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On the Fresnel zones of a circular transducer

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In classic acoustics one differs two areas around the acoustic source – near field and far field. There is a third area, called very near field, but it is mostly the subject of researches in the field of vibration mechanics. The near and the far field are often called Fresnel zones and Fraunhofer zone. In this work a criterion for determining the borders of Fresnel zones is proposed. The criterion is based on multiple examinations, researches and measurements of the spatial parameters and characteristics and some of the electrical parameters and characteristics of circular transducers (loudspeakers).

Keywords: near field, far field, Fresnel zones, Rayleigh integral.

INTRODUCTION

In order to gain deeper understanding of the nature of the sound field created by a transducer it is necessary to examine its spatial characteristics. Sound pressure p_a is the physical quantity (parameter) that gives detailed information about the sound field. Hence, in order to understand the physics of the sound field it is necessary to calculate the sound pressure level (SPL), created by transducers with different geometry (rectangular, circular, elliptical etc.), for each point of space.

In classic acoustics one differs two areas around the acoustic source – near field and far field. There is a third area, called very near field, but it is mostly the subject of researches in the field of vibration mechanics. In many references near field is called Fresnel zones and far field is called Fraunhofer zone. For each zone there are different theoretical formulas for determining the sound pressure p_a.

In this work a criterion for determining the borders of Fresnel zones is proposed. The criterion is based on a multitude of examinations, researches and measurements of the spatial parameters and characteristics [1, 2, 3] and some of the electrical parameters and characteristics [4] of circular transducers (loudspeakers). The criterion uses a normalized difference between two formulas – one for calculating the on-axis total acoustic sound pressure p_{aTotal} and the other for calculating the on-axis sound pressure in the far field p_{aFar}.

1. THEORETICAL BACKGROUND

One can calculate the acoustic pressure p_a created by the circular transducer by Huyghens-Fresnel dependence, known in acoustics as Rayleigh integral [5, 6, 7]:

$$p_a(\theta, f, r, t) = j\rho_s f v_m \int_Q \frac{1}{y} e^{j(\omega t - ky)} dQ, \tag{1}$$

where θ is directivity angle (elevation); f – transducer frequency; r – distance between the transducer and the observation point (where the sound pressure is measured); ρ_s – density of the environment; v_m – piston amplitude; Q – surface of the transducer; y – distance between elementary section (point source) and observation point; ω – angular frequency; $k = 2\pi / \lambda$ – wave number; λ – wavelength.

In [1] a modified expression for calculating the total acoustic pressure p_{aTotal} is proposed:

$$p_{aTotal}(\theta, f, r) = \rho_s f v_m e^{j\omega t} \int_0^a x dx \int_0^{2\pi} \frac{e^{-j\frac{2\pi f}{c}\left(\sqrt{r^2 + x^2 + 2rx\sin\theta\cos\alpha}\right)}}{\sqrt{r^2 + x^2 + 2rx\sin\theta\cos\alpha}} d\alpha, \tag{2}$$

where a is a circular transducer radius; x – the radial distance between the elementary section (point source) and center of the circular transducer; c – speed of sound; α – directivity angle (azimuth).

For the far field, when the distance r is significantly larger than the diameter of the transducer d ($r >> d$) the acoustic pressure p_{aFar} is [6]:

$$p_{aFar}(\theta, f, r) = \frac{\rho_s f v_m}{r} e^{j\omega t} \int_0^a x dx \int_0^{2\pi} e^{-j\frac{2\pi f}{c}(r + x\sin\theta\cos\alpha)} d\alpha. \tag{3}$$

Eq. (3) is also known as Fraunhofer solution of the Rayleigh integral.

For the near field the acoustic pressure amplitude p_a on the axis of the transducer (Eqs. (1) and (2)) can be simplified [5, 6]:

$$p_a(0, f, r) = 2\rho_s f v_m \left| \sin\left\{ \frac{\pi f}{c} \left[\sqrt{r^2 + a^2} - r \right] \right\} \right|. \tag{4}$$

Eq. (4) is a kind of on-axis Fresnel solution of the Rayleigh integral.

Study of Eq. (4) reveals that the axial acoustic pressure extremes occur (because of the *sine* function) at [2, 5, 6]:

$$\frac{\pi f}{c} \left[\sqrt{r^2 + a^2} - r \right] = m\pi, \quad m = 0, 1, 2... \tag{5}$$

Furthermore, the solution of the above expression gives the values of r at the SPL extremes [5]:

$$r_m = \frac{a^2}{m\lambda} - \frac{m\lambda}{4}. \tag{6}$$

Subsequently, if one moves toward the sound source from a given distance r, the first SPL maximum will be encountered at:

$$r_1 = \frac{a^2}{\lambda} - \frac{\lambda}{4}. \tag{7}$$

The first minimum in the SPL will appear at:

$$r_2 = \frac{a^2}{2\lambda} - \frac{\lambda}{2}. \tag{8}$$

For distances $r < r_1$ the acoustic field in the immediate proximity of the transducer is complicated (because of the SPL minimums and maximums). These extremes are typical for the Fresnel zones because the axial sound pressure p_a displays strong interference effects.

For distances $r > r_1$ the character of the axial sound pressure p_a decreases monotonically, approaching an asymptotic dependence $1/r$ [7, 8] (Fraunhofer zone). The distance r_1 could be accepted as a reasonable upper border of Fresnel zones or as a border between Fresnel zones and Fraunhofer zone. This distance r_1 has physical meaning only if (by suitable manipulation of Eq. 7):

$$f_{\min} > \frac{c}{2a}. \tag{9}$$

From Eq. (9) it is obvious that at frequencies lower that $f_{\min}$ the radiation of the transducer will be similar to the radiation of a simple source (there will be no Fresnel zones).

It should be noted that different authors [9, 10] have proposed various expressions (similar to Eq. (7) which gives the last maximum, perceived by some as the upper border distance of the near field). In [11] the hybrid intensity method is used for determining the border between near and far field (Fresnel zones and Fraunhofer zones).

Those methods have some disadvantages. The first, used in [9, 10], are too rough and not so accurate. The method used in [11] is too complicated and much less practical.

2. SOLUTION OF THE PROBLEM

In this paper the author proposes a method for determining the border between the Fresnel zones and the Fraunhofer zone similar to those used for determining the spectral density of a rectangular pulse train or RC-circuit time constant. Based on the normalized difference between Eq. (2) and Eq. (3):

$$\Delta p_a(\theta, f, r) = \frac{p_{aTotal} - p_{aFar}}{p_{aFar}} \cdot 100, \tag{10}$$

one can define the upper frequency $f_{\max Fresnel}$ limit of the Fresnel zones (the moment when both of the expressions become similar) for a given loudspeaker (with a given radius a). The frequency range of the loudspeaker must be also taken into account.

For instance, the axial sound pressure in the far field p_{aFar} created by the loudspeaker with radius $a = 0.08\,m$ is shown in Fig. 1.

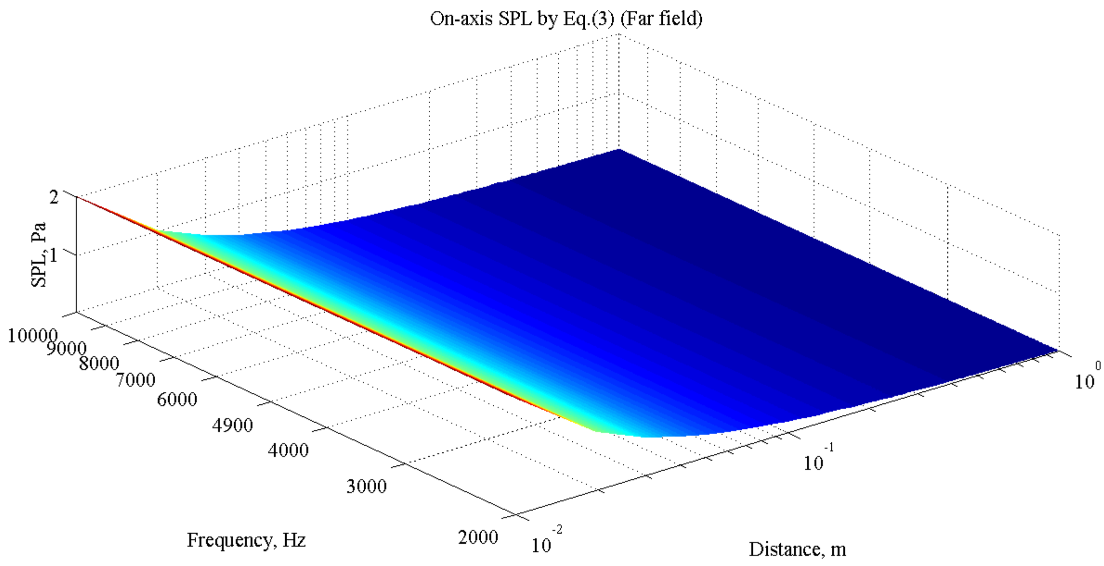

Figure 1. Axial SPL in the far field by Eq. (3)

The total axial sound pressure p_{aTotal} created by the same loudspeaker is shown in Fig. 2.

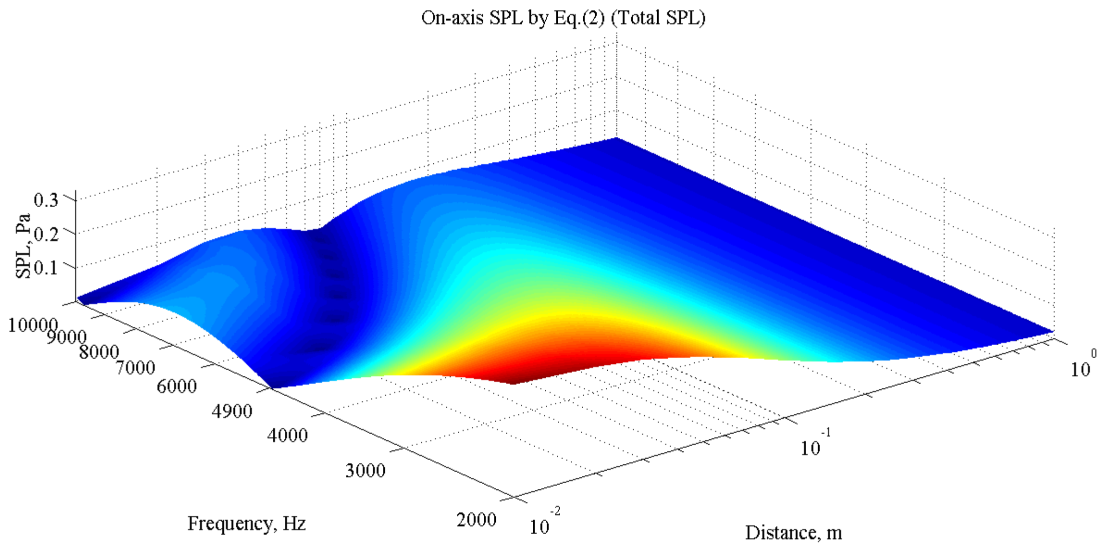

Figure 2. Total SPL by Eq. (2)

The normalized difference between the calculated SPL by Eq. (2) and the SPL calculated by Eq. (3) is shown in Fig. 3. The same correlation in percentage is presented in Fig. 4.

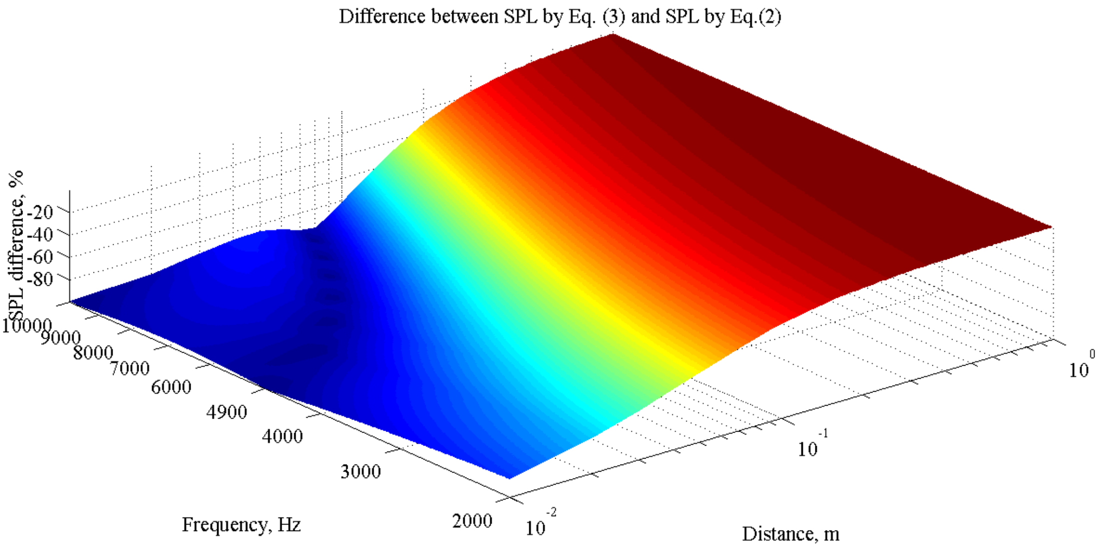

Figure 3. Normalized difference between the Total SPL and the SPL in the far field by Eq. (10)

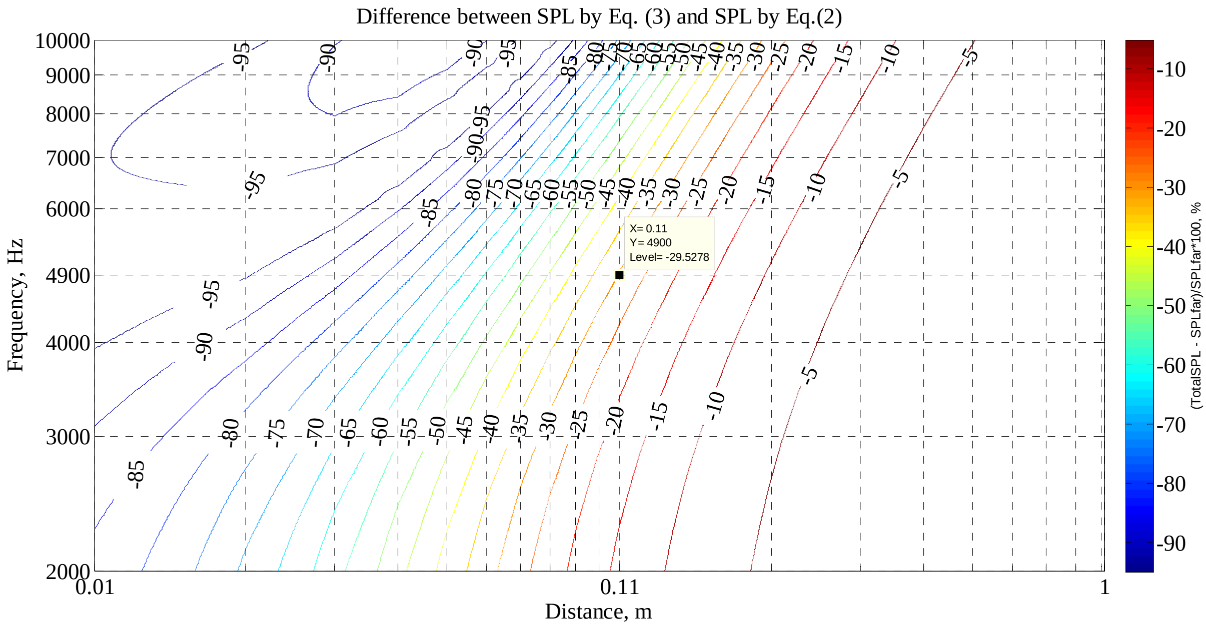

Figure 4. Normalized difference between the Total SPL and the SPL in the far field by Eq. (10) – in percentage

Fig. 4. reveals that the Fresnel zones will exists only for some frequencies f and some distances r. The author proposes the moment when $\Delta p_a < 30\%$ to be considered an upper limit border of the Fresnel zones (similar to half power point – 0.707).

3. EXPERIMENTAL RESULTS

The practical measurements were carried out with a DAQ system, a laptop, a sound-level meter, loudspeaker (radius $a = 0.08\,m$) and software product for SPL measuring. The measurements took place in an anechoic chamber at Technical University – Varna [12]. During the experiments the temperature and the intrinsic noise level in the anechoic chamber were measured with professional environment meter as follows: $t = 19.7^{\circ}\mathrm{C}$, intrinsic noise level – $26.8\,dB$.

The goal of the presented measurements is to show pattern and to gives a semiquantitative, intuitive understanding of the proposed criterion, not to claim for accuracy. More precise results must be obtained using numerical analysis techniques or finite element analysis.

For the loudspeaker at hand, one observes the Fresnel zones between frequencies $f_{\min Fresnel} > 2147\,Hz$ (by Eq. (9)) and its upper frequency range (in this case approximately $f_{\max} = f_{\max Fresnel} = 10000\,Hz$ [2]). Furthermore, for distances larger than $r > 0.065\,m$ for $f = 2200\,Hz$ and $r > 0.21\,m$ for $f = 10000\,Hz$ (Fig. 4) the sound pressure p_a drops monotonically, therefore there is a Fraunhofer zone.

Theoretical frequency response at distance $r = 0.012\,m$ is shown in Fig. 5. The plot is a slide of a 3D plot in Fig. 2. A minimum at frequency $f = 4900\,Hz$ for that distance $r = 0.012\,m$ is clearly noticeable.

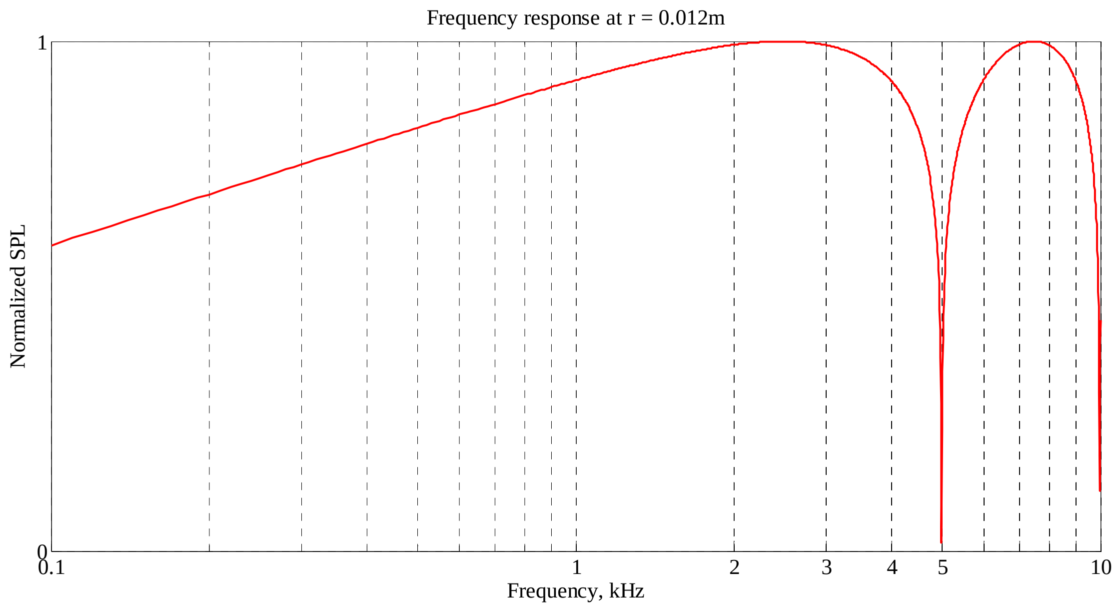

Figure 5. Theoretical frequency response at distance $r = 0.012\,m$

In Fig. 6 the measured loudspeaker's frequency response at distance $r = 0.012\,m$ is presented.

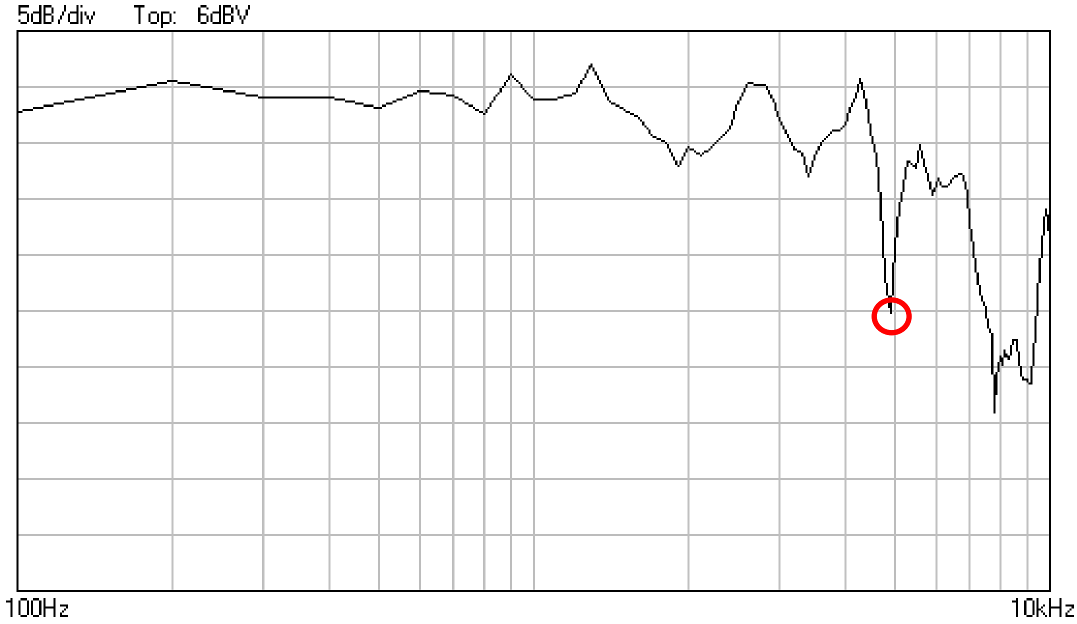

Figure 6. Measured frequency response at $r = 0.012\,m$

In Fig. 6 one can notice the sharp minimums and maximums in the frequency response for frequencies $f > 2147\,Hz$. Some of those extremes are caused by the interferences in the Fresnel zones (i.e. the sharp minimum at frequency $f = 4900\,Hz$ marked with red circle).

To express the asymptotic form of the SPL in the far field (in the Fraunhofer zone) the frequency response of a loudspeaker (radius $a = 0.08\,m$) at six different distances is measured (Fig. 7). The SPL for frequency $f = 4900\,Hz$ from all of the six measurements are visualized in Fig. 8.

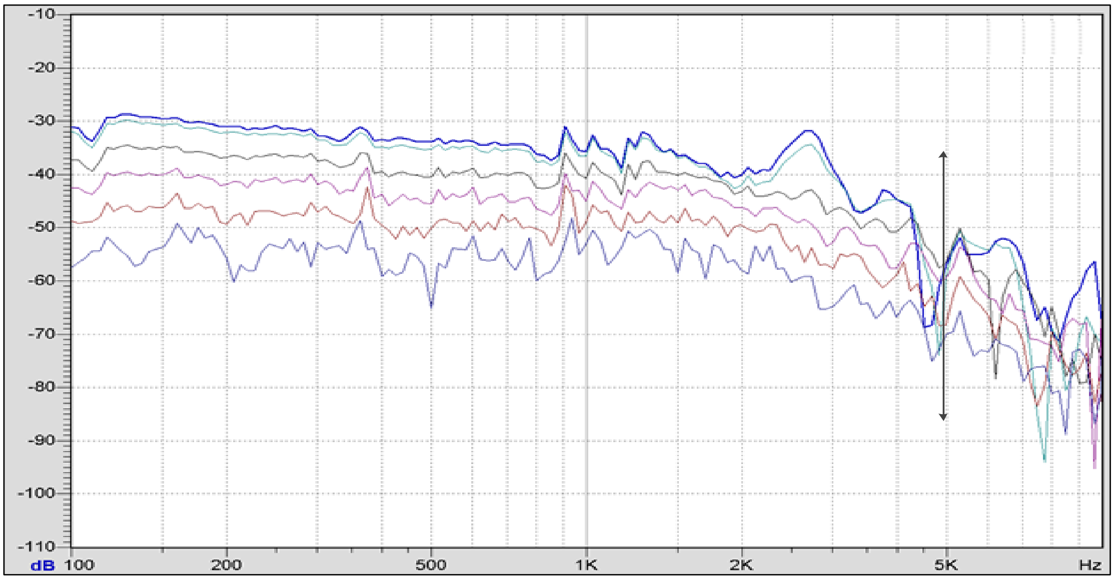

Figure 7. Measured frequency response at distances
$r = (0.002\,m, 0.012\,m, 0.06\,m, 0.12\,m, 0.24\,m, 0.48\,m)$

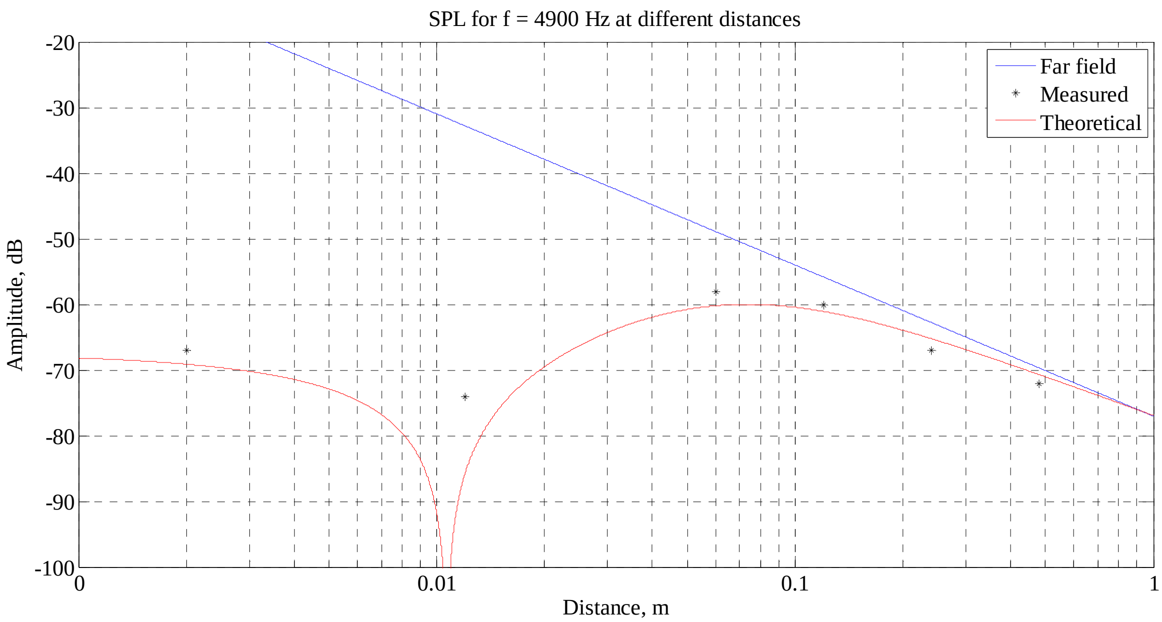

Figure 8. SPL for frequency $f = 4900\,Hz$, at distances $r = (0.002\,m, 0.012\,m, 0.06\,m, 0.12\,m, 0.24\,m, 0.48\,m)$

CONCLUSIONS

The experimentally obtained minimum in the SPL for frequency $f = 4900\,Hz$ (Fig. 6) is theoretically predicted by Fresnel solution (Fig. 5). For a given circular acoustic transducer the Fresnel zones appear for some frequencies f at some distances r.

In Fig. 8 one can notice the overlaps between the measured SPL and the theoretical SPL for frequency $f = 4900\,Hz$. For relatively large distances r the theoretical SPL and the experimental SPL decrease monotonically, approaching an asymptotic dependence $1/r$ (typical for Fraunhofer zone).

If one applies the proposed criterion for a loudspeaker with radius $a = 0.08\,m$, one will observe the Fresnel zones between frequencies $f_{\min Fresnel} > 2147\,Hz$ and the upper frequency range of the loudspeaker. After applying 30% difference between the SPL calculated by Eq. (2) and the SPL calculated by Eq. (3) one can acquire the distance r of the Fresnel zones (frequency dependent). Thus, for frequency $f = 4900\,Hz$ by means of the isobars from Fig. 4 the Fresnel zones exists only for distances $r < 0.11\,m$, for frequency $f = 2200\,Hz$ the Fresnel zones exists for distances $r < 0.065\,m$, etc.

The proposed criterion for determining the border between the Fresnel zones and Fraunhofer zone is simple, easy to use and gives relatively accurate results. If one knows the radius a and the frequency f of the transducer (loudspeaker) one can implement Eq. (10) and acquire graph similar to those in Fig. 4. By means of isobars one can choose specific conditions for stating the border between the Fresnel zones and the Fraunhofer zone (different from $\Delta p_a < 30\%$).

From the presented theoretical and experimental review of the Fresnel zones one can conclude:

- the bigger the transducer radius a, the longer the Fresnel zones;
- the higher the frequency f, the longer the Fresnel zones;
- the higher the wavelength λ, the shorter the Fresnel zones;
- the higher the sound speed c, the shorter the Fresnel zones.

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