of gas volume elasticity, V, υ, ρ — are volume, specific volume and density of gas, respectively. Volume elasticity module presents by itself a proportion coefficient.

In addition to gas volume elasticity module, compressibility coefficient and sound velocity are used for characteristic of compressibility. They are connected as follows:

$$\beta = \frac{1}{E}, \; a^2 = \frac{E}{\rho}, \qquad (2)$$

where: β — is compressibility coefficient, a — is sound velocity, ρ — is density.

Linear dependence between change of pressure and change of volume is true with constant gas volume elasticity module. Elasticity module of gases in rest depends on its pressure. And if this parameter is constant it is constant value as well. So linear Hooke's Law for gases in rest is describing accurately the connection between change of pressure and change of volume.

But for moving flow of viscous heat-conducting gas with transverse shift, this linear dependence between change of pressure and change of volume is broken because elasticity module depends on processes active in this medium (flow speed, frequency and disturbance intensity, gradients of speed and temperature and others). Let us show it now.

In work [8] the formula for sound velocity in a viscous gas flow is obtained with energy dissipation and heat exchange taken into account:

$$a^2 = a_s^2 + \frac{\boldsymbol{V}\cdot\left(a_s^2 grad\rho - gradp\right) + (k-1)\Phi}{\frac{\partial\rho}{\partial t}}, \qquad (3)$$

where a_s — is adiabatic and isentropic value of sound velocity; p, ρ — are gas pressure and density; Φ — is a function which includes energy dissipation and heat exchange:

$$\Phi = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) +$$
$$+\mu\left\{\begin{array}{l} 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \\ + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 \end{array}\right\};$$

T — is gas temperature; $\boldsymbol{V}$ — is gas velocity vector with u, v, w projections along Cartesian coordinates system of x, y, z respectively; λ — is heat-conducting coefficient; μ — is coefficient of dynamic viscosity; t — is time; k — is adiabatic index.

Let's find volume viscosity module taking into account the obtained expression for sound velocity (3):

$$E = \rho a^2 = \rho a_s^2 + \rho \frac{\boldsymbol{V} \cdot \left(a_s^2 grad\rho - gradp\right) + (k-1)\Phi}{\frac{\partial \rho}{\partial t}}. \tag{4}$$

The analysis of expression (4) shows that volume elasticity module can be presented as

$$E = Es + En, \tag{5}$$

where: $Es = \rho a_s^2$ — means adiabatic module of volume elasticity, $En = \rho \frac{\boldsymbol{V} \cdot \left(a_s^2 grad\rho - gradp\right) + (k-1)\Phi}{\frac{\partial \rho}{\partial t}}$ — is nonlinear addition of volume elasticity module. It is conditioned by energy dissipation and heat exchange in a viscous heat-conducting gas flow.

It is coming from expression (5) that volume elasticity module with nonlinear addition is a local function of flow parameters. It is changing from point to point in a flow. Qualitative analysis of dependence between nonlinear addition to module of volume elasticity and various factors shows as follows:

$$En = f\left(\lambda, \mu, \frac{\Delta T}{h^2}, \frac{u^2}{h^2}, \frac{1}{\rho_{amp}\omega}\right), \tag{6}$$

where h — is characteristic dimension, ρ_{amp} — is density disturbance amplitude, ω — is circular frequency of density disturbance.

It means that nonlinear addition is directly proportional to a square of flow speed and inversely proportional to amplitude and frequency of density disturbance.

If we take into account (5), then we find, according to (1), character changes of pressure with changes of density

$$dp = Es\frac{d\rho}{\rho} + En\frac{d\rho}{\rho}. \tag{7}$$

It comes out from expression (7) that linear Hooke's Law of change of pressure depending on density changes is broken in a flow of viscous gas with transverse shift. Because of the fact that nonlinear addition to volume elasticity module is a local function of flow parameters, small disturbances of density will result in nonproportional changes of pressure. And it brings forth turbulent pulsations.

RESULTS OF COMPUTING AND NATURAL EXPERIMENTS

A typical law of change of volume elasticity module of viscous heat-conducting gas in a fixed time moment along flat canal close to the wall is given in fig. 1. Volume elasticity module $E_{i,59}$ is given here. Energy dissipation and heat exchange are taken into account. It is defined with the help of computing model [7]. Also is given adiabatic module of volume elasticity $Es_{i,59}$ which is computed by formula $Es = \rho a_s^2$. Adiabatic module of volume elasticity is practically constant. It equals to $1.414 \cdot 10^5$ Pa. Volume elasticity module $E_{i,59}$, with dissipation energy and heat exchange taken into account, suffers from interruptions. Jumps occur relatively to adiabatic volume elasticity module.

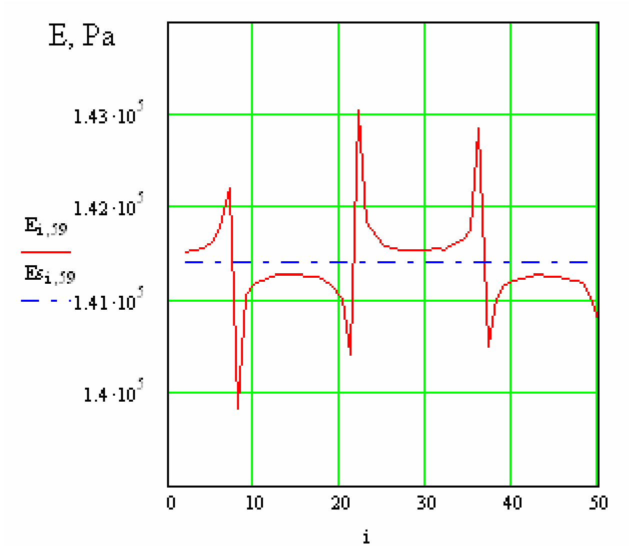

Fig. 1. $E_{i,59}$ — is volume elasticity module of viscous heat-conducting gas with energy dissipation and heat exchange, Pa; $Es_{i,59}$ — is adiabatic module of volume elasticity, Pa; i — is a node number of end-difference grid along x – axis

The results obtained during the computing experiment correspond to the derived formula (4) by character of change in volume elasticity module (fig. 1).

Rather intensive low frequency disturbances were observed in all experimental researches of «natural» transition [6].

It is underlined in work [6]: «These low frequency pulsations are sharply intensified in the region arising of turbulent spots. It points out to their significant role in transition process, … The reason of their arising is far unclear». It is concluded from the expression (6) that nonlinear addition to the volume elasticity module is inversely proportional to the frequency disturbances of density. So according to (7), low frequency acoustic disturbances will generate nonproportional pressure pulsations resulting in turbulence. To testify this position an experiment was carried out to arising turbulence in an air jet under influence of sound. The scheme of experiment is given in fig. 2. The method to carry out the experiment is developed in work [3].

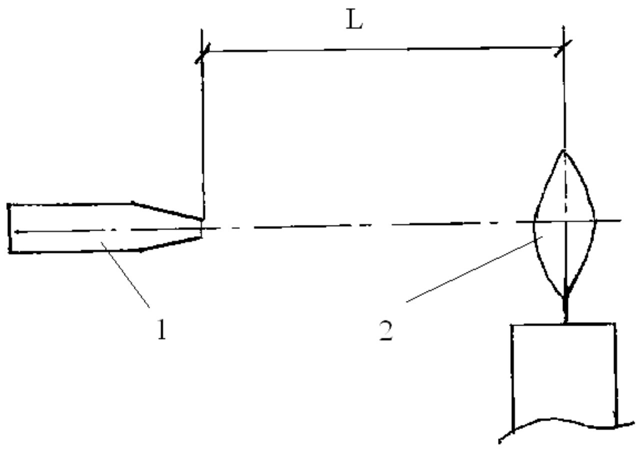

Fig. 2. Scheme of experiment of arising turbulence in an air jet under sound influence:
1 — is a nozzle of the air jet with output inner diameter at 1.5 mm; 2 — is flame of burning candle; L — is length of laminar portion of the air jet

The candle flame is used as indicator which provides an opportunity to observe visually arising turbulence. The output velocity of the air jet at the nozzle is taken as insignificant in order to get initial laminar portion of jet into candle flame region. The length of laminar portion of the jet is taken out to be L=60…90 mm. Creation of sound results in arising turbulence in the initial laminar portion of the jet which is observed visually by candle flame. To produce sound a signal generator was used as well as low frequency dynamic loudspeaker with effective operating frequency range of 70…1400 Hz. The sound pressure level was measured. The sound was perpendicular emitted to direction of jet motion and the sound pressure level was measured in the same plane. Frequency range where turbulence is arisen was analyzed. The results of measurement are presented in fig. 3. In frequency range of 70…1000 Hz creation of sound field results in arising turbulence accompanied by chaotic oscillation of candle flame. The sound frequency increased, the jet gets to be insensitive to acoustic oscillation, thus confirming dependences (6) and (7) obtained. When frequency of sound waves is increasing nonlinear addition to volume elasticity module is reduced and pressure pulsation amplitude in the jet is decreased which generate arising turbulence.

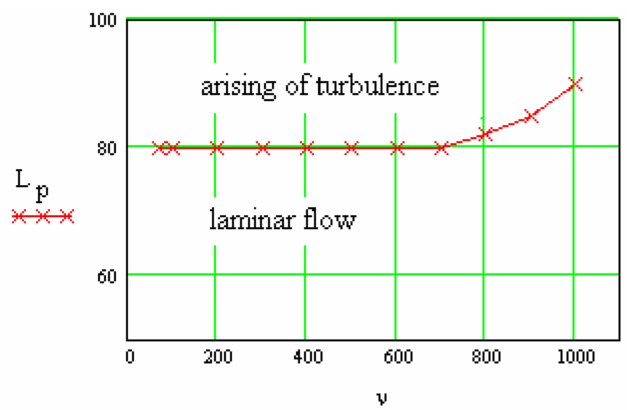

Fig. 3. Arising of turbulence in an air jet at the acoustic action:
L_p — is sound pressure level, dB; ν — is sound frequency, Hz

CONCLUSION

- It is shown that linear Hooke's Law connecting change of pressure with relative volume deformation is broken in a viscous heat-conducting gas flow. Small acoustic density disturbances arisen in viscous heat-conducting gas flow cause nonproportional pressure pulsation which brings out turbulence.
- It is stated experimentally that acoustic disturbances of low frequency of 70…1000 Hz generate arising of turbulence in an air jet.
- Analytical dependences obtained for volume elasticity module of viscous heat-conducting gas and specified to nonlinear Hooke's Law connecting change of pressure with density change are confirmed by computing and experiments.

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